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Investigation of Array Steering Algorithms for Adaptive Array Antennas

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Abstract— The performance of smart antenna greatly relies on the efficient estimation of direction of arrival (DOA) of a signal. The accurate estimation of DOA is essential so as to steer the beam of antenna such that the main lobe occurs along the direction of desired user and nulls are formed towards the interferers. This paper addresses various classes of DOA algorithms i.e. conventional methods which are Bartlett, Minimum Variance Distortionless Response (MVDR) and subspace method i.e. Multiple Signal Classification (MUSIC) and a comparison is made among them from resolution perspective. The analysis of DOA Algorithms is carried out to evaluate the performance for accurate estimation of direction-of- arrival (DOA) of signals impinging on uniform linear array (ULA). These algorithms are analyzed and simulated for various input parameters like varying Signal-to-Noise Ratio (SNR) and angular separation between the sources for uniform linear array of 8 elements. The array elements considered are assumed to be isotropic in nature with an element interspacing of 0.5λ.

Index Terms — Adaptive Beam Forming, Direction of Arrival, Minimum Variance Distortionless Response, MUltiple SIgnal Classification, Resolution, Signal-to-Noise ratio, Uniform Linear Array.

I. Introduction

Smart antennas were designed for governmental use in military applications using directed beams to hide transmissions which was implemented by very large antenna structures and time-intensive processing. Use of smart antennas to reduce network interference caused by ever increasing simultaneous users helped in increasing total number of users that wireless network could handle in a given block of spectrum [1]. But it is extremely difficult to perform complex calculations in the stringent time space available in the personal wireless communications.

Adaptive array smart antenna involves the array signal processing to manipulate the signals induced on various antenna elements in such way that the main beam is directed towards the desired user and nulls are formed towards the interferers[2, 3, 4]. This is achieved by two in-built properties of the smart antenna namely, DOA and adaptive beamforming. The DOA algorithms namely Bartlett, MVDR and MUSIC are described in Constantine A. Balanis and Panayiotis I. Ioannides [5], Lal Chand Godara [6], Jeffrey Foutz, Andreas Spanias and Mahesh K. Banavar [7], Harry L. Van Trees [8], Monson H. Hayes [9], Samuel Silver [10], Frank Gross [11] and Godara.L.C [12]. These authors provide a comprehensive study of the use of an antenna array to enhance the efficiency of mobile communication systems and provides details on the feasibility of antenna arrays for mobile communication applications. [8, 11] gives a detailed introduction of DOA algorithms for linear array antenna. Certain DOA algorithms namely Bartlett, MVDR, Linear Prediction and MUSIC are compared in a simplified manner for a linear array antenna of 6 elements in [13]. A detailed comparison of DOA algorithms by considering various statistical parameters like probability of resolution and Root Mean square Error is given in [14].

This paper depicts a systematic comparison of DOA algorithms by considering various input parameters like varying SNR and angular separation between the sources. The Direction of Arrival (DOA) estimation methods namely Bartlett, MVDR and MUSIC with their pseudo spectrum equations are analyzed and simulated in MATLAB by considering resolution and varying SNR. It was observed that MVDR and MUSIC gives the best results among Classical and Sub-Space methods respectively. Analysis was continued for other conditions like resolution, different SNR values and presence of multiple sources for a 8-element linear array antenna. The array elements considered are assumed to be isotropic in nature with an element interspacing of 0.5λ.

The remainder of the paper is organized as follows: Section II describes the received data model of the array, section III highlights the DOA algorithms used viz. Bartlett, MVDR and MUSIC, section IV gives the simulation results of algorithms and also a comparative study is made among those algorithms. Section V gives the conclusion and future scope of this paper.

II. Received Data Model

Figure 1depicts a uniform planar array considered on the yz plane of the coordinate system. The array is centered on the origin. There are M elements along any row in the y direction and N elements along any column in the z direction. Let d_y and d_z be the inter element spacing along y axis and z axis respectively. In figure 1, K signals arrive from

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K uncorrelated sources in K directions. Each received signal x_k (t) includes additive white Gaussian noise. Under this model, the received signals can be expressed as a superposition of signals from all the sources and linearly added noise represented by [8,11].

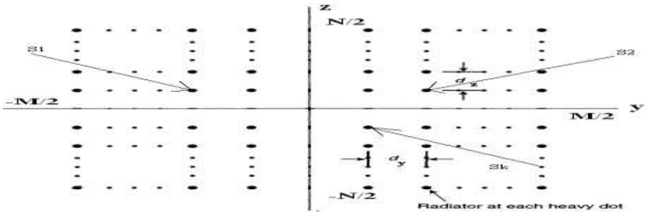

Fig. 1 Uniform planar array

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k, \Phi_k)\, s_k(t) + \mathbf{n}(t) \quad (1)$$

where $s_k(t)$ is the incoming plane wave from the k^{th} source at time t and arriving from the direction (θ_k, Φ_k) with θ_k is the elevation angle in the range $0^0 \leq \theta_k \leq 180^0$ and Φ_k is the azimuth angle, in the range $-90^0 \leq \Phi_k \leq 90^0$, $\mathbf{a}(\theta_k, \Phi_k)$ is the array steering vector for the (θ_k, Φ_k) direction of arrival, and $\mathbf{n}(t)$ represents additive white Gaussian noise. A single observation $\mathbf{x}(t)$ from the array is often referred to as a snapshot. Using a matrix notation (1) can also be written as [5, 11]

$$\mathbf{x}(t) = \mathbf{A}(\mathbf{\Theta})\, \mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

where $\mathbf{A}(\mathbf{\Theta})$ is the (M*N) x K matrix of array steering vectors. It is assumed that the arriving signals are uncorrelated and the number of arriving signals K < (M*N). The received signal covariance matrix of size (M*N) x (M*N) is given by [5, 8, 11],

$$R_{xx} = \mathbf{A}(\mathbf{\Theta})\, R_{ss}\, \mathbf{A}^H(\mathbf{\Theta}) + \sigma_n^2\, \mathbf{I} \quad (3)$$

where σ_n^2 is the noise variance and $\mathbf{I}$ is an identity matrix of size (M*N) x (M*N), R_{ss} is the source signal covariance matrix.

III. Direction Of Arrival Estimation Algorithms

The problem of localization of sources radiating energy by observing their signal received at the array antenna elements is of considerable importance occurring in many fields including radar, sonar, mobile communications, radio astronomy and seismology. In this chapter an estimation of the Direction Of Arrival (DOA) of narrowband sources of the same central frequency located in the far field of an array of antenna elements is considered and various DOA estimation methods are described. Data from an array of sensors are collected and the objective is to locate point sources assumed to be radiating energy that is detectable by the array elements.

Although most of the algorithms have been presented in the context of estimating a single angle per emitter (e.g. elevation only), generalizations to the elevation/azimuth case are relatively straight forward. Additional parameters such as frequency, polarization angle and range can also be incorporated provided that the response of the array is known as a function of these parameters. In general the DOA estimation algorithms can be categorized into two groups; the conventional algorithms and the subspace algorithms [5-11].

A. *Conventional DOA estimation methods*

Two methods are usually classified as conventional methods:

1) The Delay-and-sum method or Bartlett method.
2) Capon's Minimum Variance method or Minimum Variance Distortionless Response method

a) *Delay-and-sum method or Bartlett method*

One of the earliest methods of spectral analysis is the Bartlett method. The idea is to scan across the angular region of interest (usually in discrete steps), and whichever direction produces the largest output power is the estimate of the desired signal's direction. More specifically, as the look direction θ is varied incrementally across the space of access the array response vector $a(\theta)$ is calculated and the output power of the beamformer is measured by [5-11].

$$P_{DAS} = \frac{a^H(\theta) R_{xx} a(\theta)}{a^H(\theta) a(\theta)} \quad (4)$$

This quantity is also referred to as the spatial spectrum and the estimate of the true DOA is the angle θ that corresponds to the peak value of the output power spectrum.

b) *Minimum Variance Distortionless Response method*

The MVDR method is similar to the delay-and-sum technique described, in that it measures the power of the received signal in all possible directions. In this method, the output power is minimized with the constraint that the gain in the desired direction remains unity. Solving this constraint optimization problem for the weight vector we obtain [11]

$$W = \frac{R_{xx}^{-1} a(\theta)}{a^H(\theta) R_{xx}^{-1} a(\theta)} \quad (5)$$

which gives the MVDR spectrum [5-11]

$$P_{MVDR} = W^H R_{xx} W = \frac{1}{a^H(\theta) R_{xx}^{-1} a(\theta)} \quad (6)$$

Again, the estimate of the true direction of arrival is the angle θ that corresponds to the peak value in this spectrum. The MVDR only requires an additional matrix inversion compared to the delay-and-sum method and exhibits greater resolution in most cases.

B. *Subspace approach to DOA estimation*

The other main group of DOA estimation algorithms is called the subspace methods. Geometrically, the received signal vectors form the received signal vector space whose vector dimension is equal to the number of array elements N. The received signal space can be separated into two parts: the signal subspace and the noise subspace. The signal subspace is the subspace spanned by the columns of $A(\Theta)$ [11] and the subspace orthogonal to the signal subspace is known as the noise subspace. The subspace algorithms exploit this orthogonality to estimate the signals' DOAs.

a) The MUSIC algorithm

Within the class of the so-called signal subspace algorithms, MUSIC has been the most widely examined. MUSIC stands for Multiple SIgnal Classification. The MUSIC algorithm was developed by Schmidt by noting that the desired signal array response is orthogonal to the noise subspace. The signal and noise subspaces are first identified using Eigen decomposition of the received signal covariance matrix. Following, the MUSIC spatial spectrum is computed from which the DOAs are estimated. Inside the algorithm the general array manifold is defined to be the set

$$A = \{a(\theta_i) : \theta_i \in \Theta\} \quad (7)$$

The subspaces identified are typically achieved by Eigen decomposition of the autocovariance matrix of the received data R_{xx}. Using the model and assuming spatial whiteness, i.e., $\varepsilon\{n(t)n^H(t)\} = \sigma^2 I$, the Eigen decomposition of R_{xx} will give the Eigen values λn such that $\lambda_1 > \lambda_2 > \ldots > \lambda_K > \lambda_{K+1} = \lambda_{K+2} = \ldots = \lambda_N = \sigma_n^2$. Furthermore, it is easily shown that R_{xx} can be written in the following form [8, 11]

$$R_{xx} = \sum_{n=1}^{N} \lambda_n e_n e_n^H = E\Lambda E^H = E_s\Lambda_s E_s^H + E_n\Lambda_n E_n^H \quad (8)$$

$$R_{xx} = E_s\Lambda_s E_s^H + \sigma_n^2 E_n E_n^H = E_s\tilde{\Lambda}_s E_s^H + \sigma_n^2 I \quad (9)$$

where $E = [e_1, e_2, \ldots, e_N]$, $E_S = [e_1, e_2, \ldots, e_K]$, $E_N = [e_{K+1}, e_{k+2}, \ldots, e_N]$, $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_N\}$, $\Lambda_S = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_K\}$, $\Lambda_N = \text{diag}\{\lambda_{K+1}, \lambda_{K+2}, \ldots, \lambda_N\}$ and $\tilde{\Lambda}_S = \Lambda_S - \sigma_N^2 I$. The Eigen vectors **E** = [E_S , E_N] can be assumed to form an orthonormal basis (i.e., $EE^H = E^H E$ = **I**). The span of the K vectors E_S defines the signal subspace and the orthogonal complement spanned by E_N defines the noise subspace. For a detailed analysis of the Eigen structure properties of the signal autocovariance matrices R_{xx} and R_{SS} the reader is referred to [11]. Once the subspaces are determined the DOAs of the desired signals can be estimated by calculating the MUSIC spatial spectrum over the region of interest [5-11]

$$P_{MUSIC}(\theta) = \frac{a^H(\theta)a(\theta)}{a^H(\theta)E_n E_n^H a(\theta)} \quad (10)$$

Note that the $a(\theta)$ is the array response vectors calculated for all angles θ within the range of interest. Because the desired array response vectors **A(Θ)** are orthogonal to the noise subspace, the peaks in the MUSIC spatial spectrum represent the DOA estimates for the desired signals.

C. Flowchart to obtain varying SNR and resolution plots

The DOA is estimated for 8-element linear array antenna with Bartlett, MVDR and MUSIC algorithms. Pseudo-spectrum equation is used to obtain the plots. The flowchart of DOA for linear array antenna is given below.

Flowchart of DOA for linear array antenna

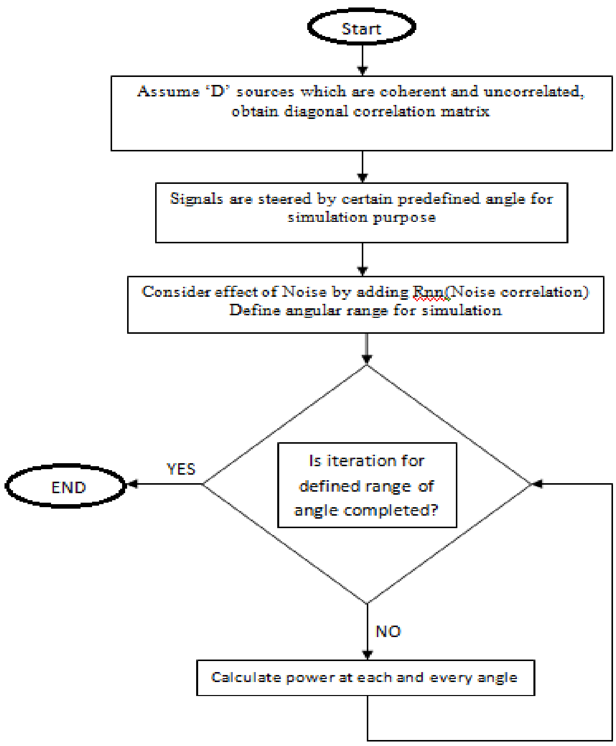

Fig. 2 Flowchart of DOA estimation for linear array antenna

Assume 'D' sources which are coherent and uncorrelated, obtain its diagonal correlation matrix. The signals are steered at specific predefined angle. Simulation with the help of pseudo spectrum equation of each method gives peak value at angle same as that of steered signal. This angle is the estimated DOA of the system.

IV. SIMULATION RESULTS OF DOA ALGORITHMS

The algorithms namely Bartlett, MVDR and MUSIC with their pseudospectrum equations are analyzed and simulated in MATLAB for 8-element linear array by varying input parameters. The array elements considered are assumed to be isotropic in nature with an element interspacing of 0.5λ. The input parameters considered are varying SNR and angular separation between the sources. The minimum angular separation that is allowed between the sources at which the algorithm is still able to distinguish them is a measure of performance of the algorithm. Thus, the resolution capability of each algorithm at a given value of SNR is determined and a comparison is brought among them.

A. Bartlett Method

The simulation of Bartlett algorithm for 8-element linear array is shown in figure 3 and figure 4. Figure 3 gives the varying SNR profile of Bartlett algorithm where the DOA angles considered are -30°, 20° and 60°. The SNR value is varied from 0dB to 30dB in steps of 10dB. It is observed that the algorithm imparts better performance at higher SNR values. Figure 4 gives the resolution profile of Bartlett algorithm for a given SNR of 30dB. It is found that the algorithm is able to resolve 14° angular separation between the sources thus the resolution capability of Bartlett algorithm at a given SNR of 30dB is 14°.

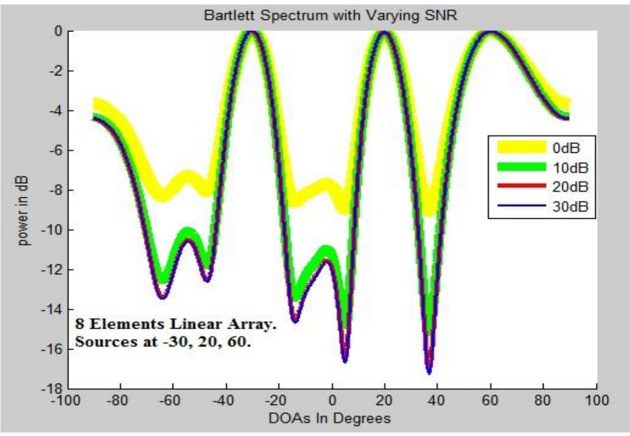

Fig.3 Varying SNR profile of Bartlett Algorithm for 8-element linear array antenna.

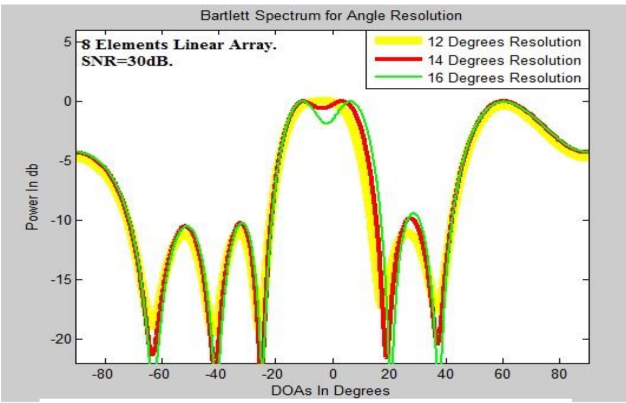

Fig.4 Resolution profile of Bartlett Algorithm for 8-element

B. MVDR Method

MVDR algorithm is simulated for 8-element linear array as shown in figure 5 and figure 6. In Figure 5, the algorithm is analyzed by giving different values of SNR (0dB, 10dB, 20dB and 30dB) for DOA angles -30°, 20° and 60°. It is observed that the SNR value is a measure of performance of the algorithm. Figure 6 gives the resolution profile of MVDR algorithm for a given SNR of 30dB. The angular separations considered between the sources are 2^0, 3^0 and 5^0. It is observed that the resolution capability of the algorithm is 3° at a SNR of 30dB.

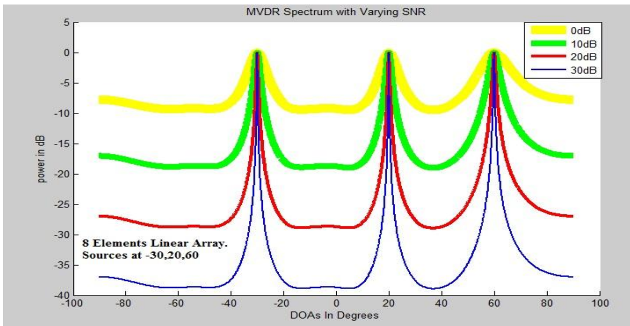

Fig.5 Varying SNR profile of MVDR Algorithm for 8-elementlinear array antenna

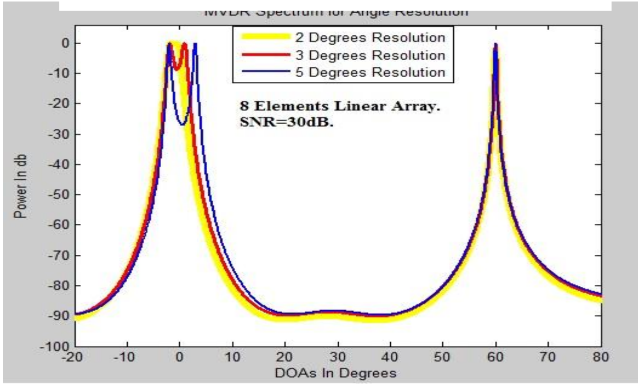

Fig.6 Resolution profile of Bartlett Algorithm for 8-element linear array antenna

C. MUSIC method

The methods discussed earlier i.e. Bartlett and MVDR are conventional methods and it can be said from the earlier obtained results that these methods have poor resolving capability. Then the analysis is continued by considering a subspace method i.e. MUSIC. Subspace methods exploit the orthogonality property between signal subspace and noise subspace thereby yielding better resolution. The simulation results obtained from MUSIC for 8-element linear array antenna are depicted in figure 7 and figure 8. Figure 7 gives the varying SNR profile of algorithm for DOA angles -30°, 20° and 60°. It is observed that the algorithm imparts better performance at higher SNR values. Figure 8 gives the resolution profile of MUSIC algorithm for a given SNR of 5dB. The angular separations considered between the sources are 2^0, 3^0 and 5^0. It is observed that the resolution capability of the MUSIC is 2° at a SNR of 30dB.

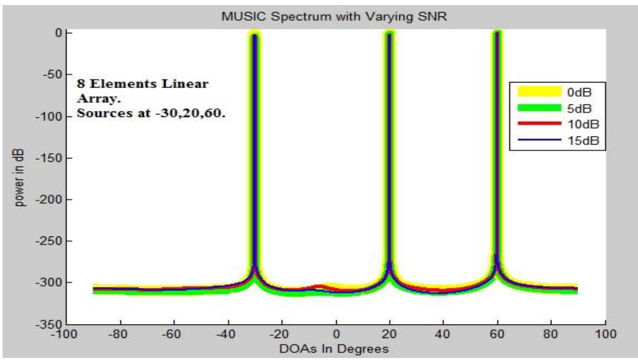

Fig.7 Varying SNR profile of MUSIC Algorithm for 8-elementlinear array antenna.

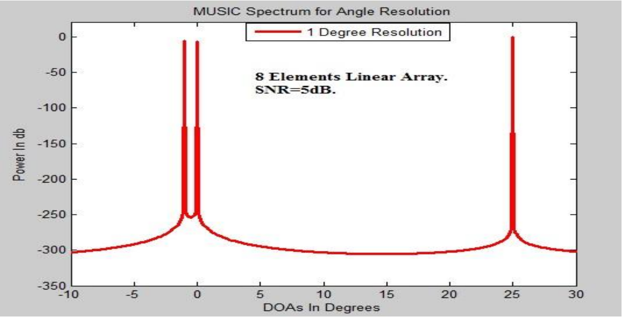

Fig.8 Resolution of MUSIC Algorithm for 8-elementlinear array antenna.

V. Tabulation of Resolution Capability of Algorithms

The resolution capability of each algorithm that is obtained earlier is tabulated as shown in Table 1. It is observed that Bartlett algorithm has a poor resolution of 14^0 at a given SNR of 30dB. In case of MVDR algorithm, at a SNR vale of 30dB the resolution achieved is 3^o. Therefore it can be inferred that MVDR algorithm has better performance among the various conventional methods of estimating DOA. MUSIC algorithm which is indeed a Subspace method has an excellent resolution of 2^0 at a SNR of 5dB. This is where Subspace methods exhibit better performance even at low SNR values when compared to that of conventional methods. So it can be concluded that MUSIC has better performance among all three algorithms that are considered under discussion.

Table 1: Comparison of DOA Algorithm Based on Resolution

Algorithms	SNR Value (in dB)	Resolution Capability (in degrees)
Bartlett	30	14
MVDR	30	3
MUSIC	5	2

VI. Conclusion

The objective of the paper is to simulate the different DOA algorithms viz. Bartlett, MVDR and MUSIC and study their performance based on varying SNR and angular resolution for 8-element linear array. The simulation results show that the performance of DOA algorithms improves with increasing SNR of signals. As far as conventional DOA methods are considered, MVDR algorithm is better as it exhibits resolution of 3^0 at a given SNR of 30dB. As the analysis was continued with subspace method i.e. MUSIC, the resolution achieved by this algorithm is 2^o at a given SNR of 5dB. This is where Subspace methods impart excellent performance even at lower values of SNR. The results show that MUSIC method is more accurate, stable and gives better resolution among all three methods that are considered under discussion.

REFERENCES

[1] Carl B. Dietrich, Jr., Warren L. Stutzman, Byung-Ki Kim, and Kai Dietze,"Smart Antennas in Wireless Communications: 13ase-Stat ion Diversity and Handset Beamforming", *IEEE Antennas and propagation magazine*, Vol. 42, Issue 5, 2000, Page(s): 142-151.

[2] S. A. Ibrahim., M. Y. Alias, N. N. Ahmad, "Joint adaptive modulation adaptive beamforming for mobile communication", *Proceedings of the 2007 IEEE International Conference on Telecommunications and Malaysia International Conference on Communications*, 14-17 May 2007, Penang, Malaysia, Page(s): 254-259.

[3] Shiyun Yu, ZhengQuan Gong, "Adaptive Digital Beamforming Techniques in Transmission Antenna Arrays", *Proceedings of fourth IEEE International Conference on Microwave and Millimeter Wave Technology*, 20054, Nanjing, China, Page(s): 207-210.

[4] Mohammad Ismat Kadir , Md. Shadiul Hoque and Saiful Islam," Direction of Arrival Algorithms for Adaptive Beamforming in Next Generation Wireless Systems", *Proceedings on 11th IEEE International conference on Computer and Information Technology(ICCIT)*, 2008, Khulna, Bangladesh, Page(s): 571-575.

[5] Constantine A. Balanis, Panayiotis I. Ioannides, "*Introduction to Smart Antennas*", Morgan & Claypool publishers, first edition, 2007, ISBN:1598291769.

[6] Lal Chand Godara, "*Smart Antennas*", CRC press publication, 2004, ISBN:084931206X.

[7] Jeffrey Foutz, Andreas Spanias, Mahesh K. Banavar, "*Narrowband Direction of Arrival Estimation for Antenna Arrays*", Morgan & Claypool publishers, 2008, ISBN:1598296507.

[8] Harry L. Van Trees, "*Optimum Array Processing-part 4 of Detection, Estimation and Modulation Theory*", Wiley-Interscience publication, 2002, ISBN:0471093904.

[9] Monson H. Hayes, "*Stastical Digital Signal Processing and Modeling*", John Wiley & Sons, 2009, ISBN:8126516100.

[10] Samuel Silver, "*Microwave Antenna Theory and Design*", McGraw Hill publications, first edition, 1949, ISBN:0863410170.

[11] Frank Gross, "*Smart Antennas for Wireless Communications with MATLAB*", McGraw-Hill publications, 2005, ISBN:007144789X.

[12] L.C. Godara, "Application of Antenna Arrays to Mobile Communications Part-I: Performance improvement, feasibility and system considerations", *proceedings of the IEEE, July 1997*, DOI:10.1109/5.611108, volume:85, issue:7, page(s):1031-1060.

[13] K. S. Shushrutha, Rohith Revindran and Suleman Nadaf, "Analysis of array steering algorithms for dielectric lens-array antennas", *International journal of Recent Advances in Engineering and Technology*", Vol. 2, Issue 2, 2014, Page(s): 17-24.

[14] Reaz, F. Haque and M. A. Matin, "A Comprehensive Analysis and Performance Evaluation of Different Direction of Arrival Estimation Algorithms", *Proceedings of IEEE symposium on Computer and Informatics*, 2012, Page(s): 256-259.

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