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A Two Species Syn-Ecological Model with a Prey and a Predator Model with a Cover for Prey and an Alternative Food for the Predator

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Abstract—**The present paper is devoted to an analytical study of two species syn-ecological model with a prey and a predator. Predator is provided with an alternative food and a partial cover is provided to prey to protect it from the attacks of the predator. All possible equilibrium points are identified and their local and global stability is carried out. Trajectories of both the populations are also carried out by using analytical techniques.**

Index Terms—**Prey predator model, Syn-ecological model, Two species pray predator model.**

I. Introduction

Ecology deals the habits and habituates of living beings. Brief idea about modeling is given by Olinck,[1]. Later Kapur, [2], Smith,[3], Colinvaux, [4], Freedman, [5] studied nteracting modelss. The stability of ecological models were discussed by May, [6] , and their exact solutions are given by Varma, [7]. Lakshmi Narayan. K et.al, [8,9] discussed different prey-predator models.

Here we considered a prey-predator model for study.

The model is characterized by coupled non-linear ordinary differential equations of order one. Possible critical points are identified and discussed their stability At each stationary point linearized equations are formed and solved completely and explained with trajectories Lypunov's function constructed and derived some threshold results developed.

II. Notation and Preliminaries

A. Nomenclature

N_1, N_2 : Strength of species,

a_1, a_2 : Natural growth rate of the species

α_{11} , α_{22} : Rates of mortality due to internal competition,

α_{12} : Death rate prey by attacks of predator

α_{21} : Growth rate of predator due to interaction with the prey

k : cover constant (0<k<1)

Here N_1 and N_2 are zero or positive and also the model parameters $a_1, a_2, \alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}, k$.

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Governing equations are

(i) *Prey*:

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 - \alpha_{12}(1-k) N_1 N_2 \quad (2.1)$$

(ii) *Predator*:

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2^2 + \alpha_{21}(1-k) N_1 N_2 \quad (2.2)$$

III. Equilibrium States

We have four critical points

I. Extinct point $\overline{N}_1 = 0; \overline{N}_2 = 0$ (3.1)

II. $\overline{N}_1 = 0; \overline{N}_2 = \frac{a_2}{\alpha_{22}}$ (3.2)

predator exists, prey extinct.

III. The state $\overline{N}_1 = \frac{a_1}{\alpha_{11}}; \overline{N}_2 = 0$ (3.3)

prey exists, predator extinct.

IV. Interior state:

$$\overline{N}_1 = \frac{a_1\alpha_{22} - a_2\alpha_{12}(1-k)}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2};$$

$$\overline{N}_2 = \frac{a_2\alpha_{11} + a_1\alpha_{21}(1-k)}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2} \quad (3.4)$$

Which possible when k > $1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}$ (3.5)

IV. Stability of Critical States

Let $N = (N_1, N_2) = \overline{N} + U = (\overline{N}_1 + u_1, \overline{N}_2 + u_2)$ (4.1)

With U = (u_1, u_2) as perturbation matrix over $\overline{N} = (\overline{N}_1, \overline{N}_2)$.

$\frac{dU}{dt} = AU$ where

$$A = \begin{bmatrix} a_1 - 2\alpha_{11}\overline{N}_1 - \alpha_{12}(1-k)\overline{N}_2 & -\alpha_{12}(1-k)\overline{N}_1 \\ \alpha_{21}(1-k)\overline{N}_2 & a_2 - 2\alpha_{22}\overline{N}_2 + \alpha_{21}(1-k)\overline{N}_1 \end{bmatrix} \quad (4.2)$$

The secular equation is $det[A - \lambda I] = 0$ (4.3)

It is stable only when the critical values negative in case they are real or complex with negative real parts.

A. Critical State I

I. a. Stability:

The deviations u_1, u_2 satisfy the equations

$$\frac{du_1}{dt} = a_1u_1 \quad \text{and} \quad \frac{du_2}{dt} = a_2u_2 \tag{4.4}$$

and the secular equation is $(\lambda - a_1)(\lambda - a_2) = 0$, (4.5)

with two positive roots results an unstable**.**

The solutions are

$$u_1 = u_{10}\, e^{a_1 t} \quad \text{and} \quad u_2 = u_{20}\, e^{a_2 t} \tag{4.6}$$

With u_{10}, u_{20} as starting strengths of u_1 and u_2. The solution explained in *Figures 1 to 5.*

Case 1: Predator's dominance continues throughout as shown in Fig.1.

Case 2: Initially predator dominates, after some time situation reverses (i.e. $a_1 < a_2$ & $u_{10} > u_{20}$). At $t = t^* = \frac{ln\{u_{10}/u_{20}\}}{(a_2 - a_1)}$ both are equal strength (4.7)

As displayed in (Fig.2).

Case 3: Initially predator dominates, after some time situation reverses (i.e. $a_1 > a_2$ & $u_{10} < u_{20}$), At $t = t^* = \frac{ln\{u_{10}/u_{20}\}}{(a_2 - a_1)}$ both are with equal strength as shown in (Fig.3). (4.8)

Case 4: Prey's dominance continues throughout as shown in Fig.4.

I. b. Trajectories of Perturbed Species:

The trajectories in the u_1-u_2 plane are

$$\left[\frac{u_1}{u_{10}}\right]^{a_2} = \left[\frac{u_2}{u_{20}}\right]^{a_1} \tag{4.9}$$

and these are shown in Fig.5.

B. Critical State II

II. a. Stability : $\overline{N}_1 = 0; \overline{N}_2 = \frac{a_2}{\alpha_{22}}$

The perturbed equations for the critical state II are

$$\frac{du_1}{dt} = a_1u_1 - \frac{\alpha_{12}(1-k)a_2}{\alpha_{22}}u_1 \quad \text{and}$$

$$\frac{du_2}{dt} = -a_2u_2 + \frac{\alpha_{21}(1-k)a_2}{\alpha_{22}}u_1 \tag{4.10}$$

The secular equation is $(\lambda + a_2)$

$$\{\lambda - [a_1 - \frac{\alpha_{12}(1-k)a_2}{\alpha_{22}}]\} = 0 \tag{4.11}$$

One root of the equation (4.11) is $(\lambda_1 = -a_2)$ is negative.

Case A:

If $k > 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}$, the another root of equation (4.11),

$\lambda_2 = a_1 - \frac{\alpha_{12}(1-k)a_2}{\alpha_{22}}$ is positive.

Hence it results an *unstable*.

The equation (4.10) gives

$u_1 = u_{10}\, e^{\lambda_2 t}$ and

$$u_2 = \frac{1}{\gamma_1}\left[u_{10}a_2\alpha_{21}(1-k)e^{\lambda_2 t} + \{u_{20}\gamma_1 - u_{10}a_2\alpha_{21}(1-k)e^{-a_2 t}\right] \tag{4.12}$$

Here $\gamma_1 = a_1\alpha_{22} + a_2[\alpha_{22} - \alpha_{12}(1-k)]$ (4.13)

The solution curves are shown in *Fig.6 & 7*

Case A1: Pray dominance continues to out number the predator (i.e. $u_{10} > u_{20}$), the prey continues out numbering the predator as shown in Fig.6.

Case A2: Predator dominates over the prey initially (i.e. $u_{10} < u_{20}$), the predator continues to out number the prey till the time-instant

$$t = t^* = ln\left[\frac{u_{20}\alpha_{22}(\lambda_2 + a_2) - u_{10}a_2\alpha_{21}(1-k)}{u_{10}[\alpha_{22}(\lambda_2 + a_2) - a_2\alpha_{21}(1-k)]}\right] \tag{4.14}$$

Then the prey out number the predator. This is shown in *Fig.7*

Case B:

If $k < 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}$**,** the another root of the equation (4.11)

$\lambda_2 = a_1 - \frac{\alpha_{12}(1-k)a_2}{\alpha_{22}}$ is negative and

hence the critical state is *stable*. The trajectories in this case are the same as in (4.12).

Case B1: Pray dominance continues to out-number the predator (i.e. $u_{10} > u_{20}$). However both converge asymptotically to the critical point $(\overline{N}_1, \overline{N}_2)$ given by (3.2). Hence the critical point is *stable.* This is shown in *Fig.8*

Case B2: If the predator dominates over the prey initially (i.e. $u_{10} < u_{20}$), the predator continues to out-number the prey and till the time instant

$$t = t^* = \frac{1}{(\lambda_2 + a_2)} ln\left[\frac{u_{20}\alpha_{22}(\lambda_2 + a_2) - u_{10}a_2\alpha_{21}(1-k)}{u_{10}[\alpha_{22}(\lambda_2 + a_2) - a_2\alpha_{21}(1-k)]}\right] \tag{4.15}$$

after which the prey out-number the predator and grows unbounded while the predator asymptotically approaches to the equilibrium value $\overline{N}_2$ given in (3.2). Hence the state is *unstable.* This is shown in *Fig.9*

Case C:

If $k = 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}$, another root of the equation (4.11) is $\lambda_2 = 0$. Hence the critical state is *"neutrally stable".*

The trajectories are $u_1 = u_{10}$ and

$$u_2 = \frac{a_1\alpha_{21}}{a_2\alpha_{12}} u_{10} + \left[u_{20} - \frac{a_1\alpha_{21}}{a_2\alpha_{12}} u_{10} \right] e^{-a_2 t} \quad (4.16)$$

Case C1: If the prey dominates over the predator initially (i.e. $u_{10} > u_{20}$) and it continues throughout its growth. In course of time $u_2 \to u_2{*} = \frac{a_1\alpha_{21}}{a_2\alpha_{12}} u_{10}$ as is clear from equation (4.16). This is shown in *Fig.10.*

Case C2: If the predator dominates over the prey initially (i.e. $u_{10} < u_{20}$), the predator continues to out-number the prey and till the time instant

$$t = t{*} = \frac{1}{a_2} ln \left[\frac{a_2 u_{20}\alpha_{12} - a_2 u_{10}\alpha_{21}}{(\alpha_{12}a_2 - a_2\alpha_{21})u_{10}} \right] \quad (4.17)$$

after which the prey out-number the predator. This is shown in *Fig.11*

II. b. Trajectories of Perturbed Species:

The trajectories in the $u_1 - u_2$ plane are given by

$$(q_1 - 1)u_2 = c u_1^{q_1} - p_1 u_1 \quad (4.18)$$

here $p_1 = \frac{a_2\alpha_{21}(1-k)}{a_1\alpha_{22} - a_2\alpha_{12}(1-k)}$; $q_1 = \frac{-a_2\alpha_{22}}{a_1\alpha_{22} - a_2\alpha_{12}(1-k)}$; $c =$ arbitrary constant and

$$k \neq 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}. \quad (4.19)$$

The solution curves are shown in *Fig.12.*

C. Critical state III:

III. a. Stability : $\overline{N}_1 = \frac{a_1}{\alpha_{11}}; \overline{N}_2 = 0$

The perturbed equations for the critical state III are

$$\frac{du_1}{dt} = -a_1 u_1 - \frac{a_1\alpha_{12}(1-k)u_2}{\alpha_{11}} \text{ and}$$

$$\frac{du_2}{dt} = \left[a_2 + \frac{a_1\alpha_{21}(1-k)}{\alpha_{11}} \right] u_2 \quad (4.20)$$

and the secular equation is

$$(\lambda + a_1)\{\lambda - [a_2 + \frac{a_1\alpha_{21}(1-k)}{\alpha_{11}}]\} = 0 \quad (4.21)$$

One root of the equation (4.21) ($\lambda_1 = -a_1$) is negative and the another root

$\lambda_2 = a_2 + \frac{a_1\alpha_{21}(1-k)}{\alpha_{11}}$ is positive. Hence the critical state is *unstable.*

The trajectories in the (u_1, u_2) plane is given by the following equations:

$$u_1 = \frac{1}{\gamma_2}\left[-u_{20} a_1\alpha_{12}(1-k)e^{ct} + \{u_{10}\gamma_2 - u_{20}a_1\alpha_{12}(1-k)e^{-a_1 t} \right]$$

and $u_2 = u_{20}e^{dt}$ (4.22)

here $d = a_2 + \frac{a_1\alpha_{21}(1-k)}{\alpha_{11}}$ and

$$\gamma_2 = a_2\alpha_{11} + a_1[\alpha_{11} + \alpha_{21}(1-k)] \quad (4.23)$$

The solution curves are shown in *Figures 13 & 14*

Case 1: If the predator dominates initially (i.e. $u_{10} < u_{20}$), then the predator species to be going away from the critical point while the prey-species would become extinct at the instant (t*) of time given by the positive root of the equation

$$e^{dt} + e^{-a_2 t} = \frac{u_{10}\gamma_2}{u_{20}a_1\alpha_{12}(1-k)} \quad (4.24)$$

As such the state is *unstable*. This is shown in *Fig.13*

Case 2: If the prey dominates initially (i.e. $u_{10} > u_{20}$.), the prey continues to out-number the predator till the time instant,

$$t = t{*} = \frac{1}{c_1 + a_1} ln \left\{ \frac{u_{10}\alpha_{11}(c_1 + a_1) + u_{20}a_1\alpha_{12}(1-k)}{u_{10}[\alpha_{11}(c_1 + a_1)] + a_1\alpha_{12}(1-k)} \right\} \quad (4.25)$$

after which the predator out-number the prey. And also the predator species is noted to be going away from the critical point while the prey-species would become extinct at the instant (t*) of time given by the positive root of the equation (4.24). As such the state is *unstable*. This is shown in *Fig.14*

III.b.Trajectories:

The trajectories in the u_1-u_2 plane are given by

$$(p_2 - 1)u_1 = c u_2^{p_2} - q_2 u_2 \quad (4.26)$$

here

$$p_2 = \frac{-a_1\alpha_{11}}{a_2\alpha_{11} + a_1\alpha_{21}(1-k)}; \quad q_2 = \frac{-a_1\alpha_{12}(1-k)}{a_2\alpha_{11} - a_1\alpha_{21}(1-k)} \quad (4.27)$$

and c is an arbitrary constant. The solution curves are shown in *Figure 15.*

D. Critical state IV i.e. the normal steady state:

IV. a. Stability :

The perturbed equations for the critical state IV are

$$\frac{du_1}{dt} = -\alpha_{11}\overline{N}_1 u_1 - \alpha_{12}(1-k)\overline{N}_1 u_2 \text{ and}$$

$$\frac{du_2}{dt} = -\alpha_{22}\overline{N}_2 u_2 + \alpha_{21}(1-k)u_1\overline{N}_2 \quad (4.28)$$

The secular equation is

$$\lambda^2 + (\alpha_{11}\overline{N}_1 + \alpha_{22}\overline{N}_2)\lambda + [\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2]\ \overline{N}_1\overline{N}_2 = 0 \quad (4.29)$$

The roots of this equation are negative. The co-existent critical state is *stable.*

The trajectories are

$$u_1 = \left[\frac{u_{10}(\lambda_1 + \alpha_{22}\overline{N}_2) - u_{20}\alpha_{12}\overline{N}_1(1-k)}{\lambda_1 - \lambda_2}\right] e^{\lambda_1 t} + \left[\frac{u_{10}(\lambda_2 + \alpha_{22}\overline{N}_2) - u_{20}\alpha_{12}\overline{N}_1(1-k)}{\lambda_2 - \lambda_1}\right] e^{\lambda_2 t} \quad (4.30)$$

$$u_2 = \left[\frac{u_{20}(\lambda_1 + \alpha_{11}\overline{N}_1) - u_{10}\alpha_{21}\overline{N}_2(1-k)}{\lambda_1 - \lambda_2}\right] e^{\lambda_1 t} + \left[\frac{u_{20}(\lambda_2 + \alpha_{11}\overline{N}_1) - u_{10}\alpha_{21}\overline{N}_2(1-k)}{\lambda_2 - \lambda_1}\right] e^{\lambda_2 t} \quad (4.31)$$

Case 1: Predator dominates initially (i.e. $u_{10} < u_{20}$), and continues to out-number the prey, it is evident that both the species converging asymptotic to the equilibrium point. Hence this state is *stable*. This is illustrated in *Fig. 16.*

Case 2: Prey dominates in natural growth rate but its initial strength is less than that of predator (i.e. $u_{10} > u_{20}$), the prey out number the predator initially and continues till the time

$$t = t^* = \frac{1}{\lambda_2 + \lambda_1}\ ln \left[\frac{(b_3 - a_5)u_{10} + (a_3 + b_1)u_{20}}{(b_2 - a_6)u_{10} + (a_4 + b_1)u_{20}}\right] \quad (4.32)$$

Here

$$a_3 = \lambda_1 + \alpha_{11}\overline{N}_1;\quad a_4 = \lambda_2 + \alpha_{11}\overline{N}_1;\quad a_5 = \lambda_1 + \alpha_{22}\overline{N}_2;\quad a_6 = \lambda_2 + \alpha_{22}\overline{N}_2;\quad b_1 = \alpha_{12}(1-k)\overline{N}_1;\quad b_2 = \alpha_{21}(1-k)\overline{N}_2. \quad (4.33)$$

after which the predator out-number the prey. As $t \to \infty$ both u_1 & u_2 approaches to the critical point. Hence the state is *stable.* This is shown in *Fig. 17*

When $(\alpha_{11}\overline{N}_1 + \alpha_{22}\overline{N}_2)^2 > 4\alpha_{12}\alpha_{21}(1-k)^2\overline{N}_1\overline{N}_2$, (4.34)

the roots are complex with negative real part. Hence the critical state is *stable.* The solution curves are shown in *Fig.18*

IV.b.Trajectories of Perturbed Species:

The trajectories in the u_1-u_2 plane are given by

$$[u_2^{(1+a)(v_1+v_2)}]d = \frac{(u_1 - u_2 v_1)^{(p_3 - av_1)}}{(u_1 - v_2 u_2)^{(p_3 - av_1)}} \quad (4.35)$$

here, $p_3 = \overline{N}_2\alpha_{22}$; v_1 and v_2 are roots of an equation

$$av^2 + bv + c_4 = 0 \quad (4.36)$$

$$a = \alpha_{21}\overline{N}_2(1-k);\quad b = \alpha_{11}\overline{N}_1 - \alpha_{22}\overline{N}_2;\quad c_4 = \alpha_{12}\overline{N}_1(1-k) \quad (4.37)$$

and d is an arbitrary constant.

V. THRESHOLD RESULTS

Employing the principle of competitive exclusion (Gause [10]), the following threshold results are established.

a. If,

$$\frac{a_1}{\alpha_{12}(1-k)} > \frac{a_2}{\alpha_{22}} \text{ and } \frac{a_2}{\alpha_{21}(1-k)} < \frac{a_1}{\alpha_{11}} \quad (5.1)$$

Only prey species survives as shown in Fig. 20

b. If,

$$\frac{a_1}{\alpha_{12}(1-k)} < \frac{a_2}{\alpha_{22}} \text{ and } \frac{a_2}{\alpha_{21}(1-k)} > \frac{a_1}{\alpha_{11}} \quad (5.2)$$

Only predator species survives as shown in Fig. 21

c. When,

$$\frac{a_1}{\alpha_{12}(1-k)} > \frac{a_2}{\alpha_{22}} \text{ and } \frac{a_2}{\alpha_{21}(1-k)} > \frac{a_1}{\alpha_{11}} \quad (5.3)$$

VI. LYAPUNOV'S FUNCTION FOR GLOBAL STABILITY

The Perturbed Equations for the model are:

$$\frac{du_1}{dt} = -\alpha_{11}\overline{N}_1 u_1 - \alpha_{12}(1-k)\overline{N}_2 u_2 \quad (6.1)$$

$$\frac{du_2}{dt} = \alpha_{21}(1-k)\overline{N}_2 u_1 - \alpha_{22}\overline{N}_2 u_2 \quad (6.2)$$

The secular equation is:

$$\lambda^2 + p\lambda + q = 0 \quad (6.3)$$

here $p = \alpha_{11}\overline{N}_1 + \alpha_{22}\overline{N}_2 > 0$ (6.4)

$$q = \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2 > 0 \quad (6.5)$$

Therefore the conditions are satisfied.

Now, define

$$E(u_1, u_2) = \frac{1}{2}(au_2 + 2bu_1u_2 + cu_2^2) \quad (6.6)$$

Here

$$a = \frac{(\alpha_{21}(1-k)\overline{N}_2)^2 + (\alpha_{22}N_2)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D} \quad (6.7)$$

$$b = \frac{\alpha_{11}\alpha_{21}(1-k)\overline{N}_1\overline{N}_2 - \alpha_{12}\alpha_{22}(1-k)\overline{N}_1\overline{N}_2}{D} \quad (6.8)$$

$$c = \frac{(\alpha_{11}\overline{N}_1)^2 + (\alpha_{12}(1-k)\overline{N}_1)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D} \text{ and}$$

(6.9)

$$D = pq = \{\alpha_{11}\overline{N}_1 + \alpha_{22}N_2\}\{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2$$

(6.10)

From equations (6.4) & (6.5) it is clear that $D > 0$ and $a > 0$. Also

$$D^2(ac-b^2) > 0 \quad (6.11)$$

Since $D^2 > 0 \Rightarrow ac - b^2 > 0$ (6.12)

∴ The function E(x, y) is positive definite.

Then

$$\frac{\partial E}{\partial u_1}\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}\frac{du_2}{dt} =$$

$$(au_1 + bu_2)[-\alpha_{11}\overline{N}_1 u_1 - \alpha_{12}(1-k)\overline{N}_1 u_2] +$$

$$(bu_1 + cu_2)[\alpha_{21}(1-k)\overline{N}_2 u_1 - \alpha_{22}\overline{N}_2 u_2]$$

$$= (b\alpha_{21}(1-k)\overline{N}_2 - a\alpha_{11}\overline{N}_1)u_1^2 - (b\alpha_{12}(1-k)\overline{N}_1 + c\alpha_{22}\overline{N}_2)u_2^2 -$$

$$\{[b\alpha_{11} + a\alpha_{12}(1-k)]\overline{N}_1 + [b\alpha_{22} - c\alpha_{21}(1-k)\overline{N}_2\}u_1u_2 \quad (6.13)$$

On substituting the values of *a*, *b* and *c* from equations (6. 7), (6.8) & (6.9) and after simplification, we get

$$\frac{\partial E}{\partial u_1}\frac{du_1}{dt} + \frac{\partial E}{\partial u_2}\frac{du_2}{dt} = -(u_1^2 + u_2^2), \quad (6.14)$$

which is clearly negative definite. So *E(x, y) is a Lyapunov function* for the linear system.

Next we prove that E (u_1, u_2) is also a Lyapunov function for the non-linear system.

If, F_1 and F_2 are defined by

$$F_1(N_1, N_2) = N_1\{a_1 - \alpha_{11}N_1 - \alpha_{12}(1-k)N_2\} \quad (6.15)$$

$$F_2(N_1, N_2) = N_2\{a_1 - \alpha_{22}N_2 - \alpha_{21}(1-k)N_1\} \quad (6.16)$$

We have to show that $\frac{\partial E}{\partial u_1}F_1 + \frac{\partial E}{\partial u_2}F_2$ is negative definite.

On putting $N_1 = \overline{N}_1 + u_1$ and $N_2 = \overline{N}_2 + u_2$ in (2.1) & (2.2) equations, we notice after simplification, that

$$F_1(u_1, u_2) = \frac{du_1}{dt} =$$

$$-\alpha_{11}\overline{N}_1 u_1 - \alpha_{12}(1-k)\overline{N}_1 u_2 + f_1(u_1, u_2) \quad (6.17)$$

and $F_2(u_1, u_2) = \frac{du_2}{dt} =$

$$-\alpha_{22}\overline{N}_2 u_2 - \alpha_{21}(1-k)\overline{N}_2 u_1 + f_2(u_1, u_2) \quad (6.18)$$

Here

$$f_1(u_1, u_2) = -\alpha_{11}u_1^2 - \alpha_{12}(1-k)u_1u_2$$

and

$$f_2(u_1, u_2) = -\alpha_{22}u_2^2 + \alpha_{21}(1-k)u_1u_2 \quad (6.19)$$

We have $\frac{\partial E}{\partial u_1} = au_1 + bu_2$ and $\frac{\partial E}{\partial u_2} = bu_1 + cu_2$ (6.20)

Now from equations (6.17) and (6.18)

$$\frac{\partial E}{\partial u_1}F_1 + \frac{\partial E}{\partial u_2}F_2 =$$

$$-(u_1^2 + u_2^2) + (au_1 + bu_2)f_1(u_1, u_2)$$

$$+(bu_1 + cu_2)f_2(u_1, u_2) \quad (6.21)$$

By introducing polar co-ordinates we get

$$\frac{\partial E}{\partial u_1}F_1 + \frac{\partial E}{\partial u_2}F_2 =$$

$$-r^2 + r[(a\cos\theta + b\sin\theta)f_1(u_1, u_2) + (b\cos\theta + c\sin\theta)f_2(u_1, u_2)] \quad (6.22)$$

Denote largest of the numbers $|a|, |b|, |c|$ by M

Our assumptions $|f_1(u_1, u_2)| < \frac{r}{6M}$ and

$$|f_2(u_1, u_2)| < \frac{r}{6M} \quad (6.23)$$

for all sufficiently small $r > 0$, so

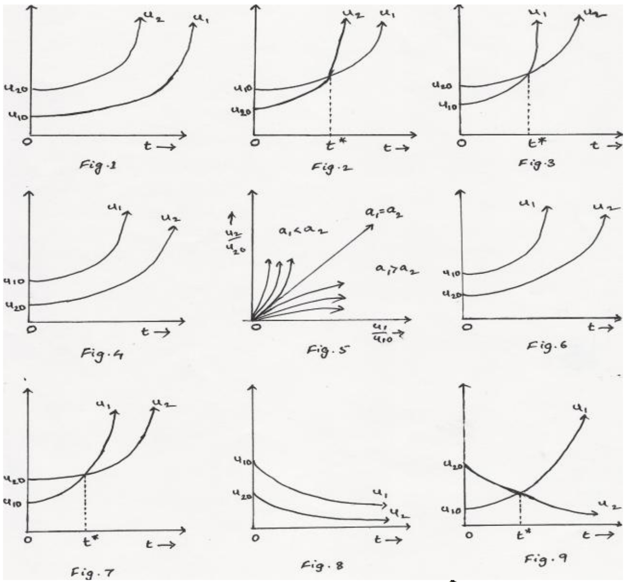
$$\frac{\partial E}{\partial u_1}F_1 + \frac{\partial E}{\partial u_2}F_2 < -r^2 + \frac{4Kr^2}{6M} = -\frac{r^2}{3} < 0 \quad (6.24)$$

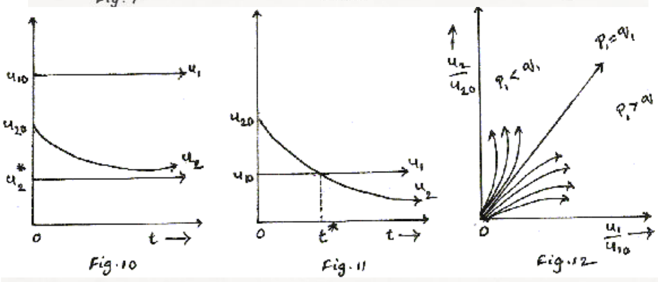
Thus E (u_1, u_2) is a positive definite function with $\frac{\partial E}{\partial u_1}F_1 + \frac{\partial E}{\partial u_2}F_2$ is negative definite.

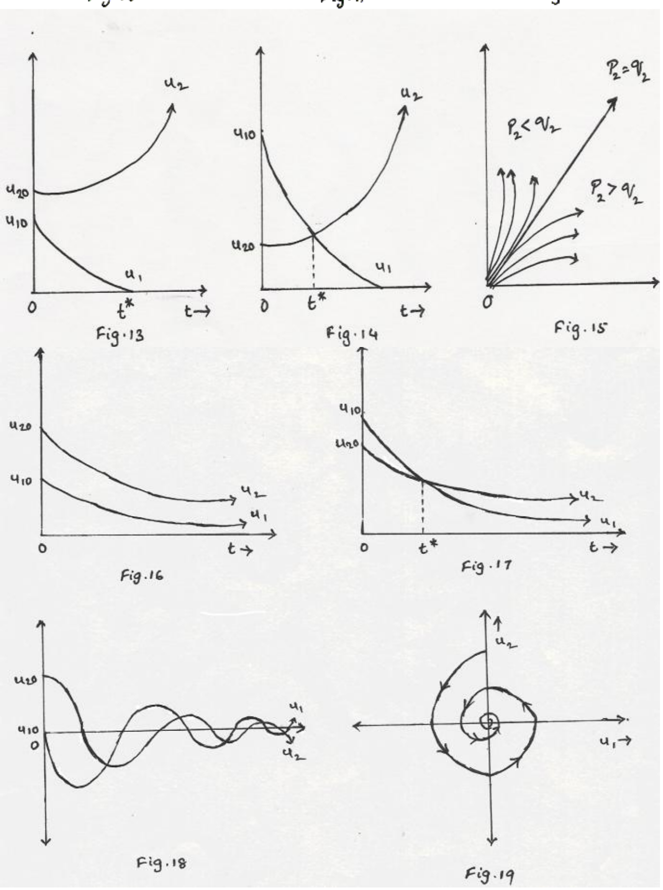
∴ The critical point is *asymptotically "stable"*.

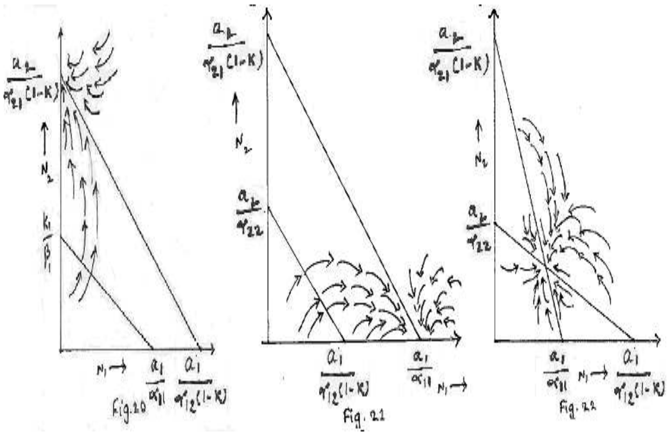
VII. Figures

Trajectories









VIII. CONCLUSION

Here we discussed a two species syn-ecological model with a prey and a predator. Predator is provided with an alternative food and a partial cover is provided to prey to protect it from the attacks of the predator. All possible equilibrium points are identified and their local and global stability is carried out. Trajectories of both the populations are also carried out by using analytical techniques.

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