PMU Based Linear State Estimator for Electric Power System

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Abstract—A suitable methodology is needed to determine the optimal allocations of PMUs in a power system. In addition to its ability to measure voltage and current phasors, a state-of-the-art PMU may include other features such as protective actions. The scope of the present paper is limited to the use of PMUs for state estimation. A power system is called completely observable only when all of its states can be uniquely determined.

The Phasor Measurement Unit (PMU) is considered to be one of the most important measuring devices in future for the power system. The distinction comes from its unique ability to provide synchronized phasor measurements of voltages and currents from widely dispersed locations in an electric power grid. The commercialization of the global positioning system (GPS) with accuracy of timing pulses in the order of one microsecond makes possible the commercial production of PMUs.

Traditional state estimators were non-linear in nature takes very high computation times. Hybrid state estimator with a measurement model combining the state-estimate from the classical state estimator with the direct state measurement from the PMUs was linear and the solution is direct and non-iterative. In Hybrid state estimator by addition of even a few PMUs can significantly increase the accuracy of the system state. Previous PMU placement research has focused primarily on network topology, with the goal of finding configurations that achieve full network observability with a minimum number of PMUs due to high cost of PMUs. The cost associated with these devices has fallen over the last several decades as computer & GPS technology has improved while the cost of PMUs has decreased.

Index Terms—PMU, GPS, Measurement Unit, Electric Power

I. INTRODUCTION

Rising dependence of human society on electrical energy is associated with a desire for reliability of the highest order in power system. Over the years, we have grown to depend on the near perfect reliability of these systems that have become a necessary part of our everyday lives. All of our household appliances, communication devices, and almost all of our tools ranging from construction sites to our offices require electricity for operation. It is not as if we assume electricity will always be available, it is that we believe electricity will always be available.

State estimation is facet of electric power engineering that has evolved out of these needs. State estimator uses information from various measurement units and monitoring systems to estimate the states of a power system. Until recently, it was not possible to measure phase angles of the bus voltages in real time due to the technical difficulties in synchronizing measurements from distant locations. The advent of phasor measurement units (PMUs) alleviated this problem by synchronizing the voltage and current waveforms at widely dispersed locations with respect to a Global Positioning System (GPS) clock [1]. A suitable methodology is needed to determine the optimal locations of PMUs in a power system. In addition to its ability to measure voltage and current phasors, a state-of-the-art PMU may include other features such as protective actions. The scope of the present paper is limited to the use of PMUs for state estimation. A power system is called completely observable only when all of its states can be uniquely determined [2], [3].

The major objective of this paper is to outline what is being done in the field of state estimation using PMU data exclusively by presenting the work completed. The end goal of the project is to development of the linear state estimation algorithms for a three phase tracking linear state estimator and then use this state estimator to drive several other applications control center including instrument transformer calibration and intelligent islanding. The topology processor, which will take breaker status and line current phasors and determine the most up-to-date topology of the network, and the actual three phase linear state estimator which will use PMU data to determine the state of the system.

This paper discusses the background information on the topic of three phase linear state estimation and then presents the work done towards developing the state-estimator. It begins with a brief history of the significance of power system state estimation followed by the presentation of the traditional form of state estimation. It continues with the development of the linear state estimation equations and then investigates the planned to design mathematical algorithms for a three phase linear state estimation and the associated research and software development.

II. PHASOR MEASUREMENT UNIT

Phasor Measurement Unit (PMU) – a device which by employing widely used satellite technology offers new opportunities in power system monitoring, protection, analysis and control. PMUs facilitate innovative solutions to traditional utility problems and offer power system engineers a whole range of potential benefits, including precise estimates of the power system state can be obtained at frequent intervals, enabling dynamic phenomena to be observed from a central location and appropriate control actions taken. Post disturbance analysis will be much improved with the precise snapshots of the system states through GPS synchronization. Advanced protection based upon synchronized phasor measurements could be implemented.
with options for improving overall system response to catastrophic events. Advanced control using remote feedback becomes possible, thereby improving controller performance. Bock diagram of a PMU shows in figure 1. In the following figure shows anti-aliasing filters is meant to filter out from the analog input waveform frequencies more than the Nyquist rate. The other block is phase-locked oscillator (PLL) converts the GPS (Global Positioning System) pulse per second into the sequence of high-speed timing pulses used in waveform sampling. The microprocessor performs the FFT Phasor calculations.

**PMU based Linear State Estimator for Electric Power System**

PMU measures voltage and current phasor in a power system. Synchronism among phasor measurements is achieved by same time sampling of voltage and current waveforms using a commonsynchronizing signal from the global positioning satellite[8]-[10]. PMUs have higher accuracy than conventional measurements. They reduce effects of time skew among measurements, useful for many other applications such as system protection, control and stability assessment, aiding topology error identification, parameter error detection and correction and improve accuracy of state estimation. The Optimal placement of PMU becomes an important problem to be solved in power system state estimation. The PMU placement problem is formulated as a binary integer linear programming, in which the binary decision variables (0, 1) determine whether to install a PMU at each bus, while preserving the system observability and lowest system metering economy. It is neither economical nor necessary to install a PMU at every node of a wide-area interconnected network.

The cost of a PMU depends on a number of factors, including the number of measuring terminals (channels), CT and PT connections, power connection, station ground connection, and GPS antenna connection. The main purpose of performing PMU placement problem is to minimize the number of installed PMUs, so that for an n-bus system the optimization problem is given as:

\[ \text{Minimize} \sum_i W_i x_i \]

Subject to \( f(X) \geq 1 \)

Where \( W_i \) is the installation cost of the PMU at bus i.

Assume \( W_i = 1 \), for a PMU installed at bus i

\[ x_i = 1, \text{if a PMU installed at bus i} \]

\[ 0, \text{Otherwise} \]

\( f(X) \) is a vector function representing the constraints is a vector whose entries are all equal to 1.

**PMU PLACEMENT RULES**

The following PMU placement rules are as follows:

**Rule 1:** Assign one voltage measurement to a bus where a PMU has been placed, including one current measurement to each branch connected to the bus itself.

**Rule 2:** Assign one voltage pseudo-measurement to each node reached by another equipped with a PMU.

**Rule 3:** Assign one current pseudo-measurement to each branch connecting two buses where voltages are known. This allows interconnecting observed zones.

**Rule 4:** Assign one current pseudo-measurement to each branch where current can be indirectly calculated by the Kirchhoff current law. This rule applies when the current balance at one node is known, i.e. if the node has no power injections (if N-1 currents incident to the node are known, the last current can be computed by difference).

**III. PMU FOR STATE ESTIMATION**

The PMUs receive signals from GPS satellites, and provide synchronized measurements from different locations to the desired destination, commonly known as the phasor data concentrator (PDC) [9]. The measurement data can be used for wide area monitoring; real time dynamics and stability monitoring; dynamic system ratings; and improvements in state estimation, protection, and control.

A state estimator estimates the voltage magnitudes and phase angles at the buses by using the available measurements in the form of power injections, power flows, voltage magnitudes, or current through the branches. The voltage and current phasors measured by the PMUs can be used in a state estimator in two ways [10]:

1) To increase the confidence in the available measurements.

2) To replace the available conventional measurements.

When a PMU is placed at a bus, it can measure the voltage phasor at that bus, as well as at the buses at the other end of all the incident lines, using the current phasor and the known line parameters. It is assumed that the PMU has a sufficient number of channels to measure the current phasors through the entire branches incident to the corresponding bus. It is to be noted here that the errors in the voltage and current measurements by the PMU and the transmission line parameters induce uncertainties in the estimated voltage phasor at the other end of the line [11]. The measurement uncertainty can be computed by the use of the classical propagation error theory, making use of the maximum measurement uncertainties provided by the manufacturer and the maximum measurement uncertainty in the line parameters [12]. The estimated quantity can be expressed as a function of the measurements and the parameters, \( X_1, \ldots, X_n \), i.e. \( y = f(x_1, x_2, \ldots, x_n) \), where \( y \) is the estimated bus voltage magnitude or the estimated phase angle and are the measurements or parameters. The measurement uncertainty can be expressed as
\[ \Delta y = \sum \left| \frac{\partial f}{\partial x_i} \right| \Delta x_i \]

Where \( \Delta x_i \) is the maximum uncertainty associated with the quantity \( x_i \).

**STATE ESTIMATION**

A phasor measurement unit (PMU) [7] equipped with a GPS receiver allows for synchronization of measurements, yielding accurately measured and time-stamped voltage phase angles. The general conclusion is that PMUs [6] have greatly improved observability and accuracy of voltage angle estimates. Despite some opinions to the contrary, PMUs will not make state estimation obsolete even if they are available at every bus in the system. As we know, measurements are not perfect; thus a redundant set of measurements will still be needed in order to identify bad data. All of these measurements can be considered dynamic since snapshots are performed every few seconds. The status of the assets (line status, breaker status etc.) as well as network parameters can be considered as static measurements. The network topology processor in Fig. 2 determines the topology of the network from the telemeter status of circuit breakers. Having an observable set of measurements is a necessary, although not sufficient condition, for EMS computer applications. While it is desired, coordination across the network quite often does not happen in real-time. The reasons for not having real time-model are varied. While many control and monitoring functions is computer based, there are still functions handled by telephone calls between the system operator and utility control centers. It is a well known fact that control room operators have a focused view of the world. Once the problem is identified, the state estimator could function with only PMU measurements. Improvement in state accuracy could benefit from the more accurate and more frequent time-tagged information of voltages and currents from phasor measurement units (PMUs). The improved state estimation would eliminate a lot of uncertainties associated with its value, thereby allowing larger power transfers, or provide a more rational basis for refusing transactions. Prior art in accounting for phasors in classical state estimation requires appending the measurement vector with bus voltage angle measurements. Classical state estimation (SE) with synchronized phasor measurements in a non-invasive fashion. The classical SE measurement vector, (the P and Q flows, and voltage magnitudes); rather they are integrated with the calculated SE estimates by means of a linear state estimation equation post priori. This proposed integration model improves on prior phasor-in-state estimation methods by accounting for the full set of current phasor data. This hybrid weighted least squares (WLS) and linear SE model could also be performed in wide area measurement system platforms and does not require communicating PMU data via SCADA channels [4].

State Estimation [5, 8] is the process of assigning a value to an unknown system variables based on the measurements obtained from the system. State estimator has been widely used as an important tool for online monitoring, analysis and control of power systems. It is also exploited to filter redundant data, to eliminate incorrect measurements and to produce reliable state estimates. Entire power system measurements are obtained through RTU (Remote Terminal Unit) of SCADA systems [4] which have both analog and logic measurements.

![State Estimation block diagram](image)

**IV. LINEAR STATE ESTIMATOR**

PMU measurements may be included by a slightly different formulation of the traditional non-linear weighted least squares or they may be taken into consideration after a preliminary system state has already been determined. Even a small number of these precise measurements can weigh heavily on the accuracy of the overall state of the system. However, a true application of PMU technology to state estimation would have all of the traditional measurements of real and reactive power injections and current and voltage magnitudes replaced by bus voltage phasors and line current phasors. If only PMU measurements are used, there are also no complications from the use of both polar and rectangular values in the state estimation process, as would be done when including PMU measurements in traditional state estimators. If a state estimator could function with only PMU measurements as inputs then many issues associated with traditional state estimators could be resolved. Because PMUs are synchronized with GPS, the problem of scan time becomes irrelevant. One could imagine looking at the state of the power system with a traditional state estimator versus one which used explicitly synchronized PMU data and comparing it to putting on a pair of spectacles for the first time and finally having a focused view of the world. Once the problem of scan time has been erased, the only issue of time is the communication and computational delay between the collection of the measurements and the employment of useful information for decision making by the operation and control applications. Additionally, when using PMUs as metering devices, the state of the system is actually being directly measured. However, estimation is still necessary for including redundancy and bad data filtering. Because of this, the placement of the PMUs is critical for achieving a fully observable system with a sufficient amount of measurement.
Mathematical Modelling of 5-Bus System

Simple 5-bus system used as an example network in this work is given below:

![Simple 5-bus network](image)

**Figure 3: Simple 5-bus network**

Branch impedances of example network:

<table>
<thead>
<tr>
<th>From Bus</th>
<th>To Bus</th>
<th>R(p.u.)</th>
<th>X(p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0.01</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.01</td>
<td>0.07</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.03</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Table 1: Branch impedances of example network**

The bus admittance matrix will then take the following form. The rules for populating the admittance matrix of a network are derived from Kirchhoff’s laws of current injections into a node. Sparing the derivation of the equations, there are two simple rules that can be used to construct the network admittance matrix by inspection.

1. The $i$th element of the admittance matrix is the sum of the admittances of all of the lines connected to bus $i$.
2. The $ij$th element of the admittance matrix is the negative of the admittance connecting bus $i$ to bus $j$.

The shunt impedances $Z_{ij}$ from each of the components are added to the diagonal elements corresponding to each of their respective busses in form of the shunt admittance $Y_{ij}$:

$Y_{ij} = 1/Z_{ij}$ (1)

$Z_{ij} = R_{ij} + X_{ij}$ (2)

$Y_{12} = 1/Z_{12} = 0.02 + 0.06i$

$Y_{13} = 1/Z_{13} = -5.0 + 15.0i$

$Y_{ij} = g_{ij} + jb_{ij}$ (3)

$y_{ij} = g_{ij} + jb_{ij}$ (4)

Bus 1 is not directly connected with Bus 3.

Then,

$Y_{13} = 0$

For all other branches we can calculate the value of admittance

$I = \begin{bmatrix}
  Y_{11} & Y_{12} & \cdots & Y_{1N} \\
  Y_{21} & Y_{22} & \cdots & Y_{2N} \\
  \vdots & \vdots & \ddots & \vdots \\
  Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix} = Y \times V$

the network admittance matrix $Y$, $Y = \begin{bmatrix}
  6.5 - 27.3i & -5.0 + 15.0i & -1.5 + 12.3i & 0 & 0 \\
  -5.0 + 15.0i & 12.7 - 36.2i & -4.0 + 8.0i & 0 & -3.7 + 13.2i \\
  0 & -4.0 + 8.0i & 6.0 - 22.0i & 0 & -2.0 + 14.0i \\
  -15 + 12.3i & 0 & 0 & 4.8 - 22.3i & -3.3 + 10.0i \\
  0 & -3.7 + 13.2i & -2.0 + 14.0i & -3.3 + 10.0i & 9.1 - 37.2i
\end{bmatrix}$

V. RESULTS

There are four main database files that represent the system model: the series impedance input file, the shunt susceptance input file, and the voltage measurement and current measurement tables. These four files contain all of the necessary indexing and topology information necessary to construct the system matrices discussed in the previous sections. Additionally, the formatting of the input files which contain the raw measurements from the field must be constructed in a predictable way such that the computer program will interpret them in the same way each time it is read. The output of each of the estimator applications must also be constructed in a similar manner so that the human-machine-interface (HMI) can present the information to the user in a valuable and accurate format.

STATE ESTIMATOR TESTING RESULTS

The desired function of any state estimator is to filter out the measurement error using redundancy and knowledge about the measurement errors. The testing script used to test the Matlab application of the three phase linear state estimator using the data created by the test to demonstrate this in several ways.

1. The true value and the estimated value of a single state variable are plotted on the complex plane for each iteration of the testing procedure. The raw voltage measurements corresponding to that particular state variable are also included on the same plot. This type of plot shows the ability of the state estimator to filter out measurement error and one in on the true value of the state variable.

2. A different plot which shows the estimators ability to filter out measurement error from the entire state vector. These plots only show results from a single iteration. Since the true value of the system state is known from the load flow, it is compared to the corresponding voltage measurements and estimated state vector and plotted to show the effects. This is done in four different plots: a plot showing the effects on the real part of the state variable, a plot showing the effects on the imaginary part of the state variable, a plot showing the effects on the magnitude of the state variable, and a plot showing the effects on the angle of the state variable.

Figure 4 and Figure 5 are two plots showing a single state variable over 100 iterations of the testing script. This particular state variable corresponds to a substation which is
monitored by a PMU. Figure 4 shows in green the true value of the state variable and in blue the estimated values of the state variable for 100 iterations. Figure 5 shows in blue the estimated values of the state variable and in red the voltage measurements that correspond to this particular state variable. Both of these plots are from the same set of iterations so they can be visually compared to each other. These plots show very clearly the ability of the estimator to filter out measurement errors and determine the best estimate of the system state. It can be seen from Figure 6 that the true value of the state variable differs from the estimated value. Figure 7 and Figure 8 are the same type of plot but show a state variable which corresponds to a substation which is not monitored by a PMU.

The next set of plots show the effects of the estimation process on the entire state vector for a single iteration of the testing script. The absolute value of the difference between the voltage measurements and the actual state vector are shown in red. The absolute value of the difference between the estimated state vector and the actual state vector are shown in blue.
It can be seen from these plots that the state estimator can effectively filter out measurement normally distributed random additive errors and estimate with a good degree of precision the system state. In the case of all four comparisons, the estimator has the ability to come within approximately 0.002 of the true value of each of the parameters.

The database files used by the topology processor application include a current measurement lookup table, a breaker location lookup table, and a bus/branch model describing the full network. This section describes the composition of each of these database files and provides examples. The current measurement lookup table for the topology processor is not as detailed as the current measurement lookup table for the state estimator. This lookup table only contains the current measurement numbers and the location of each of those current measurements in the network. The location is given as the line number the measurement originated from. Below is an example of the current measurement lookup table 2 for the topology processor application. This table corresponds to the simple 5 bus system in Figure 3.

<table>
<thead>
<tr>
<th>Current Mess</th>
<th>Line Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2: current measurement location

The breaker location lookup table is slightly more complicated. It is organized by the line numbers of each branch in the network. Following each line number is a series of breaker numbers which all must be “open” in order for the line to be out of service. For this application it has been assumed that no more than four breakers would be required to open any particular line. Other breaker configurations in the substation could yield lesser numbers. In this case, the unfilled slots in the input file should be filled with zeros. Below is an example of the breaker lookup table for the topology processor application. This also corresponds to the simple 5 bus system in Figure 3.

<table>
<thead>
<tr>
<th>Line Number</th>
<th>BS1</th>
<th>BS2</th>
<th>BS3</th>
<th>BS4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
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<td>2</td>
<td>3</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
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<td>6</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>9</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>12</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 3: Breaker Lookup Table

Finally, it is valuable for the topology processor to have access to the bus/branch model of the full network. This is contained in another database file and is formatted as follows.

Full Network Bus/Branch Model:

<table>
<thead>
<tr>
<th>Line Number</th>
<th>From Bus</th>
<th>To Bus</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Full Network Bus/Branch Model
CONCLUSION

The ability of the state estimator to achieve a high level of efficiency and numerical robustness is of paramount importance in today’s electric utility industry. A robust algorithm must be globally convergent/convergent from any starting point), and able to solve in practice both well-conditioned and ill-conditioned problems.

The objective is to provide a more reliable and robust state estimator, which can successfully cope with all kinds of errors (bad data, topological, parameter) faced in power system models.

An important advantage of this method is non-iterative which reduces the computation time and suitable to implement in the near future. In view of extension in this work, analysis can be done taking into account the loss of measurements, failure of PMU and effectiveness of PMU. With the increasing use of computers and digital information in power systems, the availability of information about a system has become easier. Online data, artificial neural networks, fast computations and real time control are beginning to take over the theoretical and analytical solutions. However, the conventional methods of power system analysis still provide the element of theory to the latest methods of computations and estimations. Within this environment, there will be an increasing need of more and more such application oriented studies which can help minimize the resources and computation time. This study was intended to show that a small amount of real time information with limited resources can be expanded into a bigger picture of the whole system with simple relationships as derived herein.

In three phase linear state estimator feedback of data comes from PMU with help of communication system. When there is any problem find found in PMU or communication system it them there is no feedback of data. Thus there should be one more source of data is required. At present we get non-linear feedback of data with help of SCADA. By using these two methods, in future we can develop a logical state estimator for overcome these type of problem

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