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Stable Coastline between Two Groins Equation

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Abstract— *In this research an equation of static equilibrium geometry shoreline on coastal segment between two groins with quadratic polynomial equations is developed. Equation coefficients were formulated based on the characteristic of stable coastline geometry of the previous study and conservation law of mass, where volume of erosion and sedimentation are identical. The equation is capable of predicting erosion and accretion for coastline between two groin. Furthermore, with the predicted erosion and sedimentation, the groins gap and the length of groin can be planned using maximum permitted erosion criteria.*
***Keywords*—Coastline between two groin, stable coastline.**

I. INTRODUCTION

Groin is a type of massive construction as coastal protection against erosion. This construction is constructed perpendicular to the coast to withstand littoral drift. At the coastal segment located between two groins (Fig.1.), erosion and sedimentation will still occur until the formation of a stable coast, i.e. a condition where net sediment transportation is zero.

There are two conditions of stability, i.e. dynamic equilibrium and static equilibrium. At static equilibrium condition, there is no sediment transportation parallel with the coast or resultant of the long shore drift is zero. For a quite long time span, this static equilibrium condition exists if the evolution observed is the result of dominant wave. At static equilibrium condition, even sediment transportation still happened, the coastline will not change due to balance of sediment transportation..

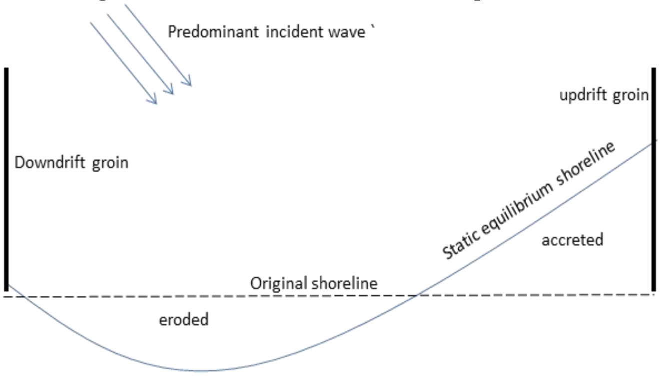

Fig.1 Evolution of Shoreline At The coast Between Two Groins

The aim of this research is to obtain coastline equation between two groins at static equilibrium condition. The equation is obtained by studying the shape of stable coastal geometry from previous researches, such as Haligan [1], half-heart bay, Silverster [2], crenulate shaped bays, Silverster and Hso [3], Hsu and Evans [4] and some other researchers.

The prediction of erosion and sedimentation at coastal segment between two groins can be done using GENESIS software or similar model. However, the analysis requires a long time. If the equation of stable coastline between two groins is obtained, the calculation can be done more practical with shorter time. Then, using the equation, the planning of the length of groin and the distance between groin can be done using the permitted erosion criteria.

II. STABLE COASTAL GEOMETRY

In this section the characteristic of stable coastal geometry will be studied. Based on the geometry characteristic, the approximation equation for stable coastal geometry between two groins will be formulated.

It has been recognized that in the nature there is stable coastal geometry in static equilibrium condition which many researches have studied to this form of stable coast. There are varieties of terminologies for the form of stable coast, i.e. Haligan [1] called it zeta bays, half-heart bay (Silverster [2]), crenulate shaped bays (Silverster dan Hso [3]) and some more. Whereas the shape and theory of the stable shoreline is as follows:

a. Parabolic Model

Hsu and Evans [4] stated a parabolic equation for distance R from point O to every point along the shoreline, whereas the shape of the equation is,

$$\frac{R_\theta}{R_{\theta_0}} = C_0 + C_1\left(\frac{\theta_0}{\theta}\right) + C_2\left(\frac{\theta_0}{\theta}\right)^2$$

$$C_0 = 0.0707 - 0.0047\theta + 0.000349\theta^2$$
$$- 0.00000875\theta^3 + 0.00000004765\theta^4$$

$$C_1 = 0.9536 + 0.0078\theta - 0.00004879\theta^2$$
$$+ 0.0000182\theta^3 - 0.000001281\theta^4$$

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$$C_2 = 0.0214 - 0.0078\theta + 0.0003004\theta^2 - 0.0000183\theta^3 + 0.000000009343\theta^4$$

R_{θ_0} = control line (line $\overline{OC}$), where the length is known

θ_0 = angle between crestline and control line (known)

R_θ = length of $\overline{OB}$ line, with θ angle toward crestline

The stable shoreline consists of two parts (Fig.2), i.e. curve part, which is shaped by diffracted wave, and the straight part which is shaped by incident wave and is parallel to it. It is known that littoral drift that is parallel to the coast is a sine function of the angle between the wave and the shoreline. Therefore, the tangent of this linear part must be parallel with the wave crestline where littoral drift is minimal or zero.

b. Logarithmic spiral model

Yasso [5] also discovered that stable coastal geometry has similar shape like the one in parabolic theory. However, he used the logarithmic spiral model equation for the stable coastal geometry, i.e. $\frac{R_2}{R_1} = e^{\theta \cot \alpha}$

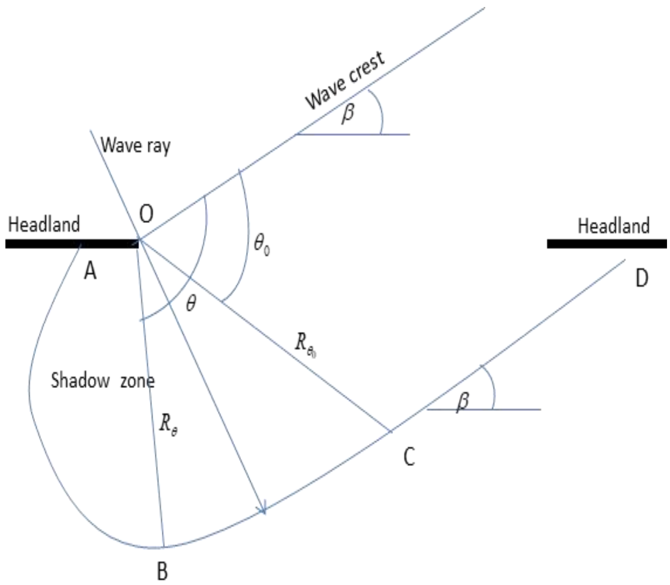

Fig.2: The Shape of Parabolic Stable Shoreline Between Two Headlands

Where (Fig.3), θ is the angle between R_2 and R_1, α is a logarithmic parameter. One thing that should be taken into account is that either parabolic model or logarithmic spiral model is equation in the diffracted wave area. Therefore, it can be stated that the curve shape of the stable coast is the result of diffracted wave.

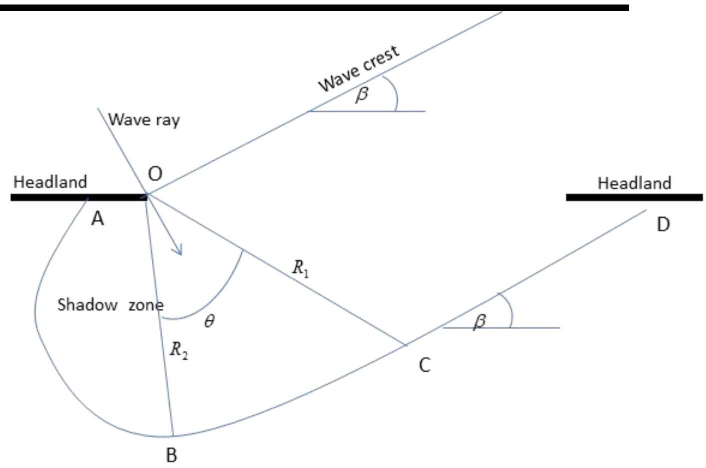

Fig.3: Logarithmic Spiral Model.

c. Shoreline Change Model

Some of the above theories on stable coastal geometry are stable coastal geometry between two headlands, whereas on stable coastal geometry between two groins, no research has been done, where at the down drift groin, coastal geometry is shaped by diffracted wave. Changes in coastline around the groin can be modeled with shoreline change model. The first shoreline change model was developed by Pelnard-Considere [6], which then was called one line model. The equation was formulated based on conservation law of mass, i.e. the volume of eroded sediment is similar to the volume of the deposited sediment, based on the main assumption that bathymetry moves parallel with the shoreline. The form of the equation is as follows:

$$\frac{\partial y}{\partial t} + \alpha \frac{\partial^2 y}{\partial x^2} = 0 \qquad \text{..............(1)}$$

Where $y(x,t)$ is the ordinate of a point on the shoreline or the distance of a point on the shoreline toward x axis which originally is parallel to the shoreline, whereas α is a coefficient which is a function of an angle of the incoming wave and wave height (Fig.4).

Furthermore, many researchers have made analytical method, both analytic and numeric, based on that one line equation to analyze shoreline evolution. Among others are GENESIS which was developed by Hanson and Krauss [7] and ONELINE (Dabees and Kamphuis [8],[9]).

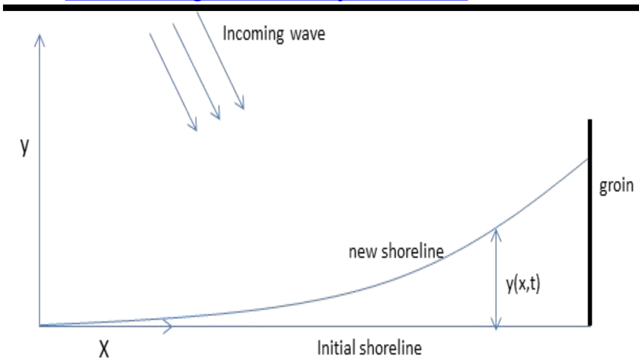

Fig.4: Definition of $y(x,t)$ on OneLine Model Equation

At static equilibrium condition (1), becomes $\frac{\partial^2 y}{\partial x^2} = 0$.

The solution of this equation is, $y(x) = c_0 + c_1 x$ which is a linear line equation. Therefore, according to one line model, shoreline geometry in static equilibrium condition is a straight line. This line should oriented parallel to crestline or perpendicular to wave direction, to avoid littoral drift. However, due to diffracted wave, shoreline in shadow zone is a curve line.

d. Diffraction at Groin

Kamphuis [10] has conducted a study on the condition of diffracted wave around groin since 1962. The diffracted wave coefficient according to Kamphuis [10] is

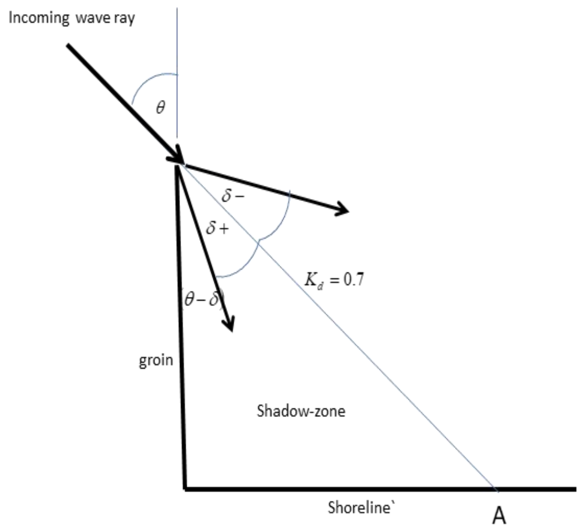

Fig.5: Diffracted Wave Around Groin.

$K_d = 0.7 - 0.0077\delta$ for $0 \le \delta \le 90^0$

$K_d = 0.7 - 0.37\sin\delta$ for $0 \le \delta \le -40^0$

$K_d = 0.83 - 0.17\sin\delta$ for $-40 \le \delta \le -90^0$

With this distribution pattern of the diffracted coefficient, it is estimated that maximum erosion at the left side of point A (Fig.5), i.e. at the end of shadow zone, whereas the point where shoreline is linear line is located at a quite far distance from point A.

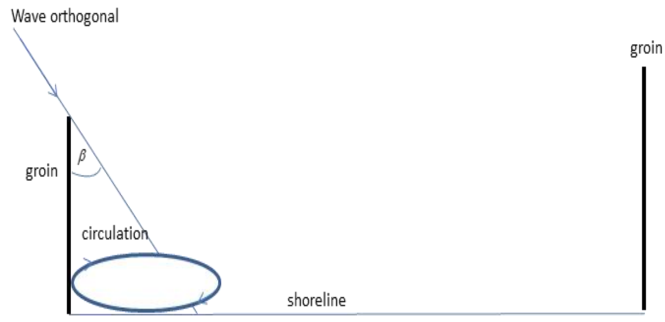

Fig.6: Secondary Circulate Longshore-current Due to Diffracted Wave

In addition, the difference in wave height as a result of diffraction, triggers longshore secondary current toward downdrift groin. As a result of the longshore current (Fig.6), there could be sedimentation or at least reducing erosion at the downstream groin (Van Rijn [11]). The volume of longshore transport is

$$Q = 0.0006\rho_s (\tan\beta)^{0.4} d_{50}^{-0.6} (H_{s,br})^{2.6} V_{wave} \quad ....(2)$$

$$V_{wave} = 0.3(gH_{s,br})^{0.5} \sin(2\theta_{br})$$

$Q =$ longshore sediment transport (kg/sec), $\rho_s =$ sediment density (kg/m^3); d_{50} = median sediment grain size (m), $\tan(\beta) =$ slope of beach surf zone $V_{wave} =$ wave induced longshore current vel. (m/sec), $\theta_{br} =$ wave angle at breakerline $(^0)$.

It should be noticed that at the longshore equation of the transport sediment, the volume of transport sediment longshore is determined by the angle of the incoming wave, and the higher the angle of the incoming wave the higher the velocity of the secondary longshore current and the bigger the littoral drift will be. Therefore, this secondary longshore current is a protector for part of the beach around down drift groin against erosion. The longer the groin, the larger the shadow zone area will be, and the bigger the secondary longshore current, the smaller the erosion or even the presence of sedimentation.

III. STATIC EQUILIBRIUM SHORELINE EQUATION BETWEEN TWO GROINS

The result of the study on section II. is that stable coastal geometry, between two headlands and also between two groins, consists of two parts, i.e. curve part and linear part. The curve part is formed by diffracted wave, whereas the linear part is formed by incident wave, where the linear part is perpendicular to wave direction or parallel with coastline.

In addition, based on a study conducted by Van Rijn [11], sedimentation could occur at the downstream groin, where the higher the angle of the incoming wave, the larger the sedimentation will be (2).

From the result of the study on part II, a hypothesis line for the static equilibrium shoreline geometry between two groins is made as presented in Fig.7, where there is linear part (straight line) and curve line. This is in accordance with parabolic theory. The straight part (BD line) is perpendicular to incident wave, which is the requirement for stable coastal line so that there will be no littoral drift, whereas the curve part is formed by diffracted wave.

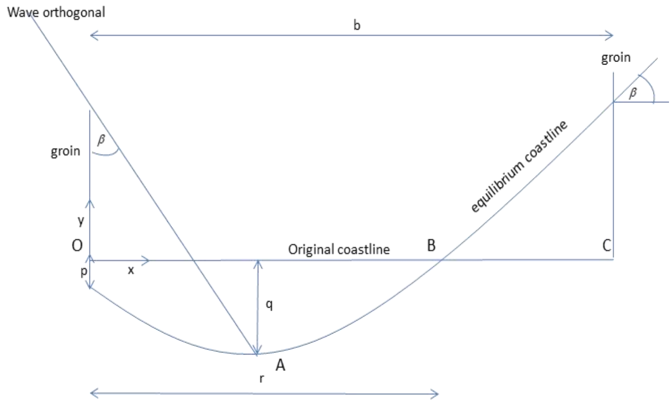

Fig.7: The Sketch of Static Equilibrium Shoreline

With an assumption that shoreline geometry is stable as shown in Fg.7, then it can be approximated with the parabolic equation, i.e.

$$y(x) = c_0 + c_1 x + c_2 x^2 \quad \text{.........................(3)}$$

Need to be determined in that equation are constant values of c_0, c_1 and c_2 and erosion at down drift groin p, the maximum erosion q, length s, with an input of incident wave direction β, length of groin L_g and distance between groin b. Sedimentation at the up drift groin side is calculated after coefficient values of c_0, c_1 and c_2 is obtained. To calculate the values of c_0, c_1, c_2, p, q and s, the following conditions are used:

A. Point Coordinate

1. $O(0,-p)$. At point O, where $x = 0$, $y = -p$, then $c_0 = -p$, coastal line equation becomes

$$y(x) = -p + c_1 x + c_2 x^2 \quad \text{................(4)}$$

2. $A(r,-q)$. At point A, i.e. at $x = r$, $y = -q$. From experiment, the approximation equation for r is $r = (L_g - q)\tan\beta$. The equation at this point is an effort to include length of groin L_g as one planning factor.

$$-p + c_1 r + c_2 r^2 = -q \quad \text{............(5)}$$

3. $B(s,0)$. At point B, i.e. at $x = s$, $y = 0$. Equation at this point is needed to determine the length of s or erosion area.

$$-p + c_1 s + c_2 s^2 = 0 \quad \text{........(6)}$$

B. The direction of line $\dfrac{dy}{dx} = c_1 + 2c_2 x$

1. At $x = x_B = b$; $\dfrac{dy}{dx} = \tan\beta$, this equation is also used as a requirement of limit at updrift groin at numeric completion of One Line model equation, i.e. as surface boundary of solid surface that no sediment goes through the groin, then

$$c_1 + 2c_2 = \tan\beta \quad \text{.....................(7)}$$

2. At $x = x_A = r$ at maximum erosion location, $\dfrac{dy}{dx} = 0$, this equation together with equation at point $A(r,-q)$, i.e. equation (5), is used to insert the influence of the length of groin.

$$c_1 + 2c_2 r = 0 \quad \text{..............(8)}$$

C. Conservation law of mass

In this section the conservation law of mass is done where the volume of erosion is similar to the volume of sedimentation, i.e.

$$\int_0^b y(x)dx = 0$$

Considering that the volume of erosion material can increase when it is submerged under water, then the relation between volume of erosion (V_e) and sedimentation (V_s) is $V_e = (1+\lambda)V_s$ where λ is sand porosity with a value of 0.2 - 0.3. The integration is divided into two parts.

$$\int_0^s y(x)dx + \left(1 + \lambda \int_s^b y(x)dx\right) = 0$$

where the first term is the volume of erosion and the second term is the volume of sedimentation. By completing the integration, and by dividing the equation with $(1+\lambda)b$, the following equation is obtained,

$$-\frac{\lambda}{(1+\lambda)b}\left(-ps + \frac{c_1}{2}s^2 + \frac{c_2}{3}s^3\right) \text{.................(9)}$$
$$+\left(-p + \frac{c_1}{2}b + \frac{c_2}{3}b^2\right) = 0$$

For $\lambda = 0$

$$-p + \frac{c_1}{2}b + \frac{c_2}{3}b^2 = 0 \quad (10)$$

Equation (4) until equation (9) and or (10) can be solved simultaneously using Newton-Rhapson iteration method for non-linear system equation, where variables to be calculated are c_1, c_2, p, q and s. To avoid the

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formation of an equation with zero diagonal, those five equations are arranged as follow

1. Equation for c_1, $x = x_B = b$, $\dfrac{dy}{dx} = \tan\beta$

$$f_1(c_1, c_2, q, r, s) = c_1 + 2c_2 b - \tan\beta$$

2. Equation for c_2, $\int_0^b y(x)dx = 0$

$$f_2(c_1, c_2, q, r, s) =$$
$$\left(-ps + \frac{c_1}{2}s^2 + \frac{c_2}{3}s^3\right)\left(-\frac{\lambda}{b}\right)$$
$$+\left(-p + \frac{c_1}{2}b + \frac{c_2}{3}b^2\right)$$

3. Equation for p, at point $A(r, -q)$

$$f_3(c_1, c_2, q, r, s) = -p + c_1 r + c_2 r^2 + q$$

4. Equation for q, $x = x_A = r$, $\dfrac{dy}{dx} = 0$

$$f_4(c_1, c_2, q, r, s) = c_1 + 2c_2(L_g - q)\tan\beta$$

5. Equation for s, $x = s$, $y = 0$

$$f_6(c_1, c_2, p, q, r, s) = -p + c_1 s + c_2 s^2$$

The Newton-Rhapton Iteration Method requires initial value. As initial value, the values on table (1) can be used.

Table.1: Initial Value for Iteration

c_1	c_2	p	q	s
$-\tan\beta$	$\dfrac{\tan\beta}{s}$	Any value	$\dfrac{\tan\beta}{s}$	$\dfrac{2}{3}b$

IV. MODEL RESULT

This section presents the result of equation for various cases. In this example, sand void ratio was not considered or $\lambda = 0$. Fig.8 shows the result of the model for wave angle incident $\beta = 15^0, 30^0, 45^0$, length of groin 40 m, distance between groin $b = 100$m. As shown on figure 8, the bigger the angle of the incoming wave, the bigger the erosion and sedimentation will be. At a big angle of incoming wave, $(\beta = 45^0)$, erosion at the down drift groin is smaller, which represents secondary effect of longshore current from Van Rijn $[11]$, as mentioned in chapter 2.

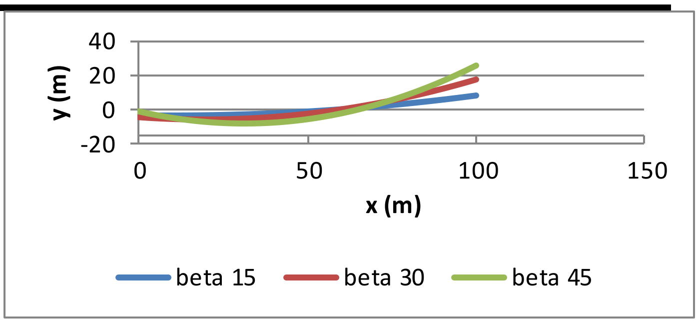

Fig.8: Example of Equation Result, for $\beta = 15^0, 30^0, 45^0$, Length of Groin $L_g = 40$ m. Distance Between Groin $b = 100$m.

Next is the presentation of model result for varied b, L_g and β as shown on table (2). At varied b with L_g and β constant, erosion and sedimentation are bigger with the expansion of distance between groins. At varied L_g with b and β constant, the longer the groin, the smaller the erosion and sedimentation will be. There is p negative at $\beta = 40^0$ and 45^0 which shows that accretion occurs at the downdrift groin. This represents the effect of secondary longshore current as stated by Van Rijn $[11]$.

In Fig. 8, there is no sedimentation at the downdrift groin although $\beta = 45^0$ is used, because the length of groin is 40 m, whereas the result on Table 2 groin length 50 m. The effect of the length of groin is also showed in table 2, i.e. reduces of the erosion. So, the length of groin and the angle of incoming wave provide secondary longshore current effect which reduces erosion at the down drift groin.

Tabel.2: The result of equation with varied L_g, b and β

Varied Distance between groin (b)							
b (m)	L_g (m)	β $(^0)$	p (m)	q (m)	r (m)	s (m)	t (m)
100	50	25	3,63	4,94	21,01	61,92	13,47
110	50	25	4,59	5,71	20,65	67,43	15,12
120	50	25	5,53	6,49	20,28	72,98	16,75
130	50	25	6,44	7,28	19,91	78,57	18,37
140	50	25	7,35	8,08	19,54	84,19	19,99
150	50	25	8,24	8,89	19,16	89,82	21,60
Varied Length of groin (L_g)							
100	30	25	5,82	6,15	11,12	59,54	14,57
100	35	25	5,32	5,82	13,60	60,05	14,32
100	40	25	4,79	5,51	16,08	60,62	14,05

100	45	25	4,23	5,21	18,55	61,24	13,77
100	50	25	3,63	4,94	21,01	61,92	13,47
100	55	25	3,00	4,68	23,46	62,67	13,16
100	60	25	2,33	4,44	25,90	63,50	12,82
100	65	25	1,62	4,23	28,33	64,42	12,47
Varied angle of incoming wave (β)							
100	50	15	3,19	3,43	12,47	59,82	8,29
100	50	20	3,64	4,24	16,65	60,75	10,92
100	50	25	3,63	4,94	21,01	61,92	13,47
100	50	30	2,97	5,535	25,67	63,42	15,92
100	50	35	1,30	6,08	30,74	65,44	18,15
100	50	40	-1,96	6,72	36,31	68,26	19,99
100	50	45	-7,73	7,73	42,26	72,15	21,13

Note : The negative value of p shows the presence of accretion

V. APPLICATION FOR THE GROIN PLANNING

The aim of shore protection using groin is to prevent erosion at coastal segment between two groins, where the actual erosion is still happening. Therefore, as the parameter for the planning of groin is the permitted maximum of the erosion.

Erosion can be limited by arranging the distance between groin and the length of groin so that the erosion that happens does not exceed the permitted erosion. For example, with the calculation on table (2) with the permitted erosion of 5 m, for angle of incoming wave of 25^{o}, then distance between groin that can be used is 100 m, with the length of groin 40m, and with this length the sand bypassing at the updrift groin will not happen since the sedimentation is only 14.0 m.

VI. CONCLUSION

At the method that is developed, there are the effects of the length of groin and the distance between groins on erosion as well as sedimentation. In addition, this method can also represent the effect of secondary longshore current from Van Rijn $[11]$, so it can be said that this method can predict erosion and sedimentation at the coastal segment between two groins and the geometry static equilibrium condition of its shoreline.

The method that is developed in this research can also be used to conduct initial estimation on the length of groin and the distance between groins, for preliminary study at a planning of coastal protection using groin.

For further development, a research should be done by comparing the equation with the result of physical model.

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