Field Theory of Metamaterials and Its Application in Antenna: A Review

Santanu Roy

***Abstract*— This paper demonstrates a review of electromagnetic field analysis of metamaterials. First it is verified that a double negative metamaterial (both permittivity and permeability negative) forms left handed triplet and consequently, in such a material, wave propagation vector is anti-parallel with Poynting vector. Furthermore, a purely left handed transmission line model has been verified and it is analytically shown that, in such a media, refractive index is negative. It is also investigated that an effective negative permittivity medium or single negative metamaterial can be analyzed by employing a simple equivalent transmission line model which enables us to formulate the wave equation for plasma medium. In the end, some of the applications of double negative and single negative metamaterials are also mentioned.**

Keywords—Double Negtaive (DNG) Metamaterial (MTM), Left Handed Material (LHM), Composte Right and Left Handed(CRLH) Transmission Line(TL).

I. INTRODUCTION

Double Negative (DNG) Metamaterials (MTM) have simultaneously negative permittivity and permeability. From the Maxwell's curl equation Veselago [1] first time (in 1968), explored that in a DNG MTM, electric field (**E**), magnetic field (**H**) and wave propagation vector (**k**) forms a left handed triplet. Thus such a material is also called as Left Handed Material (LHM). As a result the direction of wave propagation vector (**k**) in LHM is reversed with respect to Poynting vector (**S**) and refractive index in such a material is negative [1-4]. Although Veselago predicted the existence of the LHM earlier, experimental verification first time was done by D.R. Smith et.al [3], in 2001. His group at University of California San Diego (UCSD) designed LHM, comprised of repeated unit cells of Split Ring Resonators (SRRs) and thin copper strips, which exhibits negative refractive index. Since SRR structure is difficult to implement, several researchers, including Caloz and Itoh et.al, G.V. Eleftheriades et.al and A.A. Oliner simultaneously developed (in 2002) different non-resonant transmission line models of LHM [5-10] by which one can easily visualize the concept of left-handness and formulate the wave equation for LHM. The main advantage of using such non-resonant structure is that it will have lesser loss and broader

bandwidth [6] than conventional resonant structure. Later J.B.
Pendry revealed that a perfect lens [11] can be achieved by deploying LHM, which may focus entire spectrum of light onto an area smaller than the wavelength of light. Since the inception of negative refractive index transmission line, it has enabled to enhance the performance of antennas. Several scientists have employed metamaterials to achieve more directivity [13] in antenna, to miniaturize antenna structure and it can also be used in dual band [14, 15] antenna application.

Santanu Roy, Electronics Department, BITS Edu Campus, Vadodara, India, 9007690686 (e-mail: santanuroy35@gamil.com).

II. VERIFICATION OF LEFT HANDNESS IN DOUBLE NEGATIVE METAMATERIALS

In this section we investigated a material, having simultaneously negative ϵ and μ, forms left handed triplet instead of right handed triplet.

From Helmholtz's wave equation, we get

$$(\nabla^2 - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2})\psi = 0 \qquad (1.1)$$

where n is the refractive index and it can be represented by

$$n = \sqrt{\mu_r \epsilon_r} \qquad (1.2)$$

Where ϵ_r and μ_r are effective permittivity and effective permeability of the material. C is the velocity of light in vacuum and it can be represented by

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \qquad (1.3)$$

Substituting $\frac{n^2}{c^2} = \mu\epsilon$ and $\frac{\partial^2}{\partial t^2} \Leftrightarrow j^2\omega^2$ (in frequency domain), the wave equation (1.1) can also be written in the form of

$$(\nabla^2 + \omega^2\mu\epsilon)\,\psi = 0 \qquad (1.4)$$

Whereas wave propagation vector $k^2 = \omega^2\mu\epsilon$ (1.5)

In both of the equations (1.1) and (1.4), it can be observed that simultaneous change of negative ϵ and μ has no effect on those wave equations. In fact, from the equations (1.2) and (1.5), it can be observed that the refractive index and wave propagation vector both remain positive, in case of DNG metamaterial. The reason is that the material, having simultaneously negative ϵ and μ, may have some different properties from those conventional material. So it might contradict some of basic fundamental laws in nature which we do not consider. Thus we must explicitly consider those relations (Maxwell's curl equations) in which ϵ and μ appear separately. Maxwell's curl equation can be written in the form of (frequency domain)

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H} \qquad (1.6)$$

$$\nabla \text{ X } \boldsymbol{H} = j\omega\epsilon\boldsymbol{E} \tag{1.7}$$

The plane wave fields can be written in the form of

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{-jk.r} \tag{1.8}$$

$$\boldsymbol{H} = \frac{\boldsymbol{E}_0}{\eta} e^{-jk.r} \tag{1.9}$$

Lets assume

$$\boldsymbol{k} = k_x\boldsymbol{i} + k_y\boldsymbol{j} + k_z\boldsymbol{k} \tag{1.10}$$

$$and \qquad \boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} \tag{1.11}$$

Taking the dot product of two vectors $\boldsymbol{k}$ and $\boldsymbol{r}$, we get a scalar quantity.

$$\boldsymbol{k}.\boldsymbol{r} = k_x x + k_y y + k_z z \tag{1.12}$$

Therefore from equation (1.8) and (1.12) we get

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} \tag{1.13}$$

Similarly from equation (1.9) and (1.12) we get

$$\boldsymbol{H} = \frac{\boldsymbol{E}_0}{\eta} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} \tag{1.14}$$

Putting this value of $\boldsymbol{E}$ in Maxwell's equation (1.6), we get

$$\nabla \text{ X } \boldsymbol{E} = \boldsymbol{i}\left[\frac{\partial}{\partial y}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} - \frac{\partial}{\partial z}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}\right] + \boldsymbol{j}\left[\frac{\partial}{\partial z}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} - \frac{\partial}{\partial x}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}\right] + \boldsymbol{k}\left[\frac{\partial}{\partial x}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} - \frac{\partial}{\partial y}\boldsymbol{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}\right]$$

$$= \boldsymbol{i}\left[\boldsymbol{E}(-jk_y) - \boldsymbol{E}(-jk_z)\right] + \boldsymbol{j}\left[\boldsymbol{E}(-jk_z) - \boldsymbol{E}(-jk_x)\right] + \boldsymbol{k}\left[\boldsymbol{E}(-jk_x) - \boldsymbol{E}(-jk_y)\right] = (-j)[\boldsymbol{k} \text{ X } \boldsymbol{E}]$$

Therefore, we get

$$-j\omega\mu\boldsymbol{H} = (-j)[\boldsymbol{k} \text{ X } \boldsymbol{E}]$$

$$or, \boldsymbol{k} \text{ X } \boldsymbol{E} = \omega\mu\boldsymbol{H} \tag{1.15}$$

Similarly from Maxwell's equation (1.7) and equation (1.13) we get,

$$\boldsymbol{k} \text{ X } \boldsymbol{H} = -\omega\epsilon\boldsymbol{E} \tag{1.16}$$

If $\epsilon<0$ and $\mu < 0$, from the equations (1.15) and (1.16) we get

$$\boldsymbol{k} \text{ X } \boldsymbol{E} = -\omega|\mu|\boldsymbol{H} \tag{1.17}$$

$$\boldsymbol{k} \text{ X } \boldsymbol{H} = \omega|\epsilon|\boldsymbol{E} \tag{1.18}$$

From equations (1.17) and (1.18) it is observed that $\boldsymbol{E}, \boldsymbol{H}$ and $\boldsymbol{k}$ now forms a left-handed triplet which is shown in fig.1. That is why such material is called Left Handed Material (LHM).

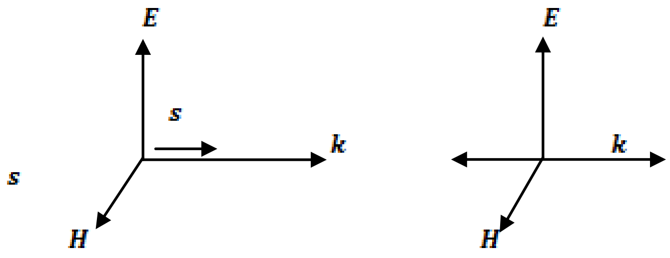

Fig. 1. A. medium with positive ϵ and μ forms right handed triplet

B. medium with negative ϵ and μ forms left handed triplet

$\boldsymbol{S}$ is known as Poynting vector or time averaged flux of energy which can be expressed by

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{E} \text{ X } \boldsymbol{H} \tag{1.19}$$

From equation (1.19), it is observed that Poynting vector $\boldsymbol{S}$ is unaffected by a simultaneous change of sign ϵ and μ. Thus in case of LHM, $\boldsymbol{E}, \boldsymbol{H}$ and $\boldsymbol{S}$ still forms a right handed triplet. Therefore in LHM, Poynting vector and wave propagation vector are anti-parallel. In other words, in such a material energy and wavefronts travel in opposite direction.

III. . VERIFICATION OF PURELY LEFT HANDED TRANSMISSION LINE MODEL

Fig.2 shows a purely Left Handed (LH) transmission line model which is hypothetical in nature. It is based on the dual version of conventional Right Handed (RH) transmission line in which the series/parallel arrangements of inductor and capacitor are interchanged. For the simplicity of calculation lossless case is considered.

Applying KVL in the circuit shown in fig.1,

$$V(z) - \frac{1}{j\omega C'}I(z)\Delta z - V(z+\Delta z) = 0 \tag{2.1}$$

Applying KCL in the above circuit,

$$I(z) - I(z+\Delta z) - \frac{V(z+\Delta z)\Delta z}{j\omega L'} = 0 \tag{2.2}$$

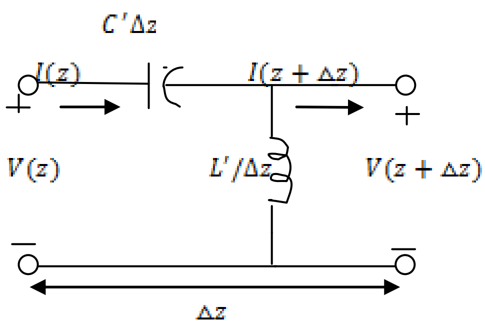

Fig. 2. Hypothetical Purely Left Handed TL model

Applying KVL in the circuit shown in fig.1,

$$V(z) - \frac{1}{j\omega C'}I(z)\Delta z - V(z+\Delta z) = 0 \tag{2.1}$$

Applying KCL in the above circuit,

ISSN: 2321-0850, Volume-4, Issue-3, March 2016

$$I(z) - I(z+\Delta z) - \frac{V(z+\Delta z)\Delta z}{j\omega L'} = 0 \quad (2.2)$$

As $\Delta z \to 0$, from equation (2.1) and (2.2) we get

$$-\frac{\partial V}{\partial z} = I(z)\frac{1}{j\omega C'} \quad (2.3)$$

$$-\frac{\partial I}{\partial z} = V(z)\frac{1}{j\omega L'} \quad (2.4)$$

Differentiating equation (2.3) with respect to z, and substituting $\partial I/\partial z$ from equation (2.4), we get Telegrapher's equation

$$\frac{\partial^2 V}{\partial z^2} = V(z)\frac{1}{j^2\omega^2 L'C'} = \gamma^2 V \quad (2.5)$$

From equation (2.5), we get propagation constant

$$\gamma = j\beta = \frac{1}{j\omega\sqrt{L'C'}}$$

$$or, \beta = -\frac{1}{\omega\sqrt{L'C'}}$$
(2.6)

Thus β is a negative real number. That means the wave propagation will be in backward direction compared to conventional right handed transmission Line. Also phase velocity and group velocity will be anti-parallel which is calculated below from the equation (2.6).

$$V_p = \frac{\omega}{\beta} = -\omega^2\sqrt{L'C'}$$
(2.7)

$$V_g = \frac{\partial\omega}{\partial\beta} = \omega^2\sqrt{L'C'} \quad (2.8)$$

Propagation constant β can be written as

$$\beta = \frac{2\pi f}{\lambda f} = \frac{1}{n}\frac{\omega}{c} \quad (2.9)$$

From equation (2.9) it is observed that if β is negative n must be negative (because frequency and velocity cannot be negative). Hence in a purely LH transmission line refractive index is negative.

Practically this model is not feasible because of unavoidable right handed parasitic components (i.e. series inductance and shunt capacitance). Therefore a Composite Right and Left Handed (CRLH) balanced transmission line model (1D) is explored by Caloz and Itoh [9-10]. This model is practically possible to implement. It is nothing but a series combination of purely RH and LH transmission line and therefore it sometimes behaves like a purely LH transmission line at lower frequencies and behaves like a purely RH transmission line at higher frequencies, which can be observed from the dispersion graph (ω-β) shown in fig.3.

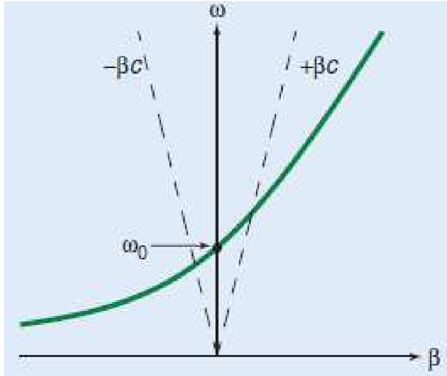

Fig. 3. Dispersion graph of lossless Balanced CRLH TL Model [10]

IV. Effective Negative Permittivity Media and Plasma Frequency

A medium which has equal positive and negative charges, of which at least one charge type is mobile, is called plasma medium. In such a medium ϵ will be negative if operating frequency (ω) is lesser than plasma frequency ω_p [12] which can be understood by the following equation.

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad (3.1)$$

A medium in which only ϵ is negative or only μ negative, is also known as Single Negative (SNG) Metamaterial (MTM). In this kind of media propagation constant β will be a purely imaginary number, [$as\ \beta = \omega\sqrt{\epsilon\mu}$] which implies that the wave will be evanescent in nature in a plasma medium.

An effective negative permittivity medium or SNG MTM can be achieved by arranging an array of thin metal wires inside a hollow waveguide and excitation of Electric field $\boldsymbol{E}$ must be parallel to z axis, which is shown in fig.4.

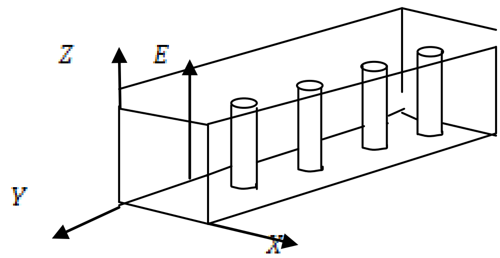

Fig. 4. An array of thin copper wires inside a hollow waveguide which forms a plasma medium

This structure shown in fig.4 can be easily analyzed by its equivalent transmission line (TL) model, which is shown in fig.5.

Applying KVL in the circuit shown in fig.5, we get

$$V(z) - L_s'\Delta z I(z) - V(z+\Delta z) = 0 \quad (3.2)$$

Applying KCL in the above circuit, we get

$$I(z) - \frac{V(z+\Delta z)\Delta z}{j\omega L_L'} - V(z+\Delta z)j\omega C_s'\Delta z - I(z+\Delta z) = 0$$
(3.3)

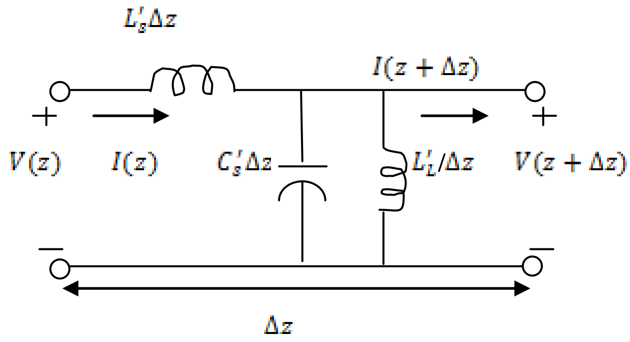

Fig. 5. Equivalent Transmission Line model of lossless Plasma medium

as $\Delta z \to 0$, from equation (3.2) and (3.3), we get the following equations.

$$-\frac{\partial V}{\partial z} = j\omega L_s' I(z) \tag{3.4}$$

$$-\frac{\partial I}{\partial z} = V(z)\left[\frac{1}{j\omega L_L'} + j\omega C_s'\right] \tag{3.5}$$

Diffrentiating equation (3.4) with respect to z and substituting the value of ∂I/∂z from equation (3.5) we get Telegrapher's equation,

$$\frac{\partial^2 V}{\partial z^2} = V(z)\left[\frac{L_s'}{L_L'} + j^2\omega^2 L_s' C_s'\right] = \gamma^2 V \tag{3.6}$$

Hence propagation constant, calculating from equation (3.6),

$$\gamma = j\beta = \sqrt{\frac{L_s'}{L_L'} - \omega^2 L_s' C_s'}$$

$$or, \beta = -j\sqrt{\frac{L_s'}{L_L'} - \omega^2 L_s' C_s'} \tag{3.7}$$

From equation (3.7) it is observed that β a is a purely imaginary no. Hence the wave will be evanescent which implies that the medium is plasmonic in nature.

V. Application of Metamaterial

Metamaterials have attracted the attention of several researchers in antenna application, because of its unique properties. CRLH-TL resonator has zeroth order and negative order resonances [9], observed from its dispersion graph. By employing zeroth order CRLH TL, antenna size can be substantially reduced, as resonance frequency does not depend on the physical length of the antenna. Rather than zeroth order mode, in CRLH TL resonator each positive (+m) mode has a corresponding negative (-m) mode, with identical field distribution. Thus a dual band meta-resonator antenna can be achieved by using pair of CRLH TL [15] with single common feeding structure. A pair of SRRs can also be used [14] instead of CRLH TL, to obtain dual band meta-resonator antenna.

A combination of DNG MTM slab and double positive material slab can be used in the substrate of a microstrip antenna to miniaturize the size of antenna [16]. The reason is that, the dispersion relation is not dependent on the sum of thickness of those slabs but it depends only on the ratio of their thickness. A Single Negative metamaterial (with only μ<0 or, ϵ<0) can be employed to enhance the directivity [13] of microstrip patch antenna. An SRR can be placed on the surface of the patch with zero refractive index (η=0). That means the electromagnetic wave, radiated from the patch, will be concentrated around the normal of the slab. Thus it is increasing the gain of the antenna.

References

[1] V.G. Veselago, "The Electrodynamics of substances with simultaneously negative values of ∈ and μ," *Soviet Physics. Uspekhi*, vol. 10, no. 4, Jan-Feb. 1968, pp- 509-514.

[2] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nassser, and S.Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Physics Review Letter*, vol. 84, no. 18, May 2000, pp. 4184–4187.

[3] R.A. Shelby, D.R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, vol. 292, no. 5514, 2001, pp.77–79

[4] R. W. Ziolkowski, E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phys. Rev. E*, vol. 64, Nov. 2001, pp. 056625/1-15.

[5] C. Caloz and T. Itoh, "Application of the transmission line theory of left handed (LH) materials to the realization of a microstrip LH transmission line," *IEEE-APS Int. Symp. Dig.*, vol. 2, June 2002, pp. 412–415,

[6] Atsushi Sanada, Christophe Caloz and Tatsuo Itoh, "Characteristics of the Composite Right/Left Handed Transmission lines", *IEEE Microwave and Wireless Components Letters*, VOL. 14, NO. 2, Feb 2001, pp. 68-70.

[7] A.A. Oliner, "A periodic-structure negative-refractive-index medium without resonant elements", *IEEE-APS Int. Symp, Dig.*, June 2002, p-41.

[8] G.V. Eleftheriades, A.K. Iyer, and P.C. Kremer, "Planar negative refractive index media using periodically L-C loaded transmission lines," *IEEE Trans Microwave Theory Tech.*, Vol. 50, Dec 2002, pp. 2702- 2712.

[9] C. Caloz and T. Itoh, "*Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*", New York: Wiley, 2004.

[10] A. Lai, C. Caloz and T. Itoh, "Composite Right/Left handed Transmission Line Metamaterial", *IEEE Microwave Magazine*, September, 2004, pp. 34-50.

[11] J.B. Pendry, "Negative Refraction makes a Perfect Lens", *Physics Review Letter*, Vol. 84, No. 18, October, 2000, pp. 3966-3969.

[12] C. Kittel, "*Introduction to Solid State Physics*", John Wiley and Sons, Inc., Singapore, New York, 2002, pp. 271-273.

[13] H. Zhou, Z. Pei, S.Qu, S. Zhang, J. Wang, Z. Duan, H.Ma, and Z. Xu, "A Novel High-Directivity Microstrip Patch Antenna Based on Zero-Index Metamaterial", *IEEE Antennas and Wireless Propagation Letters*, Vol. 8, 2009, pp. 538-541.

[14] I. K. Kim, V. V. Varadan, "Electrically Small, Millimeter Wave Dual Band Meta-Resonator Antennas", *IEEE Transactions on Antennas and Propagation*, Vol. 58, No.11, November 2010, pp. 3458-3462.

[15] Cheng-Jung Lee, Kevin M. K. H. Leong, and Tatsuo Itoh, "Compact Dual-Band Antenna Using an Anisotropic Metamaterial", *Proceedings of 36th European Microwave Conference*, Sept,2006, pp.1044-1047.

[16] G. Singh, "Double Negative Metamaterials for miniaturization of Rectangular Microstrip Antenna", *J.Electromagnetic Analysis & Applications*, Feb, 2010, Vol. 2, pp. 347-