International Journal of Engineering and Technical Research (IJETR)
ISSN: 2321-0869 (O) 2454-4698 (P) Volume-7, Issue-8, August 2017

Mathematical model of social infrastructure based on the theory of formal concept analysis (FCA)

S.P. Semenov, V.V. Slavskii, A.O. Tashkin, A.S. Tyakunov

Abstract— **In the research are investigated logic algebraic approaches to creation understanding models based on logical (truthconditional) assessments and standards. In the article describes different conceptions of logical systems in the form of algebraic structures. Research focuses on the formal description of logical systems realization and graphical illustrations concepts of the logical world. For a visual presentation results was used the Hasse diagrams to build grids of formal concept, objects and their attributes. On the basis of the theory of FCA, through the creation dynamically updated thesaurus knowledge and understanding of domain objects and representations in the form of formal concept grids is described the principle of selection necessary data for the organization of intelligent search functions and other tasks for information support people with disabilities.**

Index Terms— **Semantic, GIS, geoinformation, analysis, formal, concepts, CDAO, FCA, ontology, grid, binary, matrix, diagrams, geowheel.ru**

I. INTRODUCTION

In Russia also as in whole world, special attention paid to the problem of creation a barrier-free environment and improvement life quality of people with disabilities. Physically handicapped people are faced with two major problems in movement: the first related with physical inaccessibility objects of social infrastructure (further OSI) which is solved by installing constructions on buildings, the second problem consists in inaccessibility of information on each OSI [1,2].

Solution of information availability problem can be reached through the creation of a socially oriented geographic information system (GIS), that solving problem of satisfaction information requirements people with disabilities (DP). Similar products are created by various developers and based on mathematical models of social infrastructure objects (OSI), displaying part of the real world and the mechanisms of interaction between the OSI, the urban environment and the social sphere, presented in the form of objects and their attributes. This article describes method of development and creation mathematical model of OSI, based on the theory of formal concept analysis (FCA).

In the researched area for the scientific community are known and published the results, aimed to the problem domain description:

• Described the conceptual model of the array objects of social infrastructure, including the optimal GIS-system interfaces and the level of information support for people with disabilities. The analysis and comparison of experience in the creation of information resources published in the global Internet network, solving the problem of satisfaction information requirements of people with disabilities. [3, 4]

• Created structural-functional model of the GIS system. Proposed attributes, which allowed make group classes. Comparison of resources designed for satisfaction the information requirements of people with disabilities, revealed their positive and negative characteristics [5]. Formalized requirements to the functional of information resource for satisfaction information requirements of people with disabilities [6].

• Produced the practical research application. Designed and realized fully functional system, for example of the Khanty-Mansiysk city, published on the Internet. The system represents a socially oriented information resource with basis of GIS technologies, which display the part of the real world, describing a comprehensive assessment of the current fund urban development in terms of physical availability for people with disabilities. The system is available at www.geowheel.ru [2,7].

The purpose of this paper is an attempt to construct a mathematical model of socially-oriented GIS, designed to satisfy information requirements of people with disabilities. The research proposes to formalize the system of OSI and array of their attributes in terms of the theory of formal concept analysis (FCA) [5,7,8].

In the research have been created the binary matrix of objects and their attributes, and then using the Galois operator defined the array of formal concepts. For a visual representation of the results was used the Hasse diagrams to build formal concept grids, objects and their attributes. [9]

In this work the following notations have been introduced:

O – Objects of social infrastructure (OSI), for example shops, hospitals, municipal authorities, museums, theaters, etc.

P – Attributes, which describing OSI, for example, the number of floors in buildings or belongs the building to residential houses.

I – The relation between objects and their attributes, for example, the object endowed to certain attributes or not.

$I=\{<o_i, p_j>\}$

Couple $<o_i,p_j>$ shows that an object o_i endowed with p_j attribute.

The formal context is the three of $K=(O,P,I)$

The formal context can be presented in the form of a binary matrix (Table 1).

Table 1. Binary matrix of objects of their attributes

	P_1	P_2	P_3	P_j
O_1	1	1	0	…
O_2	0	1	1	…
O_3	1	1	1	…
O_i	…	…	…	…

Work supported by the Russian Foundation of Basic Research (project 15-41-00092).

The array of objects *O* and their attributes *P* (formal context) can be presented in the form of a binary matrix. Having two arrays (objects and their attributes), and the relationship between them, with the operators can generate Galois formal concepts.

Galois's operators:

For $A \subseteq O$ and $B \subseteq P$

$A' = \{p \in P \mid \forall o \in A: (o\ I\ p)\}$

$B' = \{o \in O \mid \forall p \in B: (o\ I\ p)\}$

A'- the array of attributes possessed by all objects of the array *A*.

B'- the array of objects that have all the attributes of the array *B*.

Formal concept (A, B) consists the array of objects $A \subseteq O$ and the array of features $B \subseteq P$, such that $B'=A$ and $A'=B$.

A - is called the volume of formal concepts (objects, written in front of all of the terms and concepts are less common than it is).

B - is called the content of the concept of formal concepts (signs written in front of this concept and the more general concepts).

The figure 1 shows a graphic representation array of objects and attributes on a plane and the areas reflecting array of formal concepts (marked with a stroke).

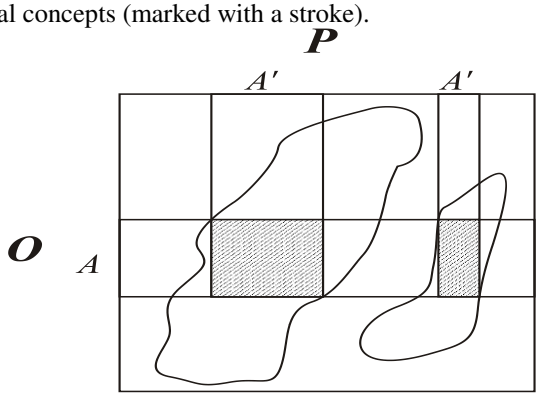

Figure 1. The areas of the formal concepts provided on the plane

The operator '' (double application of the operator ') is a closure operator: it is idempotent ($A'''' = A''$), monotone ($A \subseteq B$ implies $A'' \subseteq B''$) and extensive ($A \subseteq A''$). Array of objects $A \subseteq O$, such that $A'' = A$, called closed. Similarly, for the closed arrays of attributes - for subsets of *P*. For array of objects *A* array of their common features *A'* is a description of the similarity of objects from the array *A*, and the closed array *A''* is a cluster of similar objects (with a lot of common attributes *A'*). The relation "to be a more general concept" is defined as follows: *(A, B)* ≥ *(C, D)* if and only $A \supseteq C$. Formal context *K=(O,P,I),* ordered by inclusion volumes form grid *F (O,P,I)* is called a grid of concepts [9, 10].

Table 2. Binary matrix of objects and their attributes

	P_1	P_2	P_3	P_4
O_1	1	1	0	1
O_2	0	0	1	1
O_3	1	1	1	1
O_4	0	0	0	1

Creation of concepts grid begins with the removal of trivial attributes possessed by all objects (P_4), as well as those objects that do not possess any of considered trivial attributes (O_4). As a result of the reduction the binary matrix takes the form:

Table 3. Reduced binary matrix of objects and attributes

	P_1	P_2	P_3
O_1	1♥	1♥	0
O_2	0	0	1♣
O_3	1♥♦	1♥♦	1♣♦

Then there are the formal concepts, which is groups of objects and attributes that define each other mutually tightly. Formal concepts in the context matrix represent the maximum sub-matrix, which consisting of units, in this example the maximum submatrix *([o_1, o_3], [p_1, p_2])* and *([o_2, o_3], [p_3]).* Formal concept O_3 have volume *[o_1, o_3],* and the content of *[p_1, p_2, p_3]).* In Table 3, the formal concepts are marked by symbols ♥,♦,♣ in purple, red and green color, and presented in the Figure 2.

With Hasse diagrams, it is possible to present visually constructed grids of formal concepts. Hasse diagram is a graph of coating ratio "to be more general concept".

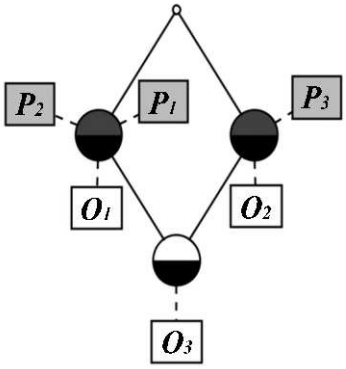

Figure 2. Example of formal concepts grid

Hasse diagram is a special case of a partially ordered array and has the following properties:

- some couples of elements are reduced in formal concepts;
- not every pair related among themselves, but only some elements of the pair;
- there are pairs not comparable among themselves;
- not all elements are comparable among themselves;
- there is at least one ordered pair.

Each top of a grid is a formal concept. Near the concept signs objects which are not in less general concepts (under this concept), and attributes which are not in less general concepts. Formal concepts can be found using the algorithm “circuiting one by one”. The function starts with the most general formal concept that includes all of the objects and often no attribute. Then there are all other concepts finds by recursive addition of attributes. The volume of a formal concept – all objects written opposite to this formal concept and all concepts, less general, than it. The content of formal concept - attributes written in front of this concept and more general concepts.

Let's consider creation a grid of formal concepts on the example of array of OSI and their attributes of the geographic information system designed to satisfy information requirements of people with disabilities.

Table 3. Table of OSI and attributes

№	*O* – Objects *P* – Attributes	P_1 Building	P_2 Road	P_3 Social	P_4 Residential	P_5 Available for disabled
O_1	County Hospital of Khanty-Masniysky region	1	0	1	0	1
O_2	Ugra State University	1	0	1	0	1
O_3	Trade center «Gostiny Dvor»	1	0	1	0	0
O_4	Mira street	0	1	1	0	1
O_5	Mira, str., 100	1	0	1	1	0
O_6	Traffic light Krasonarmeyskaya str./Chekhov, str.	0	1	1	0	1
O_7	Kalinina, str., 26	1	0	0	1	1

After accumulation enough base of social infrastructure objects and their attributes, you can build a graph using Hasse diagrams for visualization formal concept grids (Figure 3).

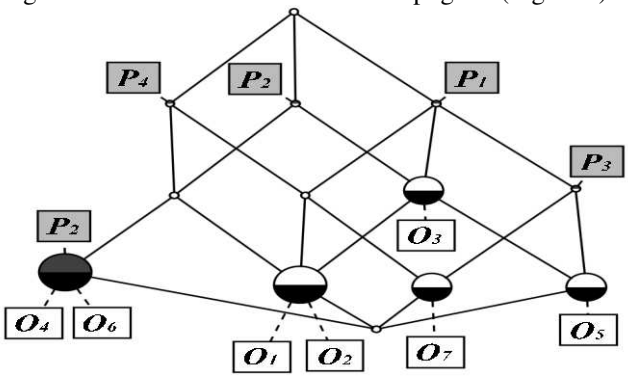

Figure 3. Formal concepts grid of geoinformational system objects

Obviously, the tops of a grid with the largest number of connections are formal concepts, created as a result of the construction of the graph. Each formal concept can assign the abbreviation. It is possible to appoint abbreviations of formal concepts in relation to GIS-system for physically disabled people as follows:

1. Objects of Social Infrastructure (OSI) not available to physically disabled people (O_3);
2. Objects of Social Infrastructure (OSI) available to physically disabled people (O_1, O_2);
3. Residential houses not available to physically disabled people (O_5);
4. Residential houses available to physically disabled people (O_7);
5. Locations available to physically disabled people (O_4, O_6).

Theoretical justification of approaches to the development of socially-oriented geographic information system based on FCA theory by creating dynamically updated thesaurus knowledge and understanding about objects of the subject area and presentation in the form of formal concept grids objects. This technique can be used in the creation of the subject area directory and the implementation possibility of heuristic search, in creation intelligent interface for instant obtaining permanently updating folksonomy data generated in accordance with the user request.

On the basis of this technique it is possible to solve problems of selection required geographical data, organization features intelligent search and other tasks for information support people with disabilities [11,12]. Developments in this area can be used as an additional tool for geospatial orientation, as a heuristic method to obtain data in information systems, as well as a mechanism to identify optimal zones, points and routes on the geographical area by the given request.

LITERATURE

[1] [Electronic resource] // World Report on Disability. Summary. 2011. WHO, the World Bank [official site] URL: http://whqlibdoc.who.int/publications/2011/9789240685215_eng.pdf?ua=1. (Date use the site: 12.15.2016).

[2] S.P. Semenov, V.V. Slavsky, A.O. Tashkin, Analysis of information resources designed to meet the information needs of people with disabilities. Bulletin of the Novosibirsk State University. Series: Information Technology. - 2016. - Volume 14, number 1. Issue - P. 115. - ISSN 1818-7900.

[3] [Electronic resource] // S.P. Semenov, A.O. Tashkin. Methodic of developing geoinformation system for people with limited mobility. Modern problems of science and education. - 2014. - № 1; [official site] URL: http://www.science-education.ru/115-12206 (Date use the site: 15.12.2016).

[4] S.P. Semenov, A.O. Tashkin. Development of geographic information systems to solve problems of information support of people with disabilities. Electronic collection of materials of the II International scientific-practical conference "The sixth technological structure: mechanisms and prospects of development" - Khanty-Mansiysk. Ugra State University Press, 2013. - 149p. p. 105 - 107

[5] S.P. Semenov, A.O. Tashkin. Application folksonomy approach in the development of socially-oriented geoinformation systems. Bulletin of the Ugra State University. - 2014, Issue 2 (33). p. 94-99 /

[6] S.P. Semenov, A.O. Tashkin. Interactive Geographic Information System for the physically challenged citizens. Collection of scientific papers of the international conference "Lomonosov Readings in Altay: fundamental problems of science and education", Barnaul, 20-24 October 2015. - Barnaul: Publishing House of the Altay University Press, 2015. - 3797 p. pp 1007-1010

[7] S.P. Semenov, A.O. Tashkin, S.P. Kononenko. Creating a socially-oriented geoinformation systems using opportunities of folksonomy approach. Proceedings of the III International scientific-practical conference sixth technological structure: mechanisms and prospects of November 13-14, 2015, Khanty-Mansiysk, Ugra State University

[8] B. Ganter, Wille R. Formal concept analysis: Mathematical Foundations. Berlin; New York:. Springer-Verl, 1999.

[9] Ignatov D.I., Kuznetsov S.O., Pulmans. Yo. Development of data resource sharing systems from three-concept to three-clusters // Mathematical methods of pattern recognition: The 15th National Conference. The collection of reports. - M:. MAKS Press, 2011. - 618 p.

[10] Ignatov D.I., Kaminska A.Yu., Bezzubtseva A.A., Blinkin K.N. Data analysis system collaborative platforms CrowDM // Image analysis, networking and text. Reports of the All-Russian scientific conference AIST'12. Models, algorithms and data analysis tools; results and opportunities for image analysis, networking and text. Ekaterinburg, 16 - 18 March 2012 - P. 16-26.

[11] Tashkin A.O. Automated system of providing services to the people of Khanty-Mansiysk city, - IV Proceedings of the International IT-Forum. Ekaterinburg, Ural University Publishing, 2011 -. P.192 -193.

[12] Tashkin A.O. Methods of creating a unified geospatial database for the study of the dynamics of environmental analysis of demographic and other processes. Materials of All-Russian scientific-practical conference "Geoinformation technologies in addressing environmental management tasks" / Ugra Research Institute of Information Technologies. - Khanty-Mansiysk: Yugrafika Ltd. Publisher, 2013. - 385 p, P. 39-41