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# On local existence and blow-up of solution for the higher-order nonlinear Kirchhoff-type equation with nonlinear strongly damped terms

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***Abstract*— In this paper ,we deal with the initial boundary value problems for higher -order kirchhoff-type equation with nonlinear strongly dissipation:**

**At first ,we prove the local existence and uniqueness of the solution by Galerkin methodthen and contracting mapping principle .Furthermore,we prove the global existence of solution , At last,we consider that blow up of solution in finite time under suitable condition .**

***Index Terms*— Higher-order nonlinear Kirchhoff wave equation;local existence; The existence and uniqueness; blow-up**

## I. INTRODUCTION

In this paper we concerned with global existence and blow-up of solution for the following for Higher-order Kirchhoff-type equation with nonlinear strongly dissipation :

$$u_{tt}+(-\Delta)^m u_t+\phi(\|\nabla^m u\|^2)(-\Delta)^m u+h(u_t)=f(u) \quad (1.1)$$

$$u(x,t)=0, \quad \frac{\partial^i u}{\partial v^i}=0, \quad i=1,2\cdots,m-1, x\in\partial\Omega, t>0, \quad (1.2)$$

$$u(x,0)=u_0, \quad u_t=u_1(x), \quad x\in\partial\Omega \quad (1.3)$$

Where $\Omega\subset R^2$ is bounded open domain with smooth boundary; $v$ is the outer norm vector; $m>1$ is a positive integer, $\phi(r)$ is a nonnegative locally Lipschitz, $h(u_t)$ is a nonlinear forcing, $f(u)$ is a nonlinear $C^1-$function, $(-\Delta)^m u_t$ is a strongly dissipation.

There have been many researches on the global and blow-up solution for Kirchhoff equation.we can see[1-8].

Kosuke Ono [9] deals with the higher-order kirchhoff-type equation with nonlinear dissipation:

$$u_{tt}+M(\|\nabla u\|^2)(-\Delta)u+|u_t|^\beta u_t=f(u), \quad x\in\Omega, \quad t>0, \quad (1.4)$$

$$u(x,0)=u_0(x), \; u_t(x,0)=u_1(x), \; and \; u(x,t)|_{\partial\Omega}=0 \quad (1.5)$$

In where $\Omega$ is a bounded domain in $R^n$ with smooth boundary $\partial\Omega$, $M(r)$ is a nonnegative $C^1$ -function for $r\geq 0$ positive satisfying $M(\|\nabla u\|^2)=a+b\|\nabla u\|^{2\gamma}$ with $a,b\geq 0, a+b>0$ and $\gamma\geq 1,$ and $f(u)$ constants is nonlinear $C^1$ -function satisfying $|f(u)|\leq k_1|u|^{\alpha+1}$ and $|f'(u)|\leq k_2|u|^\alpha$ with $k_1,k_2>0$ and $\alpha>0$ .he investigate on global existence and blow up of solution .

Yaojun Ye [10] also studied the initial-boundary value problem for a class of nonlinear higher-order kirchhoff-type equation with dissipation term

$$u_{tt}+\left\|A^{\frac{1}{2}}u\right\|^{2p}Au+a|u_t|^{q-2}u_t=b|u|^{r-2}u, \quad x\in\Omega, t>0, \quad (1.6)$$

$$u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \quad x\in\Omega. \quad (1.7)$$

$$u(x,t)=0, \quad \frac{\partial^i u(x,t)}{\partial v^i}=0, \quad i=1,2,\cdots,m-1, \quad x\in\partial\Omega, t\geq 0. \quad (1.8)$$

In a bounded domain,where $A=(-\Delta)^m$, $m>1$ is a positive integer , $a,b,p>0$ and $q,r>2$ are constants ,he obtains the global existence of solutions by construction a stable set in $H_0^m(\Omega)$ and show the energy decay estimate by applying a lemma of V.Komornik.

Takeshi Taniguchi [11] considered the existence of local solution to a weakly damped wave equation of equation of kirchhoff type the damped term and the source term

$$u_{tt}(t)-M(\|u(t)\|^2)\Delta u(t)-\gamma_2 u_t(t)+|u_t(t)|^p u_t(t)=|u(t)|^q u(t) \quad (1.9)$$

$$u(x,0)=u_0(x), \; u_t(x,0)=u_1(x), \text{ and } u(x,t)|_{\partial\Omega}=0, \; p,q,\gamma_2>0 \quad (1.10)$$

with an initial value $u(0)=u_0, \; u_t(0)=u_1$ and the Dirichlet boundary condition $u(x,t)|_{\partial\Omega}=0,$ where $\Omega$ is an open bounded domain in $R^n$ with smooth boundary and $M(r)$ is a locally Lipschitz function ,he discuss the global existence and exponential asymptotic behavior of solution.

Recently,Gongwei Liu[ [12] concerned with the study of damped wave equation of kirchhoff type

$$u_{tt}-M(\|\nabla u\|_2^2)\Delta u+u_t=g(u). \quad (1.11)$$

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in $\Omega\times(0,\infty)$, with initial and Dirichlet boundary condition.where $\Omega$ is the bound of $R^2$ have a smooth boundary $\partial\Omega$ .under the assumption that $g$ is a function with exponential growth at infinity ,he proves global existence and the decay property as well as blow-up of solutions in finite time under suitable conditions.

The paper is arranged as follows.in section 2,we state some preliminaries and some lemma ; in section 3,we obtain the existence and uniqueness of the local solution by the Banach contraction mapping principle ;in section 4,we discuss global existence and nonexistence results ;in section 5,we discuss the blow-up properties of solution for positive and negative initial energy and estimate for blow-up time $T^*$.

## II. Preliminaries

Throughout this paper ,for convenience,we denote the norm and scalar product in $L^2(\Omega)$ by $\|\cdot\|$ and (.,.); we use the next notations

$$(u,v)=\int_\Omega uvdx,$$

$$\|u\|^2=\int_\Omega |u|^2dx,$$

$$\|u\|_r=(\int_\Omega |u|^r dx)^{\frac{1}{r}}.$$

where $C_i(i=0\cdots12)$, are constants,

In this section, we present some materials needed in the proof of our results, we assume that

there exists some constants

$(G_1)$ $|f(u)|\le k_1|u|+k_2|u|^{r+1},\ k_1,k_2,r>0.$

$(G_2)$ there exist some constants

(1) $h(s)s\ge 0,\ s\in R$,

(2) $\|h(u_t)\|\le C_0(1+\|\nabla^m u_t\|)^{1-\sigma},\ 0<\sigma<1.$

(3) $|h(u_t)|\le k_3|u_t|^{p+1},\ k_3,p\ge 0.$

$(G_3)$ there exist a constant $\beta>0$, such that $f(u)u\ge(2+4\beta)G(s)$ for all $s\in R$.

In this paper ,we will use the next well-know Lemmas

Lemma2.1[10] Let $s$ be a number with $2\le s<+\infty, n\le 2m$ and $2\le s\le\frac{2n}{n-2m}$ .then there is a constant $C$ depending on $\Omega$ and $s$ such that

$$\|u\|_s\le\kappa\left\|(-\Delta)^{\frac{m}{2}}\right\|,\quad \forall u\in H_0^m(\Omega) \tag{2.1}$$

Lemma2.2[13](young inequality )Let $a,b$ and $\mu$ are positive constants,such that

satisfy $0\le r<\frac{2m}{n-2m}$ we have

$$ab\le\frac{\mu^p}{p}a^p+\frac{1}{q\mu^q}b^q\quad(\frac{1}{p}+\frac{1}{q}=1,\ p>1,q>1) \tag{2.2}$$

Lemma2.3[14] Suppose that $\delta>0$ and $B(t)$ is a nonnegative $C^2(0,+\infty)$ function such that

$$B''(t)-4(\delta+1)B'(t)+4(\delta+1)B(t)\ge 0. \tag{2.3}$$

if

$$B'(0)>r_2B(0)+K_0 \tag{2.4}$$

Then we have $\forall t>0, B'(t)>K_0$ ,where $K_0$ is a constant , $r_2=2(\delta+1)-2\sqrt{(\delta+1)\delta}$ is smaller root of the equation

$$r^2-4(\delta+1)r+4(\delta+1)=0 \tag{2.5}$$

Lemma2.4[14]If $J(t)$ is a non-increasing function on $[t_0,+\infty), t_0\ge 0$ such that

$$J'(t)^2\ge a+bJ(t)^{2+\frac{1}{\delta}},\quad \forall t_0\ge 0, \tag{2.6}$$

where $a>0, b\in R$. Then there exists a finite time $T^*$ such that

$$\lim_{t\to T^*}J(t)=0. \tag{2.7}$$

And the case that

(i) If $b>0$, $J(t_0)<\min\{1,\sqrt{\frac{a}{-b}}\}$ ,then

$$T^*\le t_0+\frac{1}{\sqrt{-b}}\ln\frac{\sqrt{\frac{a}{-b}}}{\sqrt{\frac{a}{-b}}-J(t_0)}; \tag{2.8}$$

(ii) If $b=0$ ,then $T^*\le t_0+\frac{J(t_0)}{\sqrt{a}}$. (2.9)

(iii) If $b>0$ , $T^*\le\frac{J(t_0)}{\sqrt{a}}$ or

$$T^*\le t_0+2^{\frac{3\delta+1}{2\delta}}\frac{\delta c}{\sqrt{a}}\{1-[1+cJ(t_0)]^{\frac{-1}{2\delta}}\} \tag{2.10}$$

## III. Existence of Local Solution

In this section ,we prove the existence of local solution to problem (1.1)-(1.3) for initial value $(u_0,u_1)\in H^m(\Omega)\cap H^{2m}(\Omega)\times H^m(\Omega)$

Theorem 3.1Suppose that $(G_1),(G_2)$ hold, for any give $(u_0,u_1)\in H^{2m}(\Omega)\bigcap H_0^m(\Omega)\times L^2(\Omega)$

$$u(t) \in C([0,T_0]; H_0^m(\Omega) \cap H^{2m}(\Omega))$$
$$u_t(t) \in C([0,T_0]; L^2(\Omega) \cap H^m(\Omega))$$

Proof we set for any $T > 0$,the Banach space

$$X_{T,R} = \left\{ \begin{array}{l} v(t) \in C([0,T]; H^{2m} \bigcap H_0^m), \\ v_t(t) \in C([0,T]; L^2 \cap H_0^m), \\ \rho(v(t)) \le R. \end{array} \right\},$$

Where $\rho(v(t)) = \|v_t\|^2 + \|(-\Delta)^m v\|^2$. Define distance $\rho(u,v) = \sup_{t \le 0 \le T} \rho(u(t) - v(t))$.Then $X_{T,R}$ Is a complete metric space.

We define map the $\chi$ :, $v \mapsto \chi(v) = u$. $u$ is the unique solution of the following equation:

$$u_{tt} + (-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u + h(u_t) = f(v) \qquad (3.1)$$

$$u(x,t) = 0, \quad \frac{\partial^i u(x,t)}{\partial v^i} = 0, \quad i = 1,2,\cdots,m-1, \ x \in \partial\Omega, t \ge 0. \qquad (3.2)$$

$$u(x,0) = u_0(x), \ u_t(x,0) = u_1(x), \ x \in \Omega. \qquad (3.3)$$

we define map the $T > 0$ and $R > 0$ ,such that

(1) $\chi$ maps $X_{T,R}$ into itself;

(2) $\chi$ is a contraction mapping with respect to the metric $d(.,.)$.

Step 1.we will show that $\chi$ maps $X_{T,R}$ into itself, we multiply $u_t + \varepsilon(-\Delta)^m u$ with both sides of equation (3.4).and obtain

$$(u_{tt} + (-\Delta)^m u_t + \phi(\|\nabla^m u\|^2)(-\Delta)^m u + h(u_t), u_t + (-\Delta)^m u) = (f(v), u_t + (-\Delta)^m u). \qquad (3.4)$$

$$\begin{aligned} &(u_{tt}, u_t + \varepsilon(-\Delta)^m u) \\ &= \frac{1}{2}\frac{d}{dt}\|u_t\|^2 + \frac{\varepsilon d}{dt}(u_t, (-\Delta)^m u) - \varepsilon\|\nabla^m u_t\|^2 \end{aligned} \qquad (3.5)$$

$$\begin{aligned} &((-\Delta)^m u_t, u_t + \varepsilon(-\Delta)^m u) \\ &= \|\nabla^m u_t\|^2 + \frac{\varepsilon}{2}\frac{d}{dt}\|(-\Delta)^m u\|^2 \end{aligned} \qquad (3.6)$$

$$\begin{aligned} &(\phi(\|\nabla^m v\|^2)(-\Delta)^m u, u_t + \varepsilon(-\Delta)^m u) \\ &= \frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2) \\ &- \frac{1}{2}\frac{d}{dt}(\phi\|\nabla^m v\|^2)\|\nabla^m u\|^2 + \varepsilon\phi(\|\nabla^m v\|^2)\|(-\Delta)^m u\|^2 \end{aligned} \qquad (3.7)$$

$$\begin{aligned} &\left|\frac{d}{dt}(\phi\|\nabla^m v\|^2)\|\nabla^m u\|^2\right| \\ &\le \|\phi'(\xi)\|_\infty \|(-\Delta)^m v\|\|v_t\|\|\nabla^m u\|^2 \\ &\le LR\|\nabla^m u\|^2 \\ &\le \frac{LR}{\lambda^m}\|(-\Delta)^m u\|^2 \\ &\le \kappa_0 \rho(u(t)) \end{aligned} \qquad (3.8)$$

$$(\phi(\|\nabla^m v\|^2)(-\Delta)^m u, u_t + \varepsilon(-\Delta)^m u)$$
$$\geq \frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2)$$
$$-\kappa_0 \rho(u(t)) + \varepsilon\phi(\|\nabla^m v\|^2)\|(-\Delta)^m u\|^2 \tag{3.9}$$

$$(h(u_t), u_t + \varepsilon(-\Delta)^m u)$$
$$= (h(u_t), u_t) + \varepsilon(h(u_t), (-\Delta)^m u) \tag{3.10}$$

According to young inequality and $G_2$ ,such that

$$(h(u_t), \varepsilon(-\Delta)^m u) \geq -\frac{\varepsilon\|h(u_t)\|^2}{2} - \frac{\varepsilon\|(-\Delta)^m u\|^2}{2} \tag{3.11}$$

$$\|h(u_t)\|^2$$
$$\leq C_0(1 + \|\nabla^m u_t\|)^2$$
$$\leq 2C_0 + 2C_0\|\nabla^m u_t\|^2 \tag{3.12}$$

$$(h(u_t), (-\Delta)^m u)$$
$$\geq -C_0 - C_0\varepsilon\|\nabla^m u_t\|^2 - \frac{\varepsilon}{2}\|(-\Delta)^m u\|^2 \tag{3.13}$$

According to Lemma2.1 and $(G_1)$ ,such that

$$(f(v), u_t + \varepsilon(-\Delta)^m)$$
$$\leq \int_\Omega (k_1|v| + k_2|v|^{r+1})u_t dx + \varepsilon\int_\Omega (k_1|v| + k_2|v|^{r+1})(-\Delta)^m u dx$$
$$\leq (k_1\|v\| + k_2\|v\|_{2r+2}^{r+1})\|u_t\| + \varepsilon(k_1\|v\| + k_2\|v\|_{2r+2}^{r+1})\|(-\Delta)^m u\|$$
$$\leq (\frac{k_1\sqrt{R}}{\lambda^{2m}} + k_2\kappa R^{r+1})\|u_t\| + (\frac{\varepsilon k_1\sqrt{R}}{\lambda^{2m}} + \varepsilon k_2\kappa R^{r+1})\|(-\Delta)^m u\| \tag{3.14}$$

We take $\alpha_0 = \{\frac{k_1\sqrt{R}}{\lambda^{2m}} + k_2\kappa R^{r+1}, \frac{\varepsilon k_1\sqrt{R}}{\lambda^{2m}} + \varepsilon k_2\kappa R^{r+1}\}$ ,such that

$$(f(v), u_t + \varepsilon(-\Delta)^m)$$
$$\leq \alpha_0(\|u_t\| + \|(-\Delta)^m u\|)$$
$$\leq \alpha_0(\rho(u))^{\frac{1}{2}} \tag{3.15}$$

From (3.5)and (3.15),we have

$$\frac{1}{2}\frac{d}{dt}(\|u_t\|^2 + 2\varepsilon(u_t, (-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 + \phi(\|\nabla^m v\|^2\|\nabla^m u\|^2))$$
$$+\|\nabla^m u_t\|^2 - \varepsilon\|\nabla^m u_t\|^2 - C_0\varepsilon\|\nabla^m u_t\|^2 - \frac{\varepsilon}{2}\|(-\Delta)^m u\|^2$$
$$\leq \alpha_0(\rho(u))^{\frac{1}{2}} + \kappa_0\rho(u(t)) + C_0 \tag{3.16}$$

$$\frac{dt}{d}(\frac{1}{2}\|u_t\|^2 + \varepsilon(u_t, (-\Delta)^m u) + \|(-\Delta)^m u\|^2 + \frac{1}{2}\phi(\|\nabla^m v\|^2\|\nabla^m u\|^2) + K_1)$$
$$+(1 - \varepsilon - \frac{C_0\varepsilon}{2})\|\nabla^m u_t\|^2 + \varepsilon\phi(\|\nabla^m v\|^2)\|(-\Delta)^m u\|^2$$

$$\leq \alpha_0(\rho(u))^{\frac{1}{2}} + \kappa_0\rho(u(t)) + \frac{\varepsilon}{2}\rho(u(t)) + C_0 \tag{3.17}$$

We take $\rho_1(u(t)) = \|u_t\|^2 + 2\varepsilon(u_t,(-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 + \phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2 + 2K_1$ such that

$$\|u_t\|^2 + 2\varepsilon(u_t,(-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 + \phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2 + 2K_1$$

$$\geq \|u_t\|^2 + 2\varepsilon(u_t,(-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 \tag{3.18}$$

$$2\varepsilon(u_t,(-\Delta)^m u) \geq 2\varepsilon\|u_t\|^2 - \frac{\varepsilon}{2}\|(-\Delta)^m u\|^2 \tag{3.19}$$

From (3.18) and (3.19),we have

$$\|u_t\|^2 + 2\varepsilon(u_t,(-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 + \phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2 + 2K_1 \tag{3.20}$$

$$\geq (1-2\varepsilon)\|u_t\|^2 + \frac{\varepsilon}{2}\|(-\Delta)^m u\|^2 \tag{3.21}$$

We take $\alpha_1 = \min\{1-\varepsilon, \frac{\varepsilon}{2}\}$ ,such that

$$\rho_1(u(t)) \geq \alpha_1(\|u_t\|^2 + \|(-\Delta)^m u\|^2) \tag{3.22}$$

So

$$\rho_1(u(t)) \geq \alpha_1\rho(u(t)) \tag{3.23}$$

$$\frac{dt}{d}(\|u_t\|^2 + 2\varepsilon(u_t,(-\Delta)^m u) + \varepsilon\|(-\Delta)^m u\|^2 + \phi(\|\nabla^m v\|^2)\|\nabla^m u\|^2 + 2K_1) + \alpha_1\|\nabla^m u_t\|^2$$

$$\leq \alpha_0(\rho(u))^{\frac{1}{2}} + \kappa_0\rho(u(t)) + \frac{\varepsilon}{2}\rho(u(t)) + C_0 \tag{3.24}$$

Where $\alpha_2 = 1 - \varepsilon - \frac{\varepsilon C_0}{2}$ $C_0 \leq \frac{2\kappa_0 K_1}{\alpha_1}$ .

$$\frac{d}{dt}\rho_1(u(t)) + \alpha_2\|\nabla^m u_t\|^2$$

$$\leq \frac{2\kappa_0}{\alpha_1}\rho_1(u(t)) + \frac{\varepsilon}{2}\rho_1(u(t)) + \alpha_0(\rho_1(u))^{\frac{1}{2}}. \tag{3.25}$$

So,we using Gronwall inequality,we obtain

$$\rho_1(u(t)) + \alpha_2\int_0^T \|\nabla^m u_t\|^2 ds$$

$$\leq \{\rho_1(u(0))^{\frac{1}{2}} + \alpha_0 T\}^2 \rho^{\frac{(4\kappa_0+\alpha_0\varepsilon)T}{2\alpha_1}}. \tag{3.26}$$

Where $\rho_1(u(0)) = \|u_1\|^2 + 2\varepsilon(u_1,(-\Delta)^m u_0) + \varepsilon\|(-\Delta)^m u_0\|^2 + \phi(\|\nabla^m v_0\|^2\|\nabla^m u_0\|^2) + 2K_1$

From we obtain

$$\rho(u(t)) + \alpha_2\int_0^T \|\nabla^m u_t\|^2 ds$$

$$\leq \frac{1}{\alpha_1}\rho_1(u(t)) + \frac{\alpha_2}{\alpha_1}\int_0^T \|\nabla^m u_t\|^2 ds$$

$$\le \{\rho_1(u(0))^{\frac{1}{2}} + \alpha_0 T\}^2 \rho^{\frac{(4\kappa_0+\alpha_0\varepsilon)T}{2\alpha_1}}. \tag{3.27}$$

Therefore, we choose that parameters $T$ ,and $R$ ,we obtain

$$\{\rho_1(u(0))^{\frac{1}{2}} + \alpha_0 T\}^2 \rho^{\frac{(4\kappa_0+\alpha_0\varepsilon)T}{2\alpha_1}} \le R^2. \tag{3.28}$$

So ,we finishes the proof of the first step.

Step 2 we will show that $\chi$ is a contraction in $X_{T,R}$ ,let $v_1$, $v_2 \in X_{T,R}$ ,such that $\chi(v_1) = u_1$ $\chi(v_2) = u_2$ ,Setting $w = u_1 - u_2$ ,then we have

$$w_{tt} + (-\Delta)^m w_t + \phi(\|\nabla^m v_1\|^2)(-\Delta^m)u_1 - \phi(\|\nabla^m v_2\|^2)(-\Delta^m u_2) + h(u_{t1}) - h(u_{t2}) = f(v_1) - f(v_2)$$

$$\begin{aligned}&(w_{tt}, w_t + (-\Delta)^m w)\\ &= \frac{1}{2}\frac{d}{dt}\|w_t\|^2 + \frac{\varepsilon d}{dt}(w_t, (-\Delta)^m w) - \varepsilon\|\nabla^m w_t\|^2\end{aligned} \tag{3.29}$$

$$\begin{aligned}&((-\Delta)^m w_t, w_t + \varepsilon(-\Delta)^m w)\\ &= \|\nabla^m w_t\|^2 + \frac{\varepsilon}{2}\frac{d}{dt}\|(-\Delta)^m w\|^2\end{aligned} \tag{3.30}$$

$$\begin{aligned}
&(\phi(\|\nabla^m v_1\|^2)(-\Delta^m)u_1 - \phi(\|\nabla^m v_2\|^2)(-\Delta^m)u_2, w_t + \varepsilon(-\Delta)^m w)\\
&= (\phi(\|v_1\|^2)(-\Delta)^m w, w_t + \varepsilon(-\Delta)^m w)\\
&+ (\phi(\|\nabla^m v_1\|^2)(-\Delta)^m u_2 - \phi\|\nabla^m v_2\|^2(-\Delta)^m u_2, w_t + \varepsilon(-\Delta)^m w)\\
&= \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2 + \phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2 - \frac{1}{2}\frac{d}{dt}(\phi(\|\nabla^m v_1\|^2))\|\nabla^m w\|^2\\
&+ (\phi'(\xi)(\|\nabla^m v_1\|^2 - \|\nabla^m v_2\|^2)(-\Delta)^m u_2, w_t)\\
&+ (\phi'(\xi)(\|\nabla^m v_1\|^2 - \|\nabla^m v_2\|^2)(-\Delta)^m u_2, \varepsilon(-\Delta)^m w)\\
&\ge \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2 + \varepsilon\phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2\\
&-\phi'(\|\nabla^m v_1\|^2)\int_\Omega \nabla^m v_1 \nabla^m v_{1t} dx \|\nabla^m w\|^2 + \|\phi'(\xi)\|_\infty(\|\nabla^m v_1\|^2 - \|\nabla^m v_2\|^2)(-\Delta)^m u_2, w_t)\\
&-\varepsilon\|\phi'(\xi)\|_\infty(\|\nabla^m v_1\| + \|\nabla^m v_2\|)(\|\nabla^m v_1\| - \|\nabla^m v_2\|)\|(-\Delta)^m u_2\|\|(-\Delta)^m w\|\\
&\ge \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2 + \varepsilon\phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2 - L_0\|(-\Delta)^m w\|^2\\
&-\|\phi'(\xi)\|_\infty(\|\nabla^m v_1\| + \|\nabla^m v_2\|)(\|\nabla^m v_1\| - \|\nabla^m v_2\|)\|(-\Delta)^m u_2\|\|w_t\|\\
&-\varepsilon\|\phi'(\xi)\|_\infty(\|\nabla^m v_1\| + \|\nabla^m v_2\|)(\|\nabla^m v_1\| - \|\nabla^m v_2\|)\|(-\Delta)^m u_2\|\|(-\Delta)^m w\|\\
&\ge \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2 + \varepsilon\phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2 - L_0\|(-\Delta)^m w\|^2\\
&-L_1\|(-\Delta)^m(v_1 - v_2)\|\|(-\Delta)^m u_2\|\|w_t\| - C\|(-\Delta)^m w\|^2\\
&-L_2\|(-\Delta)^m(v_1 - v_2)\|\|(-\Delta)^m u_2\|\|(-\Delta)^m w\|\\
&\ge \frac{1}{2}\frac{d}{dt}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2 + \varepsilon\phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2
\end{aligned}$$

$$-L_3[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}}-L_0\rho(w(t)) \tag{3.31}$$

From $G_2$ .we have

$$\begin{aligned}&(h(u_{t1})-h(u_{t2}),w_t+\varepsilon(-\Delta)^m w)\\ &\le L_4\|w_t\|^2+L_5\|(-\Delta)^m w\|^2\\ &\le \beta_0\rho(w(t))\end{aligned} \tag{3.32}$$

Where $\beta_0=\max\{L_4,L_5\}$ .

From $G_1$ and Lemma 2.1 we have

$$\begin{aligned}&(f(v_1)-f(v_2),w_t+\varepsilon(-\Delta)^m w)\\ &\le \int_\Omega k_1(|v_1|-|v_2|)|w_t|+k_2(|v_1|^{r+1}-|v_2|^{r+1})|w_t|dx\\ &+\int_\Omega k_1(|v_1|-|v_2|)|w_t|+k_2(|v_1|^{r+1}-|v_2|^{r+1})|(-\Delta)^m w|dx\\ &\le (k_1+k_2(\|v_1\|_{nr}^r+\|v_2\|_{nr}^r)\|v_1-v_2\|_{\frac{2n}{n-2}}\|(-\Delta)^m w\|\\ &+(k_1+k_2(\|v_1\|_{nr}^r+\|v_2\|_{nr}^r))\|v_1-v_2\|_{\frac{2n}{n-2}}\|w_t\|\\ &\le L_6\|(-\Delta)^m(v_1-v_2)\|\|w_t\|+L_7\|(-\Delta)^m(v_1-v_2)\|\|(-\Delta)^m w\|\\ &\le L_8[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}}+L_9[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w(t))]^{\frac{1}{2}}\end{aligned} \tag{3.33}$$

From (3.29)-(3.33),we obtain

$$\begin{aligned}&\frac{d}{dt}(\frac{1}{2}\|w_t\|^2+\frac{1}{2}\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2+\varepsilon\frac{1}{2}\|(-\Delta)^m w\|^2+\varepsilon(w_t,(-\Delta)^m w))\\ &+\|\nabla^m w_t\|^2-\varepsilon\|\nabla^m w_t\|^2+\varepsilon\phi(\|\nabla^m v_1\|^2)\|(-\Delta)^m w\|^2\\ &-L_3[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}}-L_0\rho(w(t))-\beta_0\rho(w(t))\\ &\le L_8[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}}+L_9[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w(t))]^{\frac{1}{2}}\end{aligned} \tag{3.34}$$

From (3.34) we obtain

$$\begin{aligned}&\frac{d}{dt}(\|w_t\|^2+\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2+\varepsilon\|(-\Delta)^m w\|^2+2\varepsilon(w_t,(-\Delta)^m w))\\ &\le (L_0+\beta_0)\rho(w(t))+(L_3+L_8+L_9)[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}}\end{aligned} \tag{3.35}$$

Where we set

$$\rho_2(w(t))=\|w_t\|^2+\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2+\varepsilon\|(-\Delta)^m w\|^2+2\varepsilon(w_t,(-\Delta)^m w) \tag{3.36}$$

$$2\varepsilon(w_t,(-\Delta)^m w)\ge 2\varepsilon)\|w_t\|^2-\frac{\varepsilon}{2}\|(-\Delta)^m w\|^2 \tag{3.37}$$

Then we have

$$\begin{aligned}&\rho_2(w(t))\\ &=\|w_t\|^2+\phi(\|\nabla^m v_1\|^2)\|\nabla^m w\|^2+\varepsilon\|(-\Delta)^m w\|^2+2\varepsilon(w_t,(-\Delta)^m w)\\ &\ge\|w_t\|^2+\varepsilon\|(-\Delta)^m w\|^2+2\varepsilon(w_t,(-\Delta)^m w)\end{aligned}$$

$$\geq (1-2\varepsilon)\|w_t\|^2 + \frac{\varepsilon}{2}\|(-\Delta)^m w\|^2$$
$$\geq \beta_1(\|w_t\|^2 + \|(-\Delta)^m w\|^2) \tag{3.38}$$

Where $\beta_1 = \min\{1-2\varepsilon, \frac{\varepsilon}{2}\}$

$$\frac{d\rho_2(w(t))}{dt}$$
$$\leq (L_0+\beta_0)\rho(w(t)) + (L_3+L_8+L_9)[\rho(v_1-v_2)]^{\frac{1}{2}}[\rho(w)]^{\frac{1}{2}} \tag{3.39}$$
$$\leq (\frac{L_0+\beta_0}{\beta_1})\rho_2(w(t)) + (L_3+L_8+L_9)\frac{1}{\beta_1}\{\rho_2(v_1(t)-v_2(t))\}^{\frac{1}{2}}\{\rho_2(w(t))\}^{\frac{1}{2}} \tag{3.40}$$

Where $\rho_2(w(0)) = 0$.

Therefore ,we use Gronwall inequality, we have

$$\rho_2(w(t)) \leq (L_3+L_8+L_9)^2 T^2 e^{\frac{(L_0+\beta_0)T}{\beta_1}} \sup_{0\leq t\leq T} \rho(v_1(t)-v_2(t)) \tag{3.41}$$

So,we have

$$\sup_{0\leq t\leq T} \rho(u_1(t)-u_2(t)) \leq C_{T,R} \sup_{0\leq t\leq T} \rho(v_1(t)-v_2(t)) \tag{3.42}$$

Where $C_{T,R} = (L_3+L_8+L_9)^2 T^2 e^{\frac{(L_0+\beta_0)T}{\beta_1}}$.

It is easy see re $C_{T,R} < 1$ for a small $T > 0$.

we finishes the proof of the second step.By applying the Banach contraction mapping theorem,the proof of Theorem 3.1 is now complete.

## IV. The Blow-up in Finite Time

In this section, we consider the blow-up of solution ,we assume that $\phi(s) := s^q$, $q \geq 0$, $h(u_t) = u_t$ such problems (1.1)-(1.3) became

$$u_{tt}(t) + (-\Delta)^m u_t(t) + \|\nabla^m u\|^{2q}(-\Delta)^m u(t) + u_t(t) = f(u(t)) \tag{4.1}$$

Next,we define the energy function of the solution $u$ of (4.1)

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2(q+1)}\|\nabla^m u\|^{2(q+1)} - \int_\Omega G(u)dx, \tag{4.2}$$

where $G(s) = \int_0^u g(s)ds.$

Then,we have

$$E(t) = E(0) - \int_0^t (\|\nabla^m u_t\|^2 + \|u_t\|^2)ds, \tag{4.3}$$

where $E(0) = \frac{1}{2}\|u_1\|^2 + \frac{1}{2(q+1)}\|\nabla^m u_0\|^{2(q+1)} - \int_\Omega G(u_0)dx.$

Definition4.1 A solution $u(t)$ of (4.1) is called a blow-up solution if there exist a finite time $T^*$ such that

$$\lim_{t\to T^*}\int_\Omega (|\nabla^m u|^2 + |u|^2)dx = +\infty. \tag{4.4}$$

For the next lemma,we define

$$Y(t)=Y(u(t))=\|u(t)\|^2+\int_0^t(\|u(s)\|^2+\|\nabla^m u(s)\|^2)ds, \quad \text{for } t\geq 0. \tag{4.5}$$

Lemma 4.1 Assume $(G_1)$, $(G_3)$ hold, Let $\frac{2+4\beta}{q+1}-2\geq 0,$ then we have

$$Y^{''}(t)-4(\beta+1)\|u_t\|^2\geq(-4-8\beta)E(0)+(4+8\beta)[\int_0^t(\|u_t(s)\|+\|\nabla^m u_t(s)\|)ds]. \tag{4.6}$$

Proof. From (4.5),we have

$$Y^{'}(t)=2\int_\Omega uu_t dx+\|u(t)\|^2+\|\nabla^m u\|^2, \tag{4.7}$$

and

$$Y^{''}(t)=2\|u_t(t)\|^2-2\|\nabla^m u\|^{2q+2}+2\int_\Omega f(u)udx. \tag{4.8}$$

From above equation and the energy identity, we have

$$Y^{''}-4(\beta+1)\|u_t\|^2=(-4-8\beta)E(0)+(4+8\beta)\int_0^t(\|u_t\|^2+\|\nabla^m u\|^2)ds$$
$$+\int_\Omega(2f(u)u-(4+8\beta)G(u))dx+\{\frac{2+4\beta}{(q+1)}\|\nabla^m u\|^{2(q+1)}-2\|\nabla^m u\|^{2q+2}\}, \tag{4.9}$$

there form the assumption $(G_3)$,we have

$$2\int_\Omega(f(u)u-(2+4\beta)G(u))dx+[\frac{2+4\beta}{q+1}\|\nabla^m u\|^{2(q+1)}-2\|\nabla^m u\|^{2q+2}]\geq 0. \tag{4.10}$$

So ,we obtain (4.6)

Now, we can consider there different cases on the sign of initial energy $E(0)$

If $E(0)<0$,from (4.6),we have

$$Y^{''}(t)\geq(-4-8\beta)E(0), \tag{4.11}$$

integration (4.11) over $[0,t]$,we have that

$$Y^{'}(t)\geq Y^{'}(0)-4(1+2\beta)E(0)t. \tag{4.12}$$

Thus ,we get $Y^{'}(t)>\|u_0\|^2+\|\nabla^m u_0\|^2$ for $t>t^*$,where

$$t^*=\max\{\frac{Y^{'}(0)-(\|u_0\|^2+\|\nabla^m u_0\|^2)}{4(1+\beta)E(0)},0\}. \tag{4.13}$$

If $E(0)=0$,from (4.6) we have

$$Y^{''}\geq 4(\beta+1)\|u_t\|^2+(4+\beta)\int_0^t(\|u_t\|^2+\|\nabla^m u\|^2)ds\geq 0 \qquad t\geq 0 \tag{4.14}$$

Integration (4.9)over[0,1],we have

$$Y^{'}\geq Y^{'}(0) \tag{4.15}$$

Furthermore,if $Y^{'}(0)>\|u_0\|^2+\|\nabla^m u_0\|^2$, $i.e\int_\Omega u_0u_1dx>0$ ,so

$$Y^{'}(t)\geq Y^{'}(0)>\|\nabla^m u_0\|^2+\|u_0\|^2, \quad t\geq 0. \tag{4.16}$$

If $E(0)>0$ we first note that

$$2(\int_0^t\int_\Omega uu_t dxdt+\int_0^t\int_\Omega \nabla^m u\nabla^m u_t dxds)=\|u\|^2+\|\nabla^m u\|^2-(\|u_0\|^2+\|\nabla^m u_0\|^2). \tag{4.10}$$

By using Holder inequality and (4.10 ) we have

$$\|u(t)\|^2+\|\nabla^m u(t)\|^2\le\|u_0\|^2+\|\nabla^m u_0\|^2+\int_0^t\|u(s)\|^2ds+\int_0^t\|u_t(s)\|^2ds+\int_0^t\|\nabla^m u(s)\|^2ds+\int_0^t\|\nabla^m u_t(s)\|^2ds$$

so,from above ,we obtain

$$Y'(t)=2\int_\Omega uu_t dx+\|u(t)\|^2+\|\nabla^m u\|^2\le\|u(t)\|^2+\|u_t(t)\|^2+\|u(t)\|^2+\|\nabla^m u(t)\|^2$$
$$\le\|u_0\|^2+\|\nabla^m u_0\|^2+\int_0^t\|u(s)\|^2ds+\int_0^t\|u_t(s)\|^2ds+\int_0^t\|\nabla^m u(s)\|^2ds+\int_0^t\|\nabla^m u_t(s)\|^2ds$$
$$+\|u(t)\|^2+\|u_t(t)\|^2 \tag{4.11}$$

$$Y'(t)\le Y(t)+\|u_t(t)\|^2+\|\nabla^m u_0\|^2+\|u_0\|^2+\int_0^t\|\nabla^m u_t(s)\|^2ds+\int_0^t\|u_t(s)\|^2ds \tag{4.13}$$

Form above inequality and (4.6),we obtain

$$Y''(t)-4(\beta+1)Y'(t)+4(1+\beta)Y(t)+Y_1\ge 0. \tag{4.14}$$

Where

$$Y_1=(4+8\beta)E(0)+4(1+\beta)(\|u_0\|^2+\|\nabla^m u_0\|^2). \tag{4.18}$$

Setting

$$B(t)=Y(t)+\frac{Y_1}{4(1+\beta)}\quad t>0 \tag{4.19}$$

Then $B(t)$ satisfies Lemma 2.3 ,if

$$Y'(0)>r_2[Y(0)+\frac{Y_1}{4(1+\beta)}]+\|u_0\|^2+\|\nabla^m u_0\|^2 \tag{4.20}$$

Then $Y'(0)>\|u_0\|^2+\|\nabla^m u_0\|^2$ for all $t>0$

Lemma 4.2Assume that $(G_1)$ and $(G_3)$ hold and that either one of the following conditions is satisfied

$E(0)<0$;

$E(0)=0$ and $\int_\Omega u_0u_1dx>0$;

$E(0)>0$ and (4.20) holds,then $Y'(0)>\|u_0\|^2+\|\nabla^m u_0\|^2$ for all $t>0$

where $t=t^*$ is given by (4.13) in case(1) ; $t_0=0$ in cases (2) and (3).

Proof .we can prove this Lemma4.2 by Lemma 4.1.

Theorem 4.1 Under the assumptions $(G_1)$ and $(G_3)$ ,and that either one of the following condition is satisfied:

$E(0)<0$;

$E(0)=0$ and $\int_\Omega u_0u_1dx>0$;

$0<E(0)<\dfrac{(\int_\Omega u_0u_1dx)^2}{2(T_1+1)(\|\nabla^m u\|^2+\|u_0\|^2)}$, and by (4.20) hold .

where $T_1$ to be chosen later

Then the solution $u$ blow-up at finite $T^*$ ,and $T^*$ can be estimate by (4.35)-(4.38), respectively,according to the sign of $E(0)$.
Proof. Let

$$J(t)=(Y(t)-(\|u_0\|^2+\|\nabla^m u_0\|^2))^{-\beta} \qquad t\in[0,T_1] \tag{4.21}$$

where $T_1$ is some certain constant which will be chosen later .Then we get

$$J^{'}(t)=-\beta J(t)^{1+\frac{1}{\beta}}(Y^{'}(t)+(T_1-t)(\|u_0\|^2+\|\nabla^m u_0\|^2)) \tag{4.22}$$

And

$$J^{"}(t)=-\beta J(t)^{1+\frac{2}{\beta}}V(t) \tag{4.23}$$

$$V(t)=Y^{"}(t)[Y(t)+(T_1-t)(\|u_0\|+\|\nabla^m u_0\|^2]-(1+\beta)(Y^{'}(t)-(\|u_0\|^2+\|\nabla^m u_0\|^2)) \tag{4.24}$$

Next we denote

$$P=\|u(t)\|^2,\quad Q=\int_0^t\|\nabla^m u(s)\|^2 ds,\quad R=\|u_t(t)\|^2,\quad S=\int_0^t\|\nabla^m u_t(s)\|^2 ds \tag{4.25}$$

By (4.7) and (4.9), and Holder inequality we have

$$\begin{aligned}Y^{'}(t)&=2\int_\Omega u(t)u_t(t)dx+\|u_0\|^2+\|\nabla^m u_0\|^2+2\int_0^t\int_\Omega(u(s)u_t(s)+\nabla^m u(s)\nabla^m u_t(s))dxds\\
&\le\|u_0\|^2+\|\nabla^m u_0\|^2+2\|u(t)\|\|u_t(t)\|+2(\lambda^{-2m}+1)(\int_0^t\|\nabla^m u_t\|^2ds)^{\frac{1}{2}}(\int_0^t\|\nabla^m u\|^2ds)^{\frac{1}{2}}\\
&\le\|u_0\|^2+\|\nabla^m u_0\|^2+2\sqrt{PR}+2(\lambda^{-2m}+1)\sqrt{QS}\\
&\le\|u_0\|^2+\|\nabla^m u_0\|^2+\gamma(\sqrt{PR}+\sqrt{QS})\end{aligned} \tag{4.26}$$

where $\gamma=2+2\lambda^{-2m}$
From (4.6),we have

$$Y^{"}(t)\ge(-4-8\beta)E(0)+(4+8\beta)(R+S) \tag{4.27}$$

From(4.24)-(4.27),we have

$$\begin{aligned}V(t)&\ge[(-4-8\beta)E(0)+4(1+\beta)(R+S)](Y(t)+(T_1-t)(\|u_0\|^2+\|\nabla^m u_0\|^2))\\
&-(1+\beta)(Y^{'}(t)-(\|u_0\|^2+\|\nabla^m u_0\|^2))\\
&\ge(-4-8\beta)E(0)J(t)^{\frac{-1}{\beta}}+4(1+\beta)(R+S)(T_1-t)(\|u_0\|^2+\|\nabla^m u_0\|^2)\\
&+4(1+\beta)[(R+S)(P+Q)-(\sqrt{PR}+\sqrt{QS})^2]\\
&\ge(-4-8\beta)E(0)J(t)^{\frac{-1}{\beta}}. \qquad t\ge t_0,\end{aligned} \tag{4.28}$$

from (4.23) we obtain

$$J^{"}(t)\le\beta(4+8\beta)E(0)J(t)^{1+\frac{1}{\beta}}. \qquad t\ge t_0 \tag{4.29}$$

Now that by Lemma2.4 that $J^{'}(t)<0$ for $t>t_0$ multiplying (4.29) by $J^{'}(t)$ and integrating it from $t_0\to t$ ,we have

$$J^{'}(t)\ge\omega+\kappa J(t)^{2+\frac{1}{\beta}}, \qquad t\ge t_0, \tag{4.30}$$

Where

$$\omega=\beta^2 J(t_0)^{2+\frac{2}{\beta}}[Y^{'}(t_0)-(\|u_0\|^2+\|\nabla^m u_0\|^2)-8E(0)J(t_0)^{\frac{-1}{\beta}}] \tag{4.31}$$

and $\kappa=8\beta^2E(0).$ (4.33)

We observe that

$\omega > 0$ if and if only $$E(0) < \frac{(Y'(t_0) - (\|u_0\|^2 + \|\nabla^m u_0\|^2))}{8[Y(t_0)] + (T_1 - t_0)(\|u_0\|^2 + \|\nabla^m u_0\|^2)}$$

Then by Lemma4.2 ,there exists a finite time $T^*$,such that $\lim_{t\to T^*} J(t) = 0$ and the upper bounds of $T^*$ are estimated ,respectively ,according to the sign of $E(0)$,we obtain

$$\lim_{t\to T^{*-}} (\|u(t)\|^2 + \int_0^t (\|u(s)\|^2 + \|\nabla^m u(s)\|^2)) = +\infty \tag{4.34}$$

The upper bounds of $T^*$ are estimate as follows by Lemma4.2

In case (1),we have

$$T^* \le t_0 - \frac{J(t_0)}{J(t_0)'} \tag{4.35}$$

Furthermore if $J(t_0) < \min\{1, \sqrt{\frac{\omega}{-\kappa}}\}$ ,then we obtain

$$T^* \le t_0 + \frac{1}{\sqrt{-\kappa}} Ln \frac{\sqrt{\frac{\omega}{-\kappa}}}{\sqrt{\frac{\omega}{-\kappa}} - J(t_0)}. \tag{4.36}$$

In case (2),

$$T^* \le t_0 - \frac{J(t_0)}{J(t_0)'} \quad \text{or} \quad T^* \le t_0 - \frac{J(t_0)}{\sqrt{\omega}} \tag{4.37}$$

In case (3)

$$T^* \le \frac{J(t_0)}{\sqrt{\omega}} \quad \text{or} \quad T^* \le t_0 + 2^{\frac{3\beta+1}{2\beta}} \frac{\beta c}{\sqrt{\omega}} \{1 - [1 + cJ(t_0)]^{\frac{-1}{2\beta}}\} \tag{4.38}$$

Where $c = (\frac{\kappa}{\omega})^{\frac{\beta}{2+\beta}}$ .note that in case(1), $t_0 = t^*$ is given in (4.13),and in case(2) and case(3) $t_0 = 0$

Remark4.1 that in we observe that the choice of $T_1$ in (4.21) is feasible under the same condition as in[15].

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