Comparison of Deflection Patterns of Simply Supported and Fixed Supported Beam Structures

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Abstract*— *When designing a building, the most of the important component is the roof. The selection of roofs for building depends on factors like type of building, type of foundation, roofing materials, roof loads, the climate, economy, the availability of materials, and the ease of construction. This study is focused on gable roof structures with two different supported structures of the ridge beam which are known as simply supported ridge beam structure and fixed supported ridge beam structure. The simply supported structure can be defined as the ends of the ridge beam are formed to stand freely on supports and the fixed supported structure can be defined as the ends of the ridge beam are supported to restrain against rotation and vertical movement. The study is considered jack wood beams which have the same length as 198 inches (5.03m) but have different cross-sectional area; 2x4inch^2, 2x5inch^2, 2x6inch^2, 3x4inch^2, 3x5inch^2 and 3x6inch^2. Further 11 Jack wood rafters which have the length 144 inches (3.66m) and cross sectional surface area 2x3 inch^2 are used. Two Jack wood king post trusses that have kingposts with the length 30 inches (0.7m) are used for fixed supported structure to fix the beam between two supporters. The roof pitch angle is approximately 30^0. According to general fact, the deflection value of simply supported beam is higher than the deflection value of fixed supported beam.

In this study, an attempt will be made to investigate the deformation of the ridge beam when it is simply supported and fixed supported and from that observe the optimum supported structure that can be used to design a roof of a building more constructive manner. Moreover by using the results, the validity of the general fact can be also proved. The major mathematical part in this study is to generate the model to calculate deflection of the ridge beam when it is simply and fixed supported by using Euler-Bernoulli Beam theory and Fourier series.

The results showed that the deflection value of simply supported beam around 0.8-0.04 m and thedeflection value of fixed supported beam around 0.008-0.001 m. According to results, the study was confirmed the general fact of the deflection values of simply supported beam structure has higher than fixed supported beam structure.

***Keywords*— Euler-Bernoulli Beam theory, Simply supported, Fixed supported, Fourier Series.**

I. INTRODUCTION

Nowadays there are several kinds of roofs. Each of those has advantages and disadvantages depending on thesame factors like climate, durability, availability of materials and ease of the construction. In this study is focused on the gable roof that has two different supporting structures of ridge beam which is known as couple roof and closed couple roof. In the couple roofs, the ridge beam laid on the end walls and so it is considered as the simple supported structure and in the closed couple roof, the ridge beam join directly to king posts by the mortise and tenon joinery at the two ends of the ridge beam. In modern days there is a huge trend on using closed couple roof when designing buildings like hotel cabanas, summerhouses and lobbies.Results of this study are proved how the variation of those two types when considering the deformation is.

II. METHODOLOGY

The Euler- Bernoulli beam theory is the one of the best method to calculate the behavior of the beam when a load is applied on the beam. By using the Euler-Bernoulli Differential Equation and Fourier series, general expressions for following structures can be constructed respectively. Then different deflection values can be obtained by applying those constructions to the real situation as stated above.

Consider the Euler-Bernoulli Differential Equation

$$\frac{d^4 y(x)}{dx^4} = \frac{1}{EI} q(x) \qquad (1)$$

where y(x) - Deflection function. (m)

q(x) - Deflection pressure per unit length at point x. (N)

E - Modulus of Elasticity of the beam. (Nm^{-2})

I - Moment of inertia of the beam. (m^4)

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Then expand q(x) = q into the Fourier sine series

$$q(x) = q = \sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L}\right) \text{ where}$$

$$q_n = \frac{2}{L}\int_0^L q \sin\left(\frac{n\pi x}{L}\right) dx \qquad n=1, 2, 3\ldots$$

$$q_n = \frac{2}{L}\int_0^L q \sin\left(\frac{n\pi x}{L}\right) dx = -\frac{2q}{L}\left[\frac{\cos\frac{n\pi x}{L}}{\frac{n\pi}{L}}\right] \quad (2)$$

$$= \frac{-2q}{n\pi}\left[1-\cos(n\pi)\right]$$

Note the direction of the y-axis, which was chosen to make y(x) positive. It is uniformly loaded q per unit length. The axis of a beam deflects from its initial position under the action of applied forces.

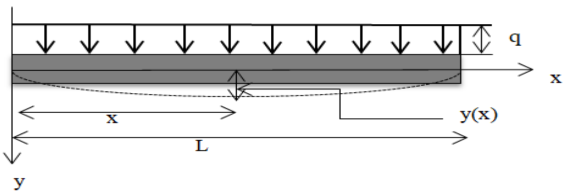

Fig. 1

As the first step this paper is focused to obtain maximum deflection values of the Jack wood beam when it is simply supported. Consider the following figure of simply supported beam.

The general expression for y(x) of simply supported beam is

$$y(x) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right) \qquad (3)$$

The fourth derivative for y(x) is

$$\frac{d^4 y}{dx^4} = \sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{L}\right)^4 \sin\left(\frac{n\pi x}{L}\right)$$

$$\sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{L}\right)^4 \sin\left(\frac{n\pi x}{L}\right) = \frac{1}{EI}\sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L}\right)$$

When n is even, $a_n = 0$ and when n is odd,

By (1), (2) and (3),

$$a_n\left(\frac{n\pi}{L}\right)^4 = \frac{1}{EI}\frac{4q}{n\pi}$$

$$a_n = \frac{4qL^4}{EIn^5\pi^5}$$

Hence

$$y(x) = \frac{4qL^4}{EI\pi^4}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^4}\left(\frac{x^2}{L^2} - \frac{x}{L} + \frac{1}{n\pi}\sin\left(\frac{n\pi x}{L}\right)\right)$$

As the second step this paper is focused to obtain maximum deflection values of the Jack wood beam when it is fixed supported.

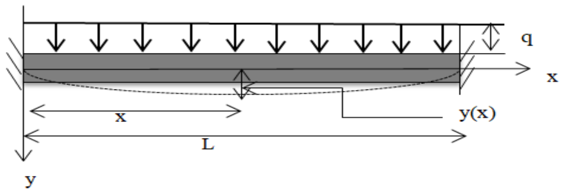

Fig. 2

The general expression for y(x) of fixed supported beam is

$$y(x) = \sum_{n=1}^{\infty} a_n \left\{-\left(\frac{x^3}{L^3} - \frac{2x^2}{L^2} + \frac{x}{L}\right) - (-1)^n\left(\frac{x^3}{L^3} - \frac{x^2}{L^2}\right) + \frac{1}{n\pi}\sin\left(\frac{n\pi x}{L}\right)\right\}$$

The fourth derivative for y(x) is

$$\frac{d^4 y}{dx^4} = \sum_{n=1}^{\infty} a_n \left(\frac{n^3\pi^3}{L^4}\right)\sin\left(\frac{n\pi x}{L}\right) \qquad (4)$$

$$\sum_{n=1}^{\infty} a_n \left(\frac{n^3\pi^3}{L^4}\right)\sin\left(\frac{n\pi x}{L}\right) = \frac{1}{EI}\sum_{n=1}^{\infty} q_n \sin\left(\frac{n\pi x}{L}\right)$$

When n is even, $a_n = 0$ and when n is odd,

By (1), (2) and (4),

$$a_n\left(\frac{n^3\pi^3}{L^4}\right) = \frac{1}{EI}\frac{4q}{n\pi}$$

$$a_n = \frac{4qL^4}{EIn^4\pi^4}$$

Hence $y(x) = \frac{4qL^4}{EI\pi^4}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^4}\left(\frac{x^2}{L^2} - \frac{x}{L} + \frac{1}{n\pi}\sin\left(\frac{n\pi x}{L}\right)\right)$

III. RESULTS

The MATLAB software is used to get deflection values that related to different situations and from that to draw deflection curves for each situation. The results were obtained simple supported structure and fixed supported structure separately.

Results for simply supported beam:

1) Beam with cross-sectional area 2x4 inch^2

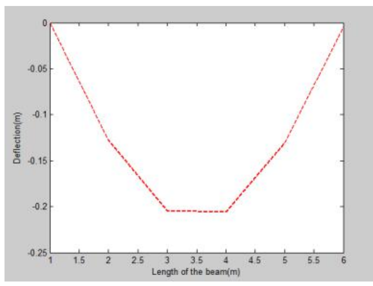

Fig.3

2) Beam with cross-sectional area 2x5 inch^2

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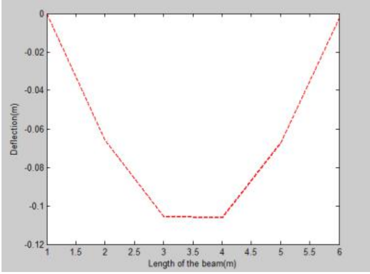

Fig.4

3) Beam with cross-sectional area 2x6inch^2

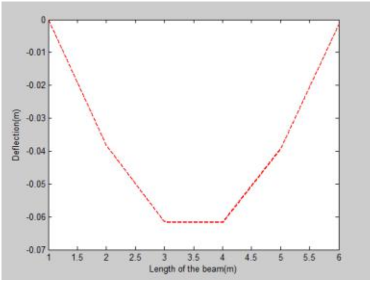

Fig.5

4) Beam with cross-sectional area 3x4 inch^2

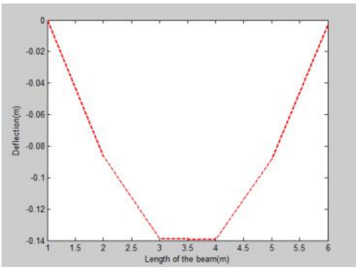

Fig.6

5) Beam with cross-sectional area 3x5 inch^2

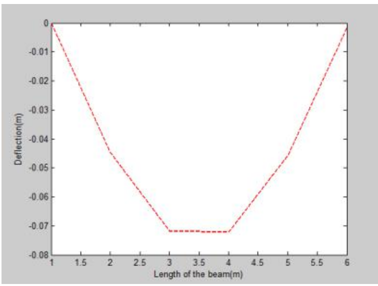

Fig.7

6) Beam with cross-sectional area 3x6 inch^2

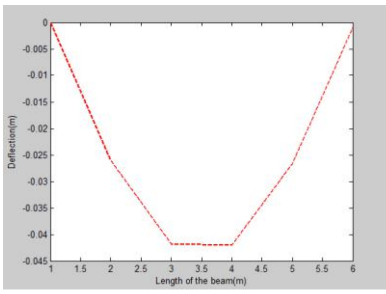

Fig. 8

Results for fixed supported beam:

1) Beam with cross-sectional area 2x4 inch^2

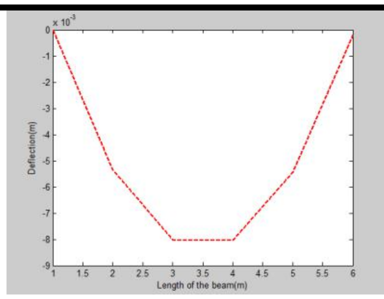

Fig. 9

2) Beam with cross-sectional area 2x5 inch^2

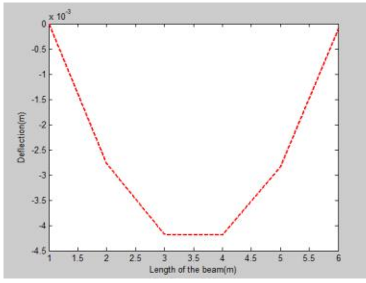

Fig. 10

3) Beam with cross-sectional area 2x6inch^2

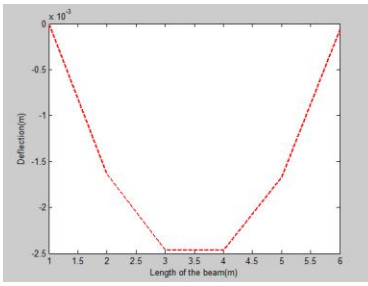

Fig. 11

4)Beam with cross-sectional area 3x4 inch^2

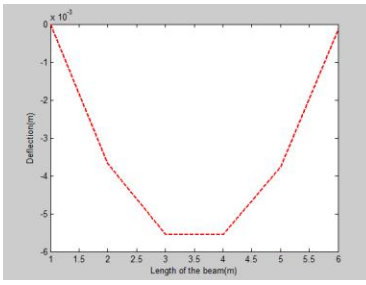

Fig. 12

5) Beam with cross-sectional area 3x5 inch^2

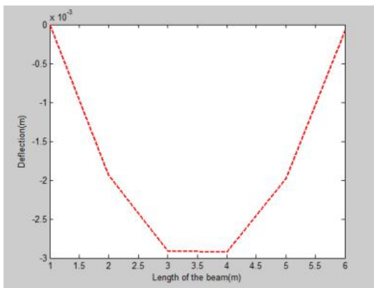

Fig. 13

By considering all graphs of deflection following tables can be constructed.

The Table 1 is represented the deflection values for simply supported beam and the Table 2 is represented the deflection values for fixed supported beam.

Table 1:

The cross-sectional area of the beam (inch^2)	The Maximum deflection (m)
2x4	0.2059
2x5	0.1062
2x6	0.6181
3x4	0.1392
3x5	0.7208
3x6	0.0421

Table 2:

The cross-sectional area of the beam (inch^2)	The Maximum deflection (m)
2x4	0.0080
2x5	0.0042
2x6	0.0025
3x4	0.0056
3x5	0.0030
3x6	0.0017

IV. CONCLUSIONS

According to above result, the maximum deflection values are occurred around 0.8-0.04 m when the beam is simply supported and the maximum deflection values are occurred around 0.008-0.001 m when the beam is fixed supported. From that the general fact which is mentioned above is proved very straightforward manner. Therefore the study shows the significance of using fixed supported structure when designing a roof.

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