The Examination of the Halo Nucleus Properties of ${}^{6\text{-}11}$Li Isotopes According to the Shell Model

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***Abstract*— Halo nuclei are separated into two as two or one neutron halo nuclei depending on the nucleon number taking place in their final orbits. These cores have three mass configurations as neutron-neutron-core (n+n+c). One neutron halo nuclei have two-mass configuration (n+c). Two mass system could be reduced to one mass system by accepting the last two neutrons of two halo nuclei called as dineutron as a single particle also by taking the phenomenon of pairing between the nucleons into consideration. In this way, it becomes easier to solve the cores transforming into one neutron halo state. The firstly observed core of this mysterious nuclear structure called as neutron halo is ^{11}Li and it still keeps its secrecy. The purpose of this study is the examination of the halo nucleus properties of ${}^{6\text{-}11}$ Li isotopes according to the shell model. To reach this purpose in the study, the halo nuclei of all Li isotopes have been suggested in different possible combinations and the ground states and excitation energies belonging to each state have been calculated.**

***Index Terms*— Halo nuclei, shell model, ${}^{6\text{-}11}$ Li isotopes, excitation energy**

I. INTRODUCTION

It is known that the neutrons and protons within the atom nucleus have been arranged with different combinations. Deviation is observed from the stability zone in the nuclei both with neutron excess and proton excess. The zero-energy line in Z-N plane is called drip line [1]. In the experimental studies conducted with radioactive nuclear beams, exotic nucleus structures have been met in the nuclei close to the drip line. Some neutrons or protons forming the nucleus may classically pass to the forbidden zone. This state that could also be considered as nuclear stratosphere zone is called as halo state. The excitation energies of the final nucleon or nucleons are too little in the nuclei close to the drip line [2-3-4]. If 6-8 MeV in the stable nuclei are met, these are around 1 MeV. Neutron intensity distribution also shows a long tail. This situation is the neutron halo state [4-5-6]. Although the mass intensity of a halo nucleus is too little, it affects the reaction impact sections very much. These and similar properties are the supports sufficient for revealing the new properties of such nuclei.

The first nucleus observed as the halo nucleus is ^{11}Li nucleus having neutron halo. The secret structure of ^{11}Li still keeps its secrecy. Some of the nuclei considered as neutron halo are ^{6}He, ^{11}Be, ^{14}Be and ^{17}B [7-8-9]. The nuclei with neutron excess are called neutron halo (such as ^{6}He, ^{8}He, ^{14}Be, ^{9}Li, ^{11}Li and ^{11}Be) and the nuclei with proton excess are called proton halo (such as ^{14}B, ^{9}C, ^{17}F, ^{17}Ne and ^{12}N) [10].

The nucleon number in the outside of the core in halo nuclei specifies how many halos that nucleus has. For instance; ^{11}Li isotope is a two neutron halo if ^{9}Li is taken as a core. Two neutron halo nuclei have three-mass configuration (n+n+core) [11-12]. One halo nuclei could be explained with two-mass (n+core) system [13-14].

In the study, possible different combinations of all Li isotopes and ^{11}Li halo nucleus have been suggested, their ground state and excitation energies have been calculated and their comparisons have been conducted with experimental studies.

II. THEORY

2.1. Neutron halo

Most of the studies related to halo nuclei have been conducted on neutron halo. The nuclei taking place in the drip line zone, which are light, unstable and with neutron excess are generally considered as neutron halo nuclei. There are various nuclei in the neutron halo state, but the nucleus which has been studied most is ^{11}Li. The first indication of the halo structure was understood with the study conducted on ^{11}Li isotope by Tanihata in 1985 for the nuclei close to the neutron drip line. The nuclei such as ^{6}He, ^{11}Be, ^{14}Be and ^{8}He out of the isotopic nuclei whose mass number is 6,8,11 and 14 are also the important neutron halo nuclei on which studies have been conducted. Moreover; there are also many nuclei candidate for the neutron halo in the drip line zone [15].

Halo nuclei are separated into two as two or one neutron halo nuclei depending on the nucleon number taking place in their final orbits. For instance; ^{11}Li isotope becomes two neutron halos if ^{9}Li is taken as a core, ^{11}Be becomes one neutron halo nucleus if ^{10}Be is taken as a core. The nuclei with neutron excess such as ^{6}He, ^{8}He, ^{14}Be and ^{17}B are two neutron halo nuclei due to the fact that they have two neurons in their final orbital. These nuclei have three mass configurations as neutron-neutron-core (n+n+c). One neutron halo nuclei have two-mass configuration (n+c). Two mass system could be reduced to one mass system by accepting the last two neutrons of two halo nuclei called as dineutron as a single particle also by taking the phenomenon of pairing between the nucleons into consideration. In this way, it becomes easier to solve the nuclei transforming into one neutron halo state [16].

2.2. Binding and Excitation Energies

Binding energy is the negative of the energy necessary to separate the E^{b} core into its protons and neutrons. The highest value of the binding energy is when the nucleus is in ground state. $E_{x}(n)$ excitation energy of n. excited state is found from E^{b} (n) binding energy.

$$E_{x}(n)=E^{b}(n)-E^{b}(0) \qquad (2.1)$$

$E^{b}(0)$ is the ground state energy. There are many terms contributing to the total binding energy of the nucleus;

$${}^{b}(core+p^{2})=2e\ +\ {}^{(1)}(\ \ {}^{2})+\ {}^{b}(core) \qquad (2.2)$$

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Here; 2e ρ is the negative of the energy necessary for extracting two particles from the potential well. It is accepted that two particles move independently in ρ orbit. This potential well is not dependent on the number of the particles out of the core.

The contribution to the binding energy occurring from the mutual interaction of two external-core particles is shown with $E_\Gamma^{(1)}(\rho^2)$. This term is not only dependent on ρ, it is also dependent on J_Γ and T_Γ. E^b(core) represents the binding energy of the particle in the core.

If the closed shell core is accepted as inert, E^b(core) term is stable, Hamiltonyen of the system with (core+two nucleons) is;

$$H=H_{core}+H_{12} \tag{2.3}$$

$$H_{kor}=\sum_{k=3}^{A}[T(k)+U(k)]+[\sum_{3=k<\lambda}^{A}W(k,\lambda)-\sum_{k=3}^{A}U(k)] \tag{2.4}$$

$$H_{12}=\sum_{k=1}^{2}[T(k)+U(k)]+[\sum_{k=1}^{2}\sum_{\lambda=3}^{A}W(k,\lambda)+W(1,2)-\sum_{k=1}^{2}U(k)] \tag{2.5}$$

Here; H$_{core}$ represents the interaction between the core particles (k=3,...A). If the core is accepted as inert, the contribution of H$_{core}$ to total energy is stable. H$_{12}$ term expresses the contribution coming from two extra-core particles.

If there are more than two nucleons outside the core; for instance, in n particle ρ shell and m particle λ shell, the binding energy is;

$$^{b}(core+\ ^{n}+\ ^{m})=\ _{c}+\ ^{b}(core)+ne\ +me\ +\ ^{1}(\ ^{n}\ \ ^{m}) \tag{2.6}$$

Here; the final term is the interaction of those more than two outside the core. Two masses are calculated with the matrix element.

$$E_\Gamma^{(1)}(\rho^n\lambda^m)\equiv\rho^n\lambda^m\left|\sum_{1=k<\lambda}^{n+m}V(k,\lambda)\right|(\rho^n\lambda^m>_\Gamma \tag{2.7}$$

$$=\sum_{\Gamma}c_\Gamma,E_\Gamma^{(1)}(\rho\lambda)$$

$$E_\Gamma^{(1)}(\rho\lambda)=<\rho,\Gamma|V(1,2)|\rho\lambda>_\Gamma \tag{2.8}$$

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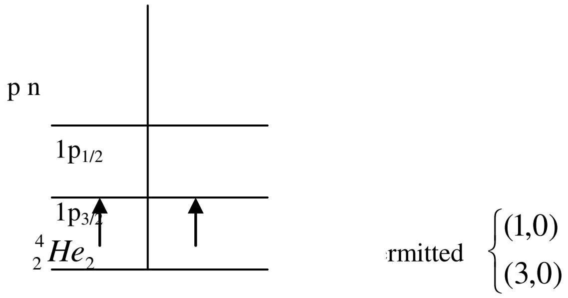
III. CALCULATIONS

3.1. $^{6}_{3}Li_3$ Isotope



(1p$_{3/2}$)^p (1p$_{3/2}$)^n, (1,0) state;

$$E_\Gamma^b(^4He+1p_{3/2})=(p)e_{p_{3/2}}+(n)e_{p_{3/2}}+E_\Gamma(1p_{p_{3/2}})+E^b(^4He)$$

$$=E^b(^4He)+(p)e_{1p_{3/2}}+(n)e_{1p_{3/2}}+\left\langle(1p_{3/2})^2\middle|V(1,2)\middle|(1p_{3/2})^2\right\rangle_{10}$$

$$E_\Gamma^b\text{=-0,53(1,2)-3(1,4)+0,2+0,887+1,967-28,297=-30,079MeV}$$

(1p$_{3/2}$)^p (1p$_{3/2}$)^n, (3,0) state;

$$E_\Gamma^b(^4He+1p_{3/2})=E^b(^4He)+(p)e_{p_{3/2}}+(n)e_{p_{3/2}}+\left\langle(1p_{3/2})^2\middle|V(1,2)\middle|(1p_{3/2})^2\right\rangle_{30}$$

(1p$_{3/2}$)^p (1p$_{3/2}$)^n, (1,0) state;

$$\boldsymbol{E_\Gamma^b=E^b(^4He)+(p)e_{p_{3/2}}+(n)e_{p_{3/2}}}+\left\langle(1\boldsymbol{p}_{3/2})^2(1\boldsymbol{p}_{1/2})^2\middle|\boldsymbol{V}(1,2)\middle|(1\boldsymbol{p}_{3/2})^2(1\boldsymbol{p}_{1/2})^2\right\rangle_{10}$$

$$E_\Gamma^b=-1{,}6A_0-3B+C+(p)e_{1p_{3/2}}+(n)e_{1p_{1/2}}$$

(1p$_{3/2}$)^p (1p$_{3/2}$)^n, (2,0) state;

$$\boldsymbol{E_\Gamma^b=E^b(^4He)+(p)e_{1p_{3/2}}+(n)e_{1p_{3/2}}}+\left\langle(1\boldsymbol{p}_{3/2})^2(1\boldsymbol{p}_{1/2})^2\middle|\boldsymbol{V}(1,2)\middle|(1\boldsymbol{p}_{3/2})^2(1\boldsymbol{p}_{1/2})^2\right\rangle_{20}$$

$$E_\Gamma^b=-1{,}2A_0-3B+C+(p)e_{1p_{3/2}}+(n)e_{1p_{1/2}}$$

When (1p$_{3/2}$)^p (1p$_{3/2}$)^n, (1,0) state and (1p$_{3/2}$)^p (1p$_{3/2}$)^n, (2,0) states are examined; it is necessary to know the excited states of ^{5}He and ^{5}Li to calculate the binding energies of (p)e$_{p1/2}$ and (n)e$_{p1/2}$ (proton and neutron). However; both experimentally and theoretically excited states of these isotopes have not been observed. Therefore; there are the levels of ^{6}Li excited only in 1$^+$ and 3$^+$ states and this has already been experimentally proven.

$$E_\Gamma=-0{,}53A_o-3B+C+(p)e_{1p_{3/2}}+(n)e_{1P_{3/2}}=-1{,}782MeV$$

$$E_\Gamma=-1{,}2A_o-3B+C+(p)e_{1p_{3/2}}+(n)e_{1P_{3/2}}=-2{,}586MeV$$

Similar calculations have been conducted for also other Lithium isotopes and the results taking place in Table 3.1 have been reached.

IV. RESULTS AND DISCUSSION

While the studies related to halo nuclei continue in full course, one of the nuclei which are most important and still worked-on is ^{11}Li and its isotopes. Binding and excitation energies of Lithium isotopes are a topic studied by the physicists. This study contains the calculation of the binding and excitation energy levels of Lithium isotopes. The energy levels of ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{10}Li and ^{11}Li nuclei have been calculated by taking into consideration some modifications conducted on the shell model. In this respect;

1. It is known that ^{6}Li nucleus is an extremely stable one and it has an experimentally excited energy level. In this study, ^{4}He nucleus has been taken as the core to calculate the energy level of ^{6}Li and an energy level has been found as a result of the calculations with the assumption that one proton and one neutron have

remained out of the core. Calculations have been made by taking the permitted states into consideration and with the assumption that both of the proton and neutron remaining out of the core are at $1p_{3/2}$ level in this nucleus.

2. ^{5}Li isotope has been taken as the core for the purpose of calculating the excited energy levels of ^{7}Li isotope and it has been assumed that two neutrons have remained outside the core. All the probabilities have been calculated by taking the levels neutrons may be at into consideration. Experimentally; the ground state of ^{7}Li isotope is $\frac{3}{2}$ and excited state is in $\frac{1}{2}$ level. There are three excited states of ^{7}Li isotope in the calculations in this study. These levels are $\frac{1}{2},\frac{3}{2}\,and\,\frac{1}{2}$. Both of these neutrons may be existent in $1p_{3/2}$ and the permitted states are (0,1) and (2,1); other probability is in $1p_{1/2}$ level in both neutrons and another probability is that one neutron may take place in $1p_{3/2}$ and one may take place in $1p_{1/2}$. Energies have been calculated for J and T values by taking these probabilities into account.

3. ^{6}Li has been taken as the core in the calculations in ^{8}Li and it has been accepted that two neutrons have remained outside the core. Both of these neutrons may take place in $1p_{3/2}$ and its permitted state may be (0,1) or both neutrons may take place in $1d_{5/2}$ level and their permitted states may be (0,1) and (2,1); another probability is that one of the neutrons may be in $1p_{3/2}$, the other one may take place in $1p_{1/2}$ level and the permitted states become (1,1) and (2,1). In the final probability, one of the neutrons takes place in $1p_{3/2}$ and the other one takes place in $1d_{5/2}$ level and the permitted states are (1,1), (2,1), (3,1) and (4,1).

4. ^{7}Li has been taken as the core to calculate the excited energy levels of ^{9}Li isotope and it has been accepted that two neutrons have remained outside the core. Both of these neutrons may be in $1p_{3/2}$ level and the permitted states for this level are (0,1) and (2,1). Another level in which neutrons may be existent is $1p_{1/2}$ and the permitted state for this level is (0,1). Another probability is that one of the neutrons may be in $1p_{3/2}$ and the other one may be in $1p_{1/2}$ and the permitted states for these levels are (1,1) and (2,1).

5. In this study, the excited energy levels of ^{10}Li have been calculated with the use of the energy level calculation method of other nuclei. Namely; ^{8}Li nucleus has been taken as the core and it has been accepted that two neutrons remain outside the core. The levels in which these two neutrons may take place either in $1p_{1/2}$ and their permitted states are (0,1), (2,1) and (4,1); or, both of them will be in $1d_{5/2}$ level and their permitted states are (0,1), (2,1) and (4,1); one of other levels neutrons may take place is $2s_{1/2}$ and the permitted state is (0,1). One of these neutrons may take place in $1p_{1/2}$ and the other one may take place in $1d_{5/2}$ and their permitted states are (2,1) and (3,1). One of other probabilities is in $1p_{1/2}$ and $1d_{5/2}$ and their permitted states are (2,1) and (3,1). One of other probabilities is that one of the neutrons is in $1p_{1/2}$ and the other one of them is in $1s_{1/2}$ and the permitted states are (0,1) and (1,1); in final estimation, one of the neutrons may take place in $1d_{5/2}$ and the other one may be in $1s_{1/2}$ and their permitted states are (2,1) and (3,1). In this study, calculations have been made by taking all of these states into consideration.

6. The excited levels of ^{11}Li nucleus are one of the issues recently studied and examined most and it still continues. In this study; ^{9}Li has been taken as the core for the purpose of calculating the energy levels of ^{11}Li and it has been accepted that two neutrons have remained outside the core. It has been accepted that this neutron has remained outside the core. These neutrons may be in the same or different levels just like other isotopes. They could be ordered as follows; both neutrons are in $1p_{1/2}$ and this level is permitted in (0,1) state, both neutrons are in $1d_{5/2}$ and the permitted states for this level are (0,1), (2,1) and (4,1). Both of these neutrons may take place in $2s_{1/2}$ and (0,1) state is permitted. If one neutron is in $1p_{1/2}$ and the other one is in $1d_{5/2}$, permitted state becomes (2,1) and (3,1); or if one of them is in $1p_{1/2}$ an the other one is in $2s_{1/2}$, the permitted states are (0,1) and (1,1); in another probability, one neutron is in $1d_{5/2}$ and the other one is in $2s_{1/2}$ and their permitted state is (2,1) and (3,1).

Table 3.1 Energies of Li isotopes calculated according to the related core

Configuration ^{6}Li	**J^π**	**T**	**Binding energies according to ^{4}He core (MeV)**	**Excitation Energies (MeV)**
$(1p_{3/2})^2$	1^+	0	-1,782	0
$(1p_{3/2})^2$	3^+	0	-2,586	-0,804
Configuration ^{7}Li	**J^π**	**T**	**Binding energies according to ^{5}Li core (MeV)**	**Excitation Energies (MeV)**
$(1p_{3/2})^2$	0	1	-14,9	0
$(1p_{3/2})^2$	2	1	-9,188	5,712
$(1p_{1/2})^2$	0	1	-6,962	6,164
$1p_{3/2})(1p_{1/2})$	1	1	-5,576	9,324
$(1p_{3/2})(1p_{1/2})$	2	1	-8,432	6,468
Configuration ^{8}Li	**J^π**	**T**	**Binding energies according to ^{6}Li core (MeV)**	**Excitation Energies (MeV)**
$(p_{3/2})^2$	2	1	-12,627	0
$(d_{5/2})^2$	0	1	-6,250	6,337
$(d_{5/2})^2$	2	1	-11,688	0,94
$(p_{3/2})\ (p_{1/2})$	1	1	-10,900	1,727
$(p_{3/2})\ (p_{1/2})$	2	1	-13,400	-0,773
$(p_{3/2})\ (1d_{5/2})$	1	1	-11,626	1,001
$(p_{3/2})\ (1d_{5/2})$	2	1	-4,126	8,501

$(p_{3/2})(1d_{5/2})$	3	1	-12,564	0,063
$(p_{3/2})(1d_{5/2})$	4	1	-4,126	8,501
$(p_{1/2})(1d_{5/2})$	2	1	-3,649	8,978
$(p_{1/2})(1d_{5/2})$	3	1	-6,337	6,29
Configuration ^{9}Li	**J^{π}**	**T**	**Binding energies according to ^{7}Li core**	**Excitation Energies (MeV)**
$(1p_{3/2})^2$	0	1	6,83	0
$(1p_{3/2})^2$	2	1	--2,94	3,89
$(1p_{1/2})^2$	0	1	-2,1	4,73
$1p_{3/2})(1p_{1/2})$	1	1	-0,31	6,52
$(1p_{3/2})(1p_{1/2})$	2	1	-2,526	4,30
Configuration ^{10}Li	**J^{π}**	**T**	**Binding energies according to ^{8}Li core (MeV)**	**Excitation Energies (MeV)**
$(1p_{1/2})^2$	0	1	-8.104	0
$(1d_{5/2})^2$	0	1	-11,144	-3,040
$(1d_{5/2})^2$	2	1	-5,794	2,310
$(1d_{5/2})^2$	4	1	-6,394	3,710
$(2s_{1/2})^2$	0	1	-3,584	4,520
$(1p_{1/2})^2(1d_{5/2})^2$	2	1	-4,624	3,480
$(1p_{1/2})^2(1d_{5/2})^2$	3	1	-6,774	1,330
$(1p_{1/2})^2(1d_{5/2})^2$	0	1	-3,344	4,760
$(1p_{1/2})^2(1d_{5/2})^2$	1	1	-4,169	3,935
$(1d_{5/2})^2(2s_{1/2})^2$	2	1	-5364	2,710
$(1d_{5/2})^2(2s_{1/2})^2$	3	1	-2,364	5,740
Configuration ^{11}Li	**J^{π}**	**T**	**Binding energies according to ^{9}Li core**	**Excitation Energies (MeV)**
$(1p_{1/2})^2$	0	1	4,546	0
$(1d_{5/2})^2$	0	1	-2,722	-7,268
$(1d_{5/2})^2$	2	1	2,142	-2,404
$(1d_{5/2})^2$	4	1	2,961	-1585
$(2s_{1/2})^2$	0	1	1,824	-2,722
$(1p_{1/2})(1d_{5/2})$	2	1	5,486	0,94
$(1p_{1/2})(1d_{5/2})$	3	1	3,531	-1,015
$(1p_{1/2})(2s_{1/2})$	0	1	6,335	1,789
$(1p_{1/2})(2s_{1/2})$	1	1	5,585	1,039
$(1p_{1/2})(2s_{1/2})$	2	1	2,218	-2,328
$(1d_{5/2})(2s_{1/2})$	3	1	4,946	0,4

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