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Estimation of The Relationship Between The Travel Time of Flood Peaks and Peak Discharge on The Poprad River by Multilinear Flood Routing

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Abstract—**The empirical relationship between travel-time of flood peaks and peak discharge was studied on a reach of the Poprad River in Slovakia. The data were fitted by regression and compared with the expected shapes as described in the literature. Further a chain of linear segments has been considered as the model of that relation. The number of segments parameters and the angles between theses in this piecewise linear model were fitted by optimisation of a conceptual multilinear flood routing model performance on a large flood wave with the help of a genetic algorithm. In the setup of the multilinear model the travel-time parameter of the model was allowed to vary with discharge according to the piecewise linear model of the travel time of flood peaks. The discrete state space representation of the Kalinin-Miljukov model was used as the basis for a multilinear discrete cascade flood routing model. The resulting relationship was compared with empirical data on travel times and used to model the variability of the time parameter in the discrete state space representation of the Kalinin and Miljukov model on three verification floods. The modelling results showed that the inclusion of empirical information on the variability of the travel-time with discharge even from one flood enables satisfactory accuracy for the prediction of the flood propagation process.**

Index Terms— **Multi-linearity, flood routing, relationship between travel-time and discharge, Poprad River.**

I. Introduction

The use of conceptual models rather than physically based (hydraulic) routing models is usually preferred for forecasting and planning purposes. There is a vast amount of information on the subject of these models. The current state-of-the-art will therefore not be discussed here in detail (see e.g. [19], [20]). Only some assumptions and limitations of their use will be touched upon. We will rather concentrate here on the practical problem of model calibration in the case of sparse data, which is often a case in real-life situations when models are used for planning purposes. In several previous studies we have shown that the empirical relationships between travel-time of flood peaks and the peak discharge can be used to parameterise a multi-linear flood routing model based on the state space representation of the classical Kalinin - Miljukov cascade [19], [18]. The time parameter of the state space model was allowed to vary with input discharge according to the travel-time peak-discharge relationship.

In this paper the direct estimation of the relationship between the travel time of flood peaks and peak discharge on the Poprad River by multilinear flood routing was attempted. A piecewise linear model of that relation has been considered consisting of a chain of linear segments. The shape of that relationship (number and length and the angles between consecutive the segments, Fig. 4) was fitted by optimisation of the performance of multilinear routing model on a recent large flood wave with the help of a genetic algorithm. The empirical relationship between travel-time of flood peaks and peak discharge was also studied using recent flood data. The fitted piecewise linear relationship was compared with the empirical data on travel times and also used in the Kalinin and Miljukov model on three verification floods.

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II. The Empirical Flood Peak Travel-Time and Flood Peak Discharge Relationship on the Poprad River

In this case study, the Poprad River basin was selected as a reach for flood routing. It is located in the northern Slovakia (Fig. 1). The reach between Matejovce and Nižné Ružbachy of length of 35.2 km was chosen.

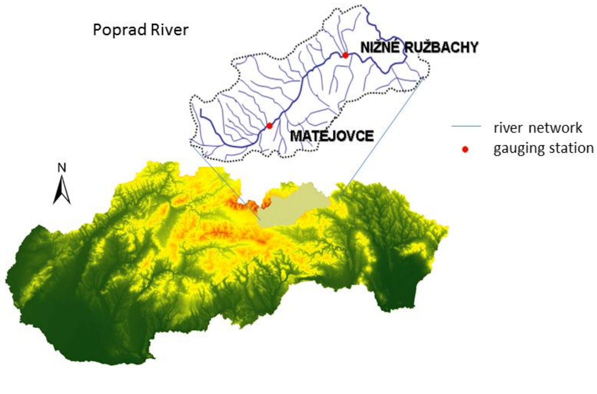

Fig. 1 The Poprad River Basin

For the analyses of the travel time of flood waves, hourly discharge data from 1992 to 2002 were used (data were provided by the Slovak Hydrometeorological Institute).

In [12] was formulated a wave speed-discharge relation, which can be interpreted as two power functions, one for the main channel and the other for over-bank flow, joined by an S-shaped transition curve (Fig. 2). Wave speed-discharge investigations carried out by [22], [23] on six Australian river reaches showed consistent variations of wave speed with discharge, which were considered as an empirical confirmation of the relation by [13]. In [6], [17], [9] similar behaviour was observed on the Danube.

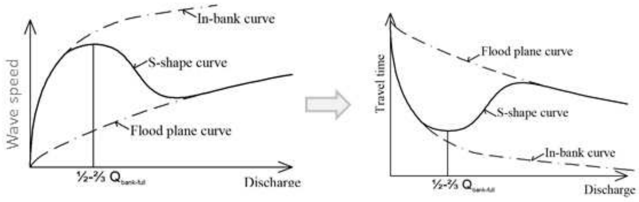

Fig. 2 Wave speed – discharge relationship (or travel time – discharge) according to [12]

Here, the physically-based derivation of such a relationship is not envisaged. To estimate the wave-speed discharge relationship from the flood data, the procedure used by [22], [23] was adopted. An estimate of the wave speed in the reach resulted from the travel-time of the main and intermediate flood peaks.

Travel time data of peak discharges for several flood waves were collected from the period 1992-2002. The resulting relationship is shown in Fig. 3.

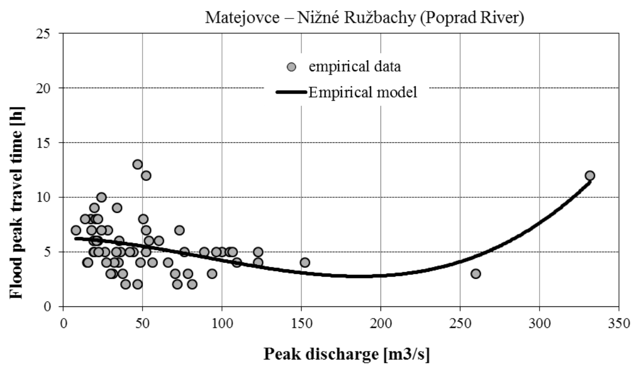

Fig. 3 Flood peak travel-time – and peak discharge relationship between Matejovce and Nižné Ružbachy fitted by regression

No theoretical model of the wave speed - discharge relationship was considered in this study, the empirical data were just fitted by regression for comparison purposes, since the mail goal of the study was the direct estimation of the relationship between the travel time of flood peaks and peak discharge on the Poprad River by multilinear flood routing. The pattern described in the literature (Fig. 2) is not clearly evident in the data due to missing travel time data for very large floods (the 100 year discharge is estimated to be 650m3/s).

III. Estimation of the Peak Flow Travel-Time and Peak Discharge Relationships by Multilinear Flood Routing

Development of conceptual non-linear reservoir type cascade models was one of the approaches how to incorporate non-linearity into the class of hydrologic routing models (see e.g. [8], [9], [14]). These models use a non-linear storage-outflow relationship in conjunction with the lumped continuity equation.

As an alternative to the use of such a relationship, the process models can be assumed to respond linearly to the input at any point in time, but with the model parameters recalculated as a function of flow values. These techniques, commonly referred to as multilinear modelling, usually distinguish different components in the input hydrograph, each of them being subsequently routed through a linear sub-model. The overall output of the non-linear system consists of the outputs from the linear sub-models. The different inflow components can be obtained by dividing the input hydrograph into segments horizontally or vertically. The former method is called the amplitude distribution scheme; the latter is the time distribution scheme. [5] gave an extensive description of the principles of these methods.

These concepts served as a basis for the development of the multiple linearization flow routing model [4] and the non-linear threshold model [1]. The multilinear discrete cascade model for channel routing based on a discrete representation of the Nash cascade as derived by [7] was used in [10], [11] and [2]. The discrete state-space version of the cascade of linear reservoirs, as derived by Szöllösi-Nagy (1982) for one input into the model of the cascade of linear reservoirs and extended by [15] for external inputs into each reservoir of the cascade, is used in this study. The model consists of a series of n linear reservoirs, each with the time constant (storage coefficient) k. The storage-discharge relationship of the ith reservoir in the series is considered in the following form:

$$S_i = kQ_i \qquad (1)$$

The $(n*1)$ state vector $\underline{S}$ of the model represents the volumes of water stored in each reservoir S_i at a given time; Q_i represents the outflow from the ith reservoir at a given instance for the corresponding storage.

The input to the cascade is given by the $(n*1)$ vector $\underline{I}$. Here, as in [15], and in contradiction to the usual formulation of the Kalinin-Miljukov-Nash cascade, each reservoir is allowed to have an external input I_i. In the first reservoir of the series, I_i accounts for the inflow to the cascade at cross section of the inlet of the modelled reach. In the subsequent reservoirs, I_i stands for the lateral inflow (or the withdrawal of water) into (from) the corresponding reservoir along the reach.

The continuity equation for the ith reservoir in the series is therefore written as:

$$\frac{dS_i}{dt} = Q_{i-1} + I_i - Q_i \qquad (2)$$

If all the inputs to the cascade $\underline{I}$ are considered to be constant during the sampling interval *(a, a+1)* of the length *T* (the input hydrograph is schematised as a stepwise constant function of time – e.g. the mean flow during the sampling interval), then the governing state-space equations of the model (the state equation and the outflow equation) can be written in the following form [15]:

$$\underline{S}(a+1) = \underline{F}(a+1,a)\underline{S}(a) + \underline{G}(a+1,a)\underline{I}(a+1,a) \quad (3)$$

$$\underline{Q}(a+1) = \underline{H}(a+1)\underline{S}(a+1) \quad (4)$$

where $\underline{S}$ and $\underline{Q}$ are the *(n*1)* vectors of the reservoir's volumes and outflows respectively, and $\underline{H}$ is the *(n*n)* matrix, which equals $\underline{Id}$**(1/k)*, where $\underline{Id}$ is the identity matrix. The elements of the *(n*n)* state and input transition matrices $\underline{F}$ and $\underline{G}$ are defined as:

$$F(i,j) = \frac{T^{i-j} e^{-T/k}}{(i-j)!k^{(i-j)}} \quad (5)$$

$$G(i,j) = k - \sum_{f=0}^{i-j} \frac{T^{f} e^{-T/k}}{f!k^{f-1}} \quad (6)$$

or *i* greater or equal to *j* and equal to *0* elsewhere.

The discrete state space version of the cascade of linear reservoirs (the classical Kalinin - Miljukov cascade [3]) was used here as the routing model [21], [12]. The fact that in the state space model the state variables vector contains a complete past history of the modelled process at a given time, is used here for building the multilinear discrete cascade model according to the time distribution scheme in the following way.

Following in [5], [1] the time distribution scheme for input division in multilinear models was selected. It is based on the algorithm for dividing the input *I* into a series of consecutive non overlapping series I_1, I_2, ... I_t and a set of distinct linear sub-models driven by the respective inputs I_i. The response *Q* of the (in general non-linear) multilinear model consists of the composition of responses Q_1, Q_2, ... Q_t of the sub-models to the corresponding input signals I_1, I_2, ... I_t.

It is known that the product *n.k*, where n is the number of linear reservoirs in the series and k is the storage coefficient, can be regarded as the travel-time of the modelled reach in the Kalinin-Miljukov scheme [3]. In several previous studies it was shown that the relationship between travel-time of flood peaks and the peak discharge could be used to parametrise this model. The time parameter of the state space model was allowed to vary with input discharge according to the travel-time peak-discharge relationship (e.g. [17], [9]).

In this paper a piecewise linear model of that relation has been considered.

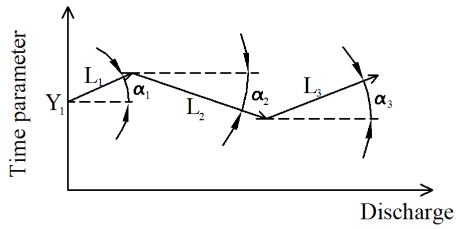

Fig. 4 The general shape of the peak flow travel-time and peak flow relationship as considered in this study

The shape and parameters of that relationship were estimated by optimisation of the multilinear routing model performance on a recent flood wave with the help of a genetic algorithm. The Nash-Sutcliffe criterion was used as an objective function. The following two assumptions were made:

1) A large flood wave has to be selected in order to cover the entire interval of flows and their corresponding travel-times.
2) To remain flexible in the determination of the shape of the travel-time discharge relationship formed by a chain of consecutive linear segments (piecewise linear function), the maximum number of sections has to be selected a-priori and kept rather high. Up to eight segments were considered in this study.

The resulting relationship estimated from one flood wave (flood on the 6.7.2001 – 17.8. 2001) is shown in Fig. 5.

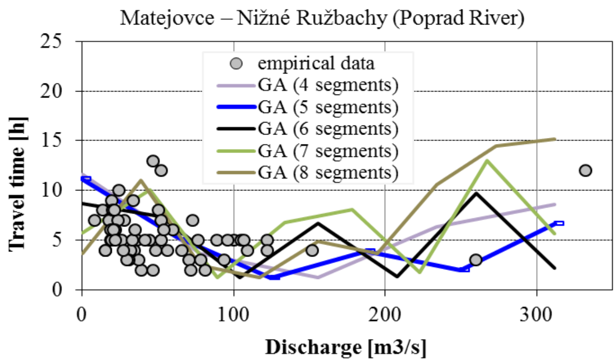

Fig. 5 Piecewise linear relationship fitted by optimising the multilinear model performance

The relationships fitted the empirical data surprisingly well. The contradictory behaviour of the relationship at small and very large discharges is just an artefact introduced by the optimisation, because there were no constraints introduced and the interval of small and very large discharge was not sufficiently covered by the discharge data. This result can be considered as a first step toward an (indirect) proof for the possibility of using time variable storage parameter in the Kalinin-Miljukov model, which had to be verified empirically for each application so far. For the verification of the concept the model consisting of five segments was selected and compared with the empirical data fitted by regression (Fig. 6).

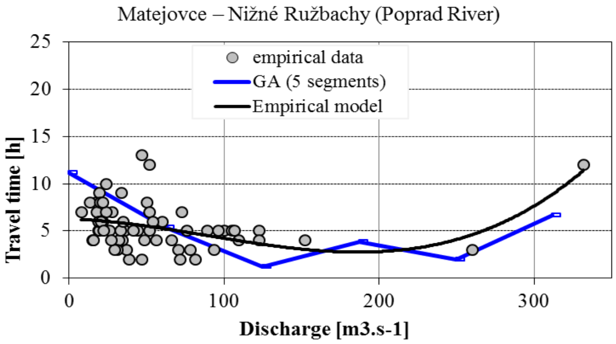

Fig. 6 The comparison of the relationships between travel-time and discharge estimated by genetic optimization with data from the Poprad River reach consisting of five segments with regression of the data (empirical model)

The performance of the respective multilinear models was verified on a set of 5 verification floods. Model performance was evaluated with the Nash Sutcliffe criterion and compared with that of the optimal linear model estimated for each flood separately (see results in Table I).

IV. Verification of the Travel-Time Discharge Relationship

For verification of the proposed multilinear scheme and the three empirical relationships between the travel time and the discharge, 5 floods were selected for model verification (Table I). The performance of the multilinear model was assessed by the Nash-Sutcliffe criterion:

$$Coef_{Nash-Sutcliffe} = \frac{\sum_{i=1}^{N}(Q_{Obs_i} - Q_{Comp_i})^2}{\sum_{i=1}^{N}(Q_{Obs_i} - \overline{Q}_{Obs})^2} \quad (7)$$

where N is the number of data, Q_{Obs} are the observed flows, Q_{Comp} the simulated flows, and $\overline{Q}_{Obs}$ is the mean value of Q_{Obs}.

The illustration of measured and simulated discharges for one flood is shown in Fig. 8. The values of this criterion are compared in Table I (for all floods).

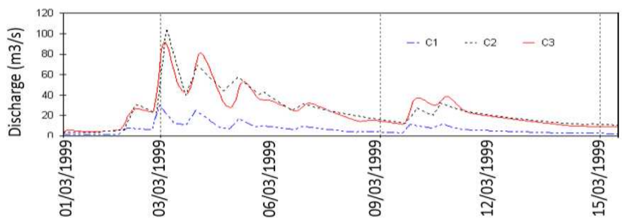

Fig. 7 Comparison of measured and simulated flows for the optimised travel-time discharge relationship (GA5) for the 25.2.1999 – 09.03.1999 verification flood. (C1 - measured input discharge, C2 - measured output discharge, C3 – simulated output discharge)

In principle it can be said that the multilinear model based on the variable travel-time discharge relationship estimated in this case study on the basis of only one large flood performed nearly as well as the optimal linear model calibrated separately for each flood wave by the genetic algorithm.

Table I. Nash Sutcliffe coefficients obtained for the verification runs of the multilinear model compared to the optimal linear model and the empirical regression model

No.	Flood duration	Q_{peak} [$m^3.s^{-1}$]	Model		
			Optimal linear	5 segments (considered)	Empirical polynomial
1	01.11. - 29.11.1991	52.1	0.940	0.941	0.933
2	25.02. - 18.03.1999	105	0.972	0.972	0.963
3	09.04. - 22.04.1994	152	0.972	0.974	0.959
4	18.03. - 25.04.2000	192	0.945	0.943	0.955
5	02.07. - 12.08.1997	332	0.756	0.759	0.756
Average Nash-Sutcliffe coefficient			0.9170	**0.9178**	0.9132

V. Conclusions

The relationship between travel-time of flood peaks and peak discharge was studied on a reach of the Poprad River. The discrete state space representation of the Kalinin-Miljukov model was used as a multilinear flood routing model. The time distribution scheme of model inputs was employed in the multilinear model and the travel-time parameter of the model was allowed to vary with discharge. A piecewise linear model of that relation has been considered. The shape and parameters of that model were fitted by optimisation of the multilinear model performance on one large flood wave with the help of a genetic algorithm.

The resulting relationship fitted empirical data on travel-times of flood peaks and was partially consistent with the findings in the literature regarding both, the physical interpretation of the factors determining the relation and empirical evidence. The fitted empirical piecewise linear model was used to model the variability of the time parameter in the discrete state space representation of the Kalinin and Miljukov model on 5 verification floods. The modelling results showed that the inclusion of empirical information on the variability of the travel-time with discharge even from one flood enabled satisfactory accuracy for the prediction of the flood propagation process.

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