Computing Density of States of One-Dimensional Photonic Crystal under P-Polarized Incident Wave

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***Abstract*— In this paper, density of states (DOS) profile of one-dimensional finite photonic crystal is analytically calculated under the incidence of p-polarized electromagnetic wave for $Al_xGa_{1-x}N/GaN$ material composition. For generalization, normalized wavelength range is considered, and structural parameters of periodic arrangement are varied to observe the effect on DOS function. Following Adachi's model, refractive indices of the materials are considered as function of mole composition and operating wavelength. The position of density peaks indicates the possible emission/detection wavelength for photonic crystal based optical emitter/detector. Fine wavelength tuning by changing layer dimensions or material compositions is characterized by blueshift and redshift. The analysis presented in this paper will be useful in designing micro-laser and optical memory devices.**

***Index Terms*— Photonic crystal, Density of states, Electromagnetic wave incidence, P-polarization, Structural parameters**

I. Introduction

One-dimensional photonic crystal is an arrangement of materials with spatial periodicity in dielectric constant along the direction of propagation of electromagnetic wave [1]. This structure exhibits photonic bandgap under certain conditions and that makes it efficient band pass/reject filter [2-3] for photonic integrated circuit applications [4]. Density of states function of a photonic crystal is very important for calculating emission and absorption properties of an atom or molecule [5]. This can tune the threshold of a laser or can modulate the capacity of photonic crystal based optical memory. Hence the accurate evaluation of density of states and its dependence on structural parameters are very important when the structure is subjected to polarized wave incidence.

Asatryan [6] calculated local density of states (LDOS) for two-dimensional photonic crystals composed of a finite cluster of circular cylinders. Local and spectral density of states functions of a two-dimensional photonic crystal is computed as a function of frequency by McPhendran [7] etc. and it is related with band structure. Kano [8] calculated DOS for anisotropic 3D photonic crystal using Maxwell's equation for thermally pumped terahertz emission. Dios-Leyva [9] investigated the dispersion relation of a N-period crystal and compared the result with that of an infinite one for finite and large values of unit cells. Thickness dependency and Q-values are also analytically characterized by Ohtaka [10] for complex dielectric constant. Rudziński [11] also showed the mode spectrum characteristics for 1D structure.

In this paper, dependency of density of states of 1D photonic crystal on structural parameters is analytically computed. Blueshift and redshift are shown for different dimensional variations as well as with change in material composition. Results are important for photonic crystal based optical emitter/detector applications.

II. Mathematical Modeling

We consider 1D photonic crystal where change in transverse propagation vector is written as

$$\delta k_t = \sqrt{\delta K^2 + \delta\beta^2} \tag{1}$$

where

$$\beta = \sqrt{k_y^2 + k_z^2} \tag{2}$$

Now variation in normal unit propagation vector k_n is

$$\delta k_n = \frac{\delta\omega}{|\nabla_k \omega|} \tag{3}$$

where

$$|\nabla_k \omega| = \sqrt{\left[\left(\frac{\partial\omega}{\partial\beta}\right)^2\Bigg|_\omega + \left(\frac{\partial\omega}{\partial k}\right)^2\Bigg|_\beta\right]} \tag{4}$$

Volume in k-space within the range ω_k is

$$\int_{\omega_k} \delta V_k = 2\pi\delta\omega \int_{\omega_k} \beta\left(\frac{\partial K}{\partial\omega}\right)\Bigg|_\beta \delta\beta \tag{5}$$

Considering dependence of propagation vectors k_{1x} and k_{2x} on refractive indices in different media as

$$k_{1x} = \frac{\omega}{c}\sqrt{n_1^2 - n^2} \tag{6.1}$$

$$k_{2x} = \frac{\omega}{c}\sqrt{n_2^2 - n^2} \tag{6.2}$$

We can write

Manuscript received March 14, 2014.
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$$\cos(K_B L) = \cos(k_{1x}d_1)\cos(k_{2x}d_2) - \frac{1}{2}\left(\frac{k_{1x}}{k_{2x}}\frac{n_2^2}{n_1^2} + \frac{k_{2x}}{k_{1x}}\frac{n_1^2}{n_2^2}\right) \times \sin(k_{1x}d_1)\sin(k_{2x}d_2) \tag{7}$$

which can be written as

$$K = \frac{1}{L}\cos^{-1}\left[F(\omega,\beta)\right] \tag{8}$$

Differentiating with relation to ω for constant β

$$\left(\frac{\delta K}{\delta \omega}\right)_\beta = \frac{\left(-\frac{1}{L}\right)\left(\frac{\partial F}{\partial \omega}\right)\Big|_\beta}{\sqrt{1-F^2(\omega,\beta)}} \tag{9}$$

Thus density of states

$$\rho(\omega) = \frac{dN}{d\omega} = \frac{V\omega^2}{4\pi^2c^2}\int_0^{n_1}\left[\frac{n\left(-\frac{1}{L}\right)\left(\frac{\partial F}{\partial \omega}\right)\Big|_\beta}{\sqrt{1-F^2(\omega,\beta)}}\right]dn \tag{10}$$

where $\rho_0(\omega) = 2V\omega^2 / 4\pi^2c^3$ is the ideal density of states function.

From Eq. (10), we can write

$$\left(-\frac{1}{L}\right)\left(\frac{\partial F}{\partial \omega}\right)\Big|_n = \frac{1}{c}\frac{d_1}{L}\sqrt{n_1^{\,2}-n^2} \times \left[\sin(k_{1x}d_1)\cos(k_{2x}d_2) + \frac{1}{2}\left(\frac{n_2^2k_{1x}}{n_1^2k_{2x}} + \frac{n_1^2k_{2x}}{n_2^2k_{1x}}\right)\cos(k_{1x}d_1)\sin(k_{2x}d_2)\right] + \frac{1}{c}\frac{d_2}{L}\sqrt{n_2^{\,2}-n^2} \times \left[\sin(k_{2x}d_2)\cos(k_{1x}d_1) + \frac{1}{2}\left(\frac{n_2^2k_{1x}}{n_1^2k_{2x}} + \frac{n_1^2k_{2x}}{n_2^2k_{1x}}\right)\cos(k_{2x}d_2)\sin(k_{1x}d_1)\right] \tag{11}$$

This can be written in the following form

$$\left(-\frac{1}{L}\right)\left(\frac{n}{\omega}\right)\left(\frac{\partial F}{\partial \omega}\right)\Big|_n = \frac{1}{c}\frac{d_1}{L}\frac{n^2}{\sqrt{n_1^{\,2}-n^2}} \times \left[\sin(k_{1x}d_1)\cos(k_{2x}d_2) + \frac{1}{2}\left(\frac{n_2^2k_{1x}}{n_1^2k_{2x}} + \frac{n_1^2k_{2x}}{n_2^2k_{1x}}\right)\cos(k_{1x}d_1)\sin(k_{2x}d_2)\right] + \frac{1}{c}\frac{d_2}{L}\frac{n^2}{n_2^{\,2}-n^2} \times \left[\sin(k_{2x}d_2)\cos(k_{1x}d_1) + \frac{1}{2}\left(\frac{n_2^2k_{1x}}{n_1^2k_{2x}} + \frac{n_1^2k_{2x}}{n_2^2k_{1x}}\right)\cos(k_{2x}d_2)\sin(k_{1x}d_1)\right] - \frac{1}{2L}\left(\frac{n}{\omega}\right)\frac{\partial}{\partial n}\left(\frac{n_2^2k_{1x}}{n_1^2k_{2x}} + \frac{n_1^2k_{2x}}{n_2^2k_{1x}}\right) \times \sin(k_{1x}d_1)\sin(k_{2x}d_2) \tag{12}$$

This is the governing equation for calculation of density of states of 1D photonic crystal under p-polarization condition.

III. RESULTS & DISCUSSION

Using Eq. (12), for different structural parameters of the dielectric layers the density of states of one-dimensional photonic crystal is first computed and then plotted as a function of normalized wavelength. In Fig. 1, density of states is plotted for different thicknesses of GaN layer keeping $Al_xGa_{1-x}N$ layer thickness as constant. If dimension of the higher bandgap material ($Al_xGa_{1-x}N$) is kept large compared to the other layer, then density of states shifts towards lower wavelength on, decreasing the dimension of GaN layer i.e. with the decreasing magnitude of the peak value redshift is observed in the spectrum in considerable proportions, as shown in Fig. 1a

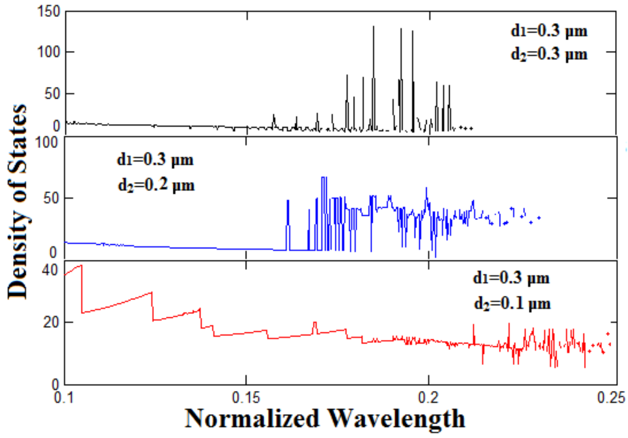

Figure 1a: Density of states with normalized wavelength for different thicknesses of GaN layer keeping dimension of $Al_xGa_{1-x}N$ constant and higher than GaN layer (0.3 µm)

Similarly if the thickness of $Al_xGa_{1-x}N$ layer is kept lower, then decreasing the thickness of GaN layer causes redshift again but this time with higher magnitude of shift in terms of wavelength, plotted in Fig. 1b.

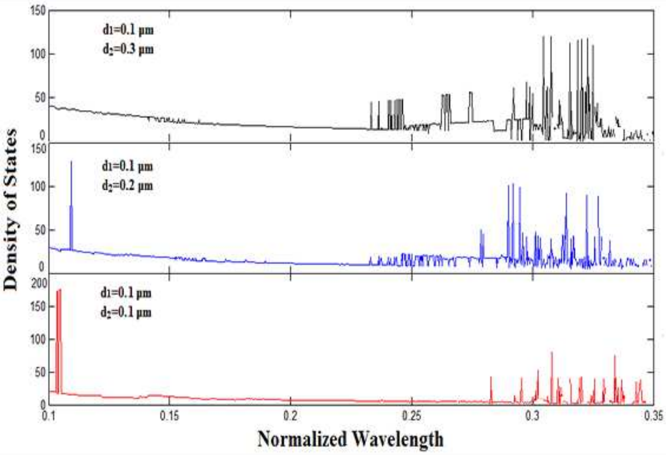

Figure 1b: Density of states with normalized wavelength for different thicknesses of GaN layer keeping dimension of AlxGa1-xN constant and lower than GaN layer (0.1 µm)

In Fig. 2, the density of states profile is plotted as function of normalized wavelength for different thicknesses of lower refractive index material. Keeping the dimension of GaN layer (higher refractive index material) higher, thickness of $Al_xGa_{1-x}N$ is varied to observe its effect on density of states. In Fig. 2a, blueshift is observed for p-polarized incident waves when difference in layer thicknesses decreases. We can observe that the magnitude of the shift is very close for the p-polarized incident wave as shown in Fig. 2a.

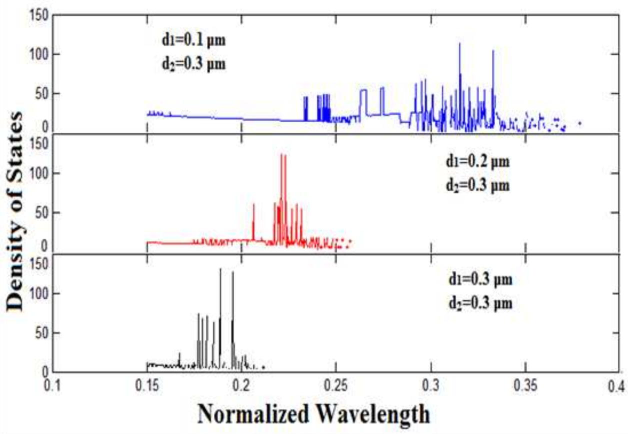

Figure 2a: Density of states with normalized wavelength for different thicknesses of AlxGa1-xN layer keeping dimension of GaN constant and higher than of AlxGa1-xN layer (0.3 µm)

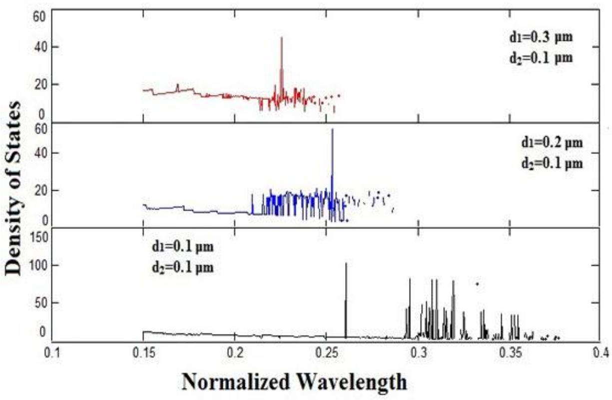

Figure 2b: Density of states with normalized wavelength for different thicknesses of AlxGa1-xN layer keeping dimension of GaN constant and lower than of AlxGa1-xN layer (0.1 µm)

In Fig. 2b, similar blueshift is obtained even when the dimension of GaN is kept lower. But the distinguishable feature is that with decrease of layer thickness differences for p-polarized wave under this condition, DOS almost vanishes.

Fig 3 shows the effect of material composition of higher bandgap material on optoelectronic properties of the photonic crystal. For three different percentage of AlN in $Al_xGa_{1-x}N$ material, density of states is plotted as a function of normalized wavelength. It may be seen from the plot that with increase of AlN percentage from 0.05 to 0.1, redshift is observed. But further increment form 0.1 to 0.15 shows blueshift.

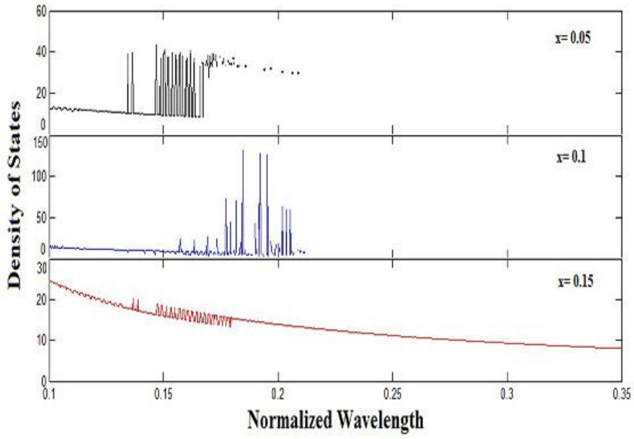

Figure 3: Density of states with normalized wavelength for different mole fraction (x) of Al keeping layer dimensions constant

IV. Conclusion

Structural parameters of one-dimensional photonic crystal can tune the peak positions of density of states profiles when plotted with relation to the wavelength. This causes the possible blueshift or redshift, important for designing optical emitter/detector. The analysis is useful for fabricating $Al_xGa_{1-x}N$/GaN composition based micro-laser, which can be embedded in photonic integrated circuit.

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