International Journal of New Technology and Research (IJNTR)
ISSN:2454-4116, Volume-3, Issue-10, October 2017 Pages 115-122

A Mathematical Study of Prey-Predator Model With Cover and Alternative Food

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***Abstract*— An analytical study of two specie syn-ecological model with cover for prey and alternative food for predator is taken up. The model is governed by coupled first order non-linear ordinary differential equations. Stability of possible equilibrium points is studied and results are compared with numerical illustrations. Lypunov's function was constructed to discuss the global stability.**

***Index Terms*— Prey-Predator Model, Lypunov's function.**

I. INTRODUCTION

Olinck,[1] gave an introduction to Mathematical modeling in life sciences. Kapur, [2], Smith,[3], Colinvaux, [4], Freedman, [5] discussed some of the prey-predator ecological models. May, [6] discussed stability and complexity of ecological models, Varma, [7] discussed about their exact solutions. Lakshmi Narayan.K, [8,9] discussed different interacting species models.

II. BASIC EQUATIONS

Nomenclature:

N_1, N_2 : strength of species,

a_1, a_2 : natural growth rate of the species,

α_{11}, α_{22} : rates of mortality due to internal competition,

α_{12} : prey's death rate due to attacks of predator,

α_{21} : growth rate of predator due to interaction with the prey,

k : cover constant (0<k<1),

here N_1 and N_2 are non negative and also the model parameters $a_1, a_2, \alpha_{11}, \alpha_{12}, \alpha_{21}$, α_{22}, k.

governing equations are

$$\frac{dN_1}{dt} = a_1(1-k_1)N_1 - \alpha_{11}N_1^2 - \alpha_{12}(1-k)N_1N_2. \qquad (2.1)$$

$$\frac{dN_2}{dt} = a_2(1-k_2)N_2 - \alpha_{22}N_2^2 + \alpha_{21}(1-k)N_1N_2 \qquad (2.2)$$

III. STATIONARY POINTS:

The system under consideration have four stationary points :

I. extinct point $\overline{N}_1 = 0; \overline{N}_2 = 0$

(3.1)

II. The state $\overline{N}_1 = 0; \overline{N}_2 = \frac{a_2(1-k_2)}{\alpha_{22}}$

(3.2)

predator exists, prey extinct.

III. The state $\overline{N}_1 = \frac{a_1(1-k_1)}{\alpha_{11}}; \overline{N}_2 = 0$

(3.3)

prey exists, predator extinct.

IV. interior state:

$$\overline{N}_1 = \frac{a_1(1-k_1)\alpha_{22} - a_2(1-k_2)\alpha_{12}(1-k)}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2};$$

$$\overline{N}_2 = \frac{a_2(1-k_2)\alpha_{11} + a_1(1-k_1)\alpha_{21}(1-k)}{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2} \qquad (3.4)$$

Which possible when k > $1 - \frac{a_1(1-k_1)\alpha_{22}}{a_2(1-k_2)\alpha_{12}}$

(3.5)

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IV. STABILITY AT STATIONARY POINTS:

Let $N = (N_1, N_2) = \bar{N} + U = (\bar{N}_1 + u_1, \bar{N}_2 + u_2)$ (4.1)

with U = (u_1, u_2) as perturbation matrix over $\bar{N} = (\bar{N}_1, \bar{N}_2)$.

The basic equations (2.2), (2.4) are quasi-linearized to obtain the equations for the perturbed state $\frac{dU}{dt} = AU$ where

$$A = \begin{bmatrix} a_1(1-k_1) - 2\alpha_{11}\bar{N}_1 - \alpha_{12}(1-k)\bar{N}_2 & -\alpha_{12}(1-k)\bar{N}_1 \\ \alpha_{21}(1-k)\bar{N}_2 & a_2(1-k_2) - 2\alpha_{22}\bar{N}_2 + \alpha_{21}(1-k)\bar{N}_1 \end{bmatrix}$$

(4.2) The secular equation for the system is

$$det[A - \lambda I] = 0 \quad (4.3)$$

Which is stable when the roots are either negative real or complex with negative real part.

4. 1. Stability at stationary point I:

The trajectories extinct state are

$$u_1 = u_{10}\, e^{a_1(1-k_1)t} \quad \text{and}$$

$$u_2 = u_{20}\, e^{a_2(1-k_2)t}$$

(4.4)

here u_{10}, u_{20} are starting values of u_1 and u_2. The solution curves are given in **Figures 1 to 5**

Case 1: predator's dominance throughout as shown in Fig.1

Case 2: Initially predator dominates, after some time situation reverses (i.e. $a_1 < a_2$ & $u_{10} > u_{20}$). At

$$t = t^* = \frac{ln\,\{u_{10} / u_{20}\}}{(a_2 - a_1)}$$

(4.5)

both are with equal strength as displayed in (Fig.2).

Case 3: Initially predator dominates, after some time situation reverses (i.e. $a_1 > a_2$ & $u_{10} < u_{20}$). At $t = t^* = \frac{ln\,\{u_{10} / u_{20}\}}{(a_2 - a_1)}$ both are with equal strength as displayed in (Fig.3).

Case 4: prey's dominance throughout as shown in Fig.4.

4. 2. Trajectories of perturbed species of stationary point I: The trajectories in the u_1-u_2 plane are

$$\left[\frac{u_1}{u_{10}}\right]^{a_2} = \left[\frac{u_2}{u_{20}}\right]^{a_1} \quad (4.6)$$

and these are given in Fig.5.

4. 3. Stability of the stationary point II:

The trajectories for the prey washed out state are

$u_1 = u_{10}\, e^{\lambda_2 t}$ and

$$u_2 = \frac{1}{\gamma_1}\left[u_{10}a_2\alpha_{21}(1-k)e^{\lambda_2 t} + \{u_{20}\gamma_1 - u_{10}a_2\alpha_{21}(1-k)e^{-a_2 t}\right]$$

(4.7)

here $\gamma_1 = a_1\alpha_{22} + a_2[\alpha_{22} - \alpha_{12}(1-k)]$

(4.8)

The solution curves are given in figures 6 & 7

Case 1: prey's dominance throughout as shown in Fig.6.

Case 2: Initially predator dominates, after some time situation reverses (i.e. $u_{10} < u_{20}$), the predator continues to out number the prey till the time-instant

$$t = t^* = ln\left[\frac{u_{20}\alpha_{22}(\lambda_2 + a_2) - u_{10}a_2\alpha_{21}(1-k)}{u_{10}[\alpha_{22}(\lambda_2 + a_2) - a_2\alpha_{21}(1-k)]}\right]$$

(4.9)

after that the prey out number the predator. This is given in Fig.7

Case B: If $k < 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}$ stationary point is stable.

Case B$_1$: Prey's dominance continues throughout (i.e. $u_{10} > u_{20}$) but both converge asymptotically to the stationary point $(\overline{N}_1, \overline{N}_2)$ given by (3.2). Hence the stationary point is **stable.** This is given in Fig.8

CaseB$_2$: Initially predator dominates, after some time situation reverses (i.e. $u_{10} < u_{20}$), the predator out numbers the prey till the time instant

$$t = t^* = \frac{1}{(\lambda_2 + a_2)} ln\left[\frac{u_{20}\alpha_{22}(\lambda_2+a_2)-u_{10}a_2\alpha_{21}(1-k)}{u_{10}[\alpha_{22}(\lambda_2+a_2)-a_2\alpha_{21}(1-k)]}\right]$$

then situation reverses and prey grows unbounded while the predator asymptotically approaches to the stationary value $\overline{N}_2$ given in (3.2). Hence the state is **unstable.** This is given in Fig.9

Case C: If $k=1-\frac{a_1\alpha_{22}}{a_2\alpha_{12}}$ stationary point is **"neutrally stable"**.

The trajectories are $u_1 = u_{10}$ and $u_2 = \frac{a_1\alpha_{21}}{a_2\alpha_{12}}u_{10} +$

$$\left[u_{20} - \frac{a_1\alpha_{21}}{a_2\alpha_{12}}u_{10}\right] e^{-a_2 t} \qquad (4.11)$$

Case C$_1$: If the prey dominates initially (i.e. $u_{10} > u_{20}$) and it continues through out its growth. In course of time $u_2 \rightarrow$

$u_2{}^* = \frac{a_1\alpha_{21}}{a_2\alpha_{12}}u_{10}$ as is given in Fig.10.

Case C$_2$: If the predator dominates initially (i.e. $u_{10} < u_{20}$), the predator continues to out number the prey and till the time instant

$$t = t^* = \frac{1}{a_2} ln\left[\frac{a_2u_{20}\alpha_{12}-a_2u_{10}\alpha_{21}}{(\alpha_{12}a_2-a_2\alpha_{21})u_{10}}\right]$$

(4.12)

then which the prey out number the predator. This is given in Fig.11

4.4. Trajectories at stationary point II:

The trajectories in the $u_1 - u_2$ plane are given by

$$(q_1 - 1)u_2 = c\,u_1{}^{q_1} - p_1u_1$$

(4.13)

here $p_1 = \frac{a_2\alpha_{21}(1-k)}{a_1\alpha_{22} - a_2\alpha_{12}(1-k)}$; $q_1 = \frac{-a_2\alpha_{22}}{a_1\alpha_{22} - a_2\alpha_{12}(1-k)}$ (4.10) ; $c =$ constant and

$$k \neq 1 - \frac{a_1\alpha_{22}}{a_2\alpha_{12}}. \qquad (4.14)$$

The solution curves are given in Fig.12**.**

4. 5. Stability at stationary point III:

The trajectories for predator washed state are

$$u_1 = \frac{1}{\gamma_2}\left[-u_{20}a_1\alpha_{12}(1-k)e^{ct} + \{u_{10}\gamma_2 - u_{20}a_1\alpha_{12}(1-k)e^{-a_1 t}\right]$$

and $u_2 = u_{20}e^{dt}$ (4.15)

here $d = a_2 + \frac{a_1\alpha_{21}(1-k)}{\alpha_{11}}$ and $\gamma_2 = a_2\alpha_{11} +$

$$a_1[\alpha_{11} + \alpha_{21}(1-k)] \qquad (4.16)$$

The solution curves are given in Figures 13 & 14

Case 1: If the predator dominates initially (i.e. $u_{10} < u_{20}$), then the predator species to be going away from the stationary point while the prey-species would become extinct at the instant (t*) of time given by the positive root of the equation

$$e^{dt} + e^{-a_2 t} = \frac{u_{10}\gamma_2}{u_{20}a_1\alpha_{12}(1-k)}$$

(4.17)

Then the state is **unstable**. This is given in Fig.13

Case 2: The prey dominances (i.e. $u_{10} > u_{20}$.), continues till

$$t = t^* = \frac{1}{c_1 + a_1} ln \left\{ \frac{u_{10}\alpha_{11}(c_1+a_1)+u_{20}a_1\alpha_{12}(1-k)}{u_{10}[\alpha_{11}(c_1+a_1)]+a_1\alpha_{12}(1-k)} \right\} \quad (4.18)$$

then the situation reverses. The prey-species would become extinct at the instant (t*) of time given by the positive root of the equation (4.17). Then the state is **unstable**. This is given in Fig.14

4.6.Trajectories at stationary point III:

The trajectories in the u_1-u_2 plane are given by

$$(p_2 - 1)u_1 = c\,u_2^{p_2} - q_2 u_2$$

(4.19)

here $p_2 = \frac{-a_1\alpha_{11}}{a_2\alpha_{11} + a_1\alpha_{21}(1-k)}$; $q_2 = \frac{-a_1\alpha_{12}(1-k)}{a_2\alpha_{11} - a_1\alpha_{21}(1-k)}$ (4.20)

and c is a constant. The solution curves are given in Fig. 15**.**

4. 7. Stability at the interior stationary state:

The trajectories for co-existence state are

$$u_1 = \left[\frac{u_{10}(\lambda_1 + \alpha_{22}\overline{N}_2) - u_{20}\alpha_{12}\overline{N}_1(1-k)}{\lambda_1 - \lambda_2}\right] e^{\lambda_1 t} + \left[\frac{u_{10}(\lambda_2 + \alpha_{22}\overline{N}_2) - u_{20}\alpha_{12}\overline{N}_1(1-k)}{\lambda_2 - \lambda_1}\right] e^{\lambda_2 t}$$

(4.21)

$$u_2 = \left[\frac{u_{20}(\lambda_1 + \alpha_{11}\overline{N}_1) - u_{10}\alpha_{21}\overline{N}_2(1-k)}{\lambda_1 - \lambda_2}\right] e^{\lambda_1 t} + \left[\frac{u_{20}(\lambda_2 + \alpha_{11}\overline{N}_1) - u_{10}\alpha_{21}\overline{N}_2(1-k)}{\lambda_2 - \lambda_1}\right] e^{\lambda_2 t}$$

(4.22)

Case 1: If the predator dominates initially (i.e. $u_{10} < u_{20}$), and predator continues to out number the prey, it is evident that both the species converging asymptotic to the stationay point. Then this state is **stable**. This is given in Fig. 16.

Case 2: If the prey dominates in natural growth rate but its initial strength is less than that of predator (i.e. $u_{10} > u_{20}$), the prey out number the predator initially and this continues till the time $t = t^* = \frac{1}{\lambda_2 + \lambda_1} ln$

$$\left[\frac{(b_3 - a_5)u_{10} + (a_3 + b_1)u_{20}}{(b_2 - a_6)u_{10} + (a_4 + b_1)u_{20}}\right] \quad (4.23)$$

here $a_3 = \lambda_1 + \alpha_{11}\overline{N}_1$; $a_4 = \lambda_2 + \alpha_{11}\overline{N}_1$;

$a_5 = \lambda_1 + \alpha_{22}\overline{N}_2$;

$a_6 = \lambda_2 + \alpha_{22}\overline{N}_2$ $\quad b_1 = \alpha_{12}(1-k)\overline{N}_1$;

$b_2 = \alpha_{21}(1-k)\overline{N}_2$. (4.24)

then which the predator out number the prey. As $t \to \infty$ both u_1 & u_2 approaches to the stationary point. Then the state is **stable.** This is given in Fig. 17

If $(\alpha_{11}\overline{N}_1 + \alpha_{22}\overline{N}_2)^2 > 4\alpha_{12}\alpha_{21}(1-k)^2\overline{N}_1\overline{N}_2$,

(4.25)

the roots are complex with negative real part. Hence the stationary point is **stable.** The solution curves are given in Fig.18

4.7. Trajectories for normal steady state:

The trajectories in the u_1-u_2 plane are given by

$$[u_2^{(1+a)(v_1+v_2)}]d = \frac{(u_1 - u_2 v_1)^{(p_3 - av_1)}}{(u_1 - v_2 u_2)^{(p_3 - av_1)}}$$

(4.26)

here, $p_3 = \overline{N}_2 \alpha_{22}$; v_1 and v_2 are roots of quadratic

equation $av^2 + bv + c_4 = 0$ (4.27)

$$a = \alpha_{21}\overline{N}_2(1-k);\ b = \alpha_{11}\overline{N}_1 - \alpha_{22}\overline{N}_2;$$

$c_4 = \alpha_{12}\overline{N}_1(1-k)$ (4.28)

and d is a constant.

V. Liapunov's Function for Global Stability

The linearized basic equations for co-existence state are:

$$\frac{du_1}{dt} = -\alpha_{11}\overline{N}_1 u_1 - \alpha_{12}(1-k)\overline{N}_2 u_2$$

(5.1) $$\frac{du_2}{dt} = -\alpha_{21}(1-k)\overline{N}_2 u_1 - \alpha_{22}\overline{N}_2 u_2$$

(5.2)

The secular equation is:

$$(\lambda + \alpha_{11}\overline{N}_1)(\lambda + \alpha_{22}\overline{N}_2) + \alpha_{12}\alpha_{21}(1-k)^2\overline{N}_1\overline{N}_2 = 0$$

$$\Rightarrow \lambda^2 + p\lambda + q = 0 \quad (5.3)$$

here $p = \alpha_{11}\overline{N}_1 + \alpha_{22}\overline{N}_2 > 0$ (5.4)

$$q = \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2 > 0$$

(5.6)

Therefore the conditions for Liapunovs function are satisfied.

Now define $E(u_1, u_2) = \frac{1}{2}(au_2 + 2bu_1u_2 + cu_2{}^2)$

(5.7)

here

$$a = \frac{(\alpha_{21}(1-k)\overline{N}_2)^2 + (\alpha_{22}N_2)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D}$$

(5.8)

$$b = \frac{\alpha_{11}\alpha_{21}(1-k)\overline{N}_1\overline{N}_2 - \alpha_{12}\alpha_{22}(1-k)\overline{N}_1\overline{N}_2}{D}$$

(5.9)

$$c = \frac{(\alpha_{11}\overline{N}_1)^2 + (\alpha_{12}(1-k)\overline{N}_1)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D} \text{ and}$$

(5.10)

$$D = pq = \{\alpha_{11}\overline{N}_1 + \alpha_{22}N_2\}\{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2$$

(5.11)

From equations (6.6)&(6.7) it is clear that $D > 0$ and $a > 0$. Also

$$D^2(ac - b^2) =$$

$$D^2\Big\{\frac{(\alpha_{21}(1-k)\overline{N}_2)^2 + (\alpha_{22}N_2)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D} \times \frac{(\alpha_{11}\overline{N}_1)^2 + (\alpha_{12}(1-k)\overline{N}_1)^2 + \{\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}(1-k)^2\}\overline{N}_1\overline{N}_2}{D}$$

$$- \frac{\alpha_{11}{}^2\alpha_{21}{}^2(1-k)^2\overline{N}_1{}^2\overline{N}_2{}^2 + \alpha_{12}{}^2\alpha_{22}{}^2(1-k)^2\overline{N}_1{}^2\overline{N}_2{}^2 - 2\alpha_{11}\alpha_{12}\alpha_{21}\alpha_{22}(1-k)^2\overline{N}_1{}^2\overline{N}_2{}^2}{D^2}\Big\} > 0$$

(5.12)

$$\Rightarrow D^2(ac - b^2) > 0 \quad (5.13)$$

Since $D^2 > 0 \Rightarrow ac - b^2 > 0$ (5.14)

∴ The function E(x, y) is positive definite.

Then

$$\frac{\partial E}{\partial u_1}\frac{du_1}{dt}+\frac{\partial E}{\partial u_2}\frac{du_2}{dt}=$$

$$(au_1+bu_2)[-\alpha_{11}\overline{N}_1u_1-\alpha_{12}(1-k)\overline{N}_1u_2]+$$
$$(bu_1+cu_2)[\alpha_{21}(1-k)\overline{N}_2u_1-\alpha_{22}\overline{N}_2u_2]$$

$$=$$

$$(b\alpha_{21}(1-k)\overline{N}_2 - a\alpha_{11}\overline{N}_1)u_1^2-(b\alpha_{12}(1-k)\overline{N}_1+c\alpha_{22}\overline{N}_2)u_2^2-$$
$$\{[b\alpha_{11}+a\alpha_{12}(1-k)]\overline{N}_1+[b\alpha_{22}-c\alpha_{21}(1-k)\overline{N}_2\}u_1u_2$$
(5.15)

On substituting the values of a, b and c from equations (5.8), (5.9) & (5.10) and after much algebraic simplification, we get

$$\Rightarrow \frac{\partial E}{\partial u_1}\frac{du_1}{dt}+\frac{\partial E}{\partial u_2}\frac{du_2}{dt}=-(u_1^2+u_2^2),$$

(5.16)

which is clearly negative definite. So **E(x, y) is a Lyapunov function** for the linear system.

Next we prove that E (u_1, u_2) is also a Lyapunov function for the non linear system.

If, F_1 and F_2 are defined by

$$F_1(N_1,N_2)=N_1\{a_1-\alpha_{11}N_1-\alpha_{12}(1-k)N_2\}$$

(5.17)

$$F_2(N_1,N_2)=N_2\{a_1-\alpha_{22}N_2-\alpha_{21}(1-k)N_1\}$$

(5.18)

We have to show that $\frac{\partial E}{\partial u_1}F_1+\frac{\partial E}{\partial u_2}F_2$ is negative definite.

On putting $N_1=\overline{N}_1+u_1$ and $N_2=\overline{N}_2+u_2$ in (5.17) & (5.18) equations, we notice after much simplification, that

$$F_1(u_1,u_2) = \frac{du_1}{dt}=$$

$$-\alpha_{11}\overline{N}_1u_1-\alpha_{12}(1-k)\overline{N}_1u_2+f_1(u_1,u_2)$$

(5.19)

and $F_2(u_1,u_2)=\frac{du_2}{dt}=$

$$-\alpha_{22}\overline{N}_2u_2-\alpha_{21}(1-k)\overline{N}_2u_1+f_2(u_1,u_2)$$

(5.20)

here $f_1(u_1,u_2) = -\alpha_{11}u_1^2-\alpha_{12}(1-k)u_1u_2$

and $f_2(u_1,u_2) = -\alpha_{22}u_2^2+\alpha_{21}(1-k)u_1u_2$

(5.21)

We have $\frac{\partial E}{\partial u_1}=au_1+bu_2$ and $\frac{\partial E}{\partial u_2}=bu_1+cu_2$

(5.22)

Now $\frac{\partial E}{\partial u_1}F_1+\frac{\partial E}{\partial u_2}F_2= -(u_1^2+u_2^2)$

$$+(au_1+bu_2)f_1(u_1,u_2)+(bu_1+cu_2)f_2(u_1,u_2) \quad (5.23)$$

By introducing polar co-ordinates we get

$$\frac{\partial E}{\partial u_1}F_1+\frac{\partial E}{\partial u_2}F_2=$$

$$-r^2+r[(a\cos\theta+b\sin\theta)f_1(u_1,u_2)+(b\cos\theta+c\sin\theta)f_2(u_1,u_2)]$$

(5.24)

Denote largest of the numbers $|a|,|b|,|c|$ by M

Our assumptions $|f_1(u_1,u_2)|<\frac{r}{6M}$ and

$$|f_2(u_1,u_2)|<\frac{r}{6M} \quad (5.25)$$

for all sufficiently small $r>0$, so $\frac{\partial E}{\partial u_1}F_1+\frac{\partial E}{\partial u_2}F_2$

$$<-r^2+\frac{4Kr^2}{6M}=-\frac{r^2}{3}<0 \quad (5.26)$$

Thus E (u_1, u_2) is a positive definite function with the property that $\frac{\partial E}{\partial u_1}F_1+\frac{\partial E}{\partial u_2}F_2$ is negative definite.

∴ The stationary point is **asymptotically "stable".**

VI. Trajectories

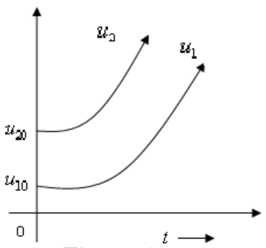


Figure 1

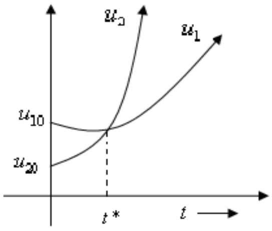
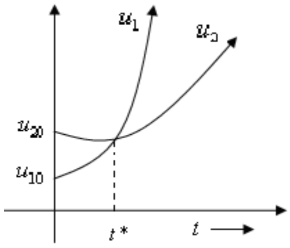


Figure 2



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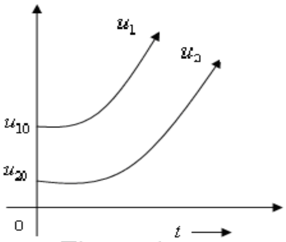

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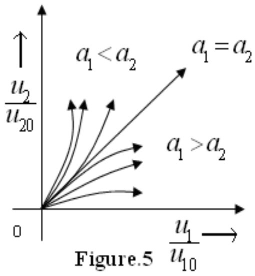


Figure.5

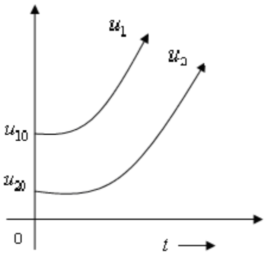


Figure 6

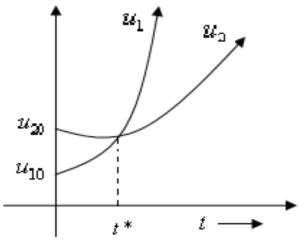


Figure 7

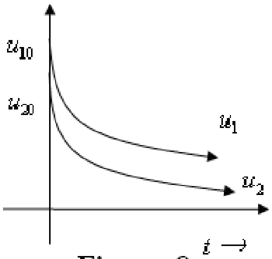


Figure 8

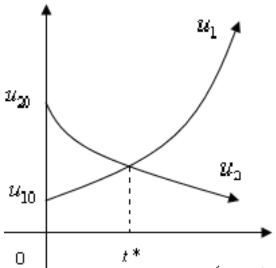


Figure 9

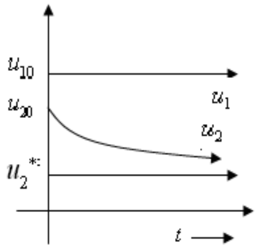
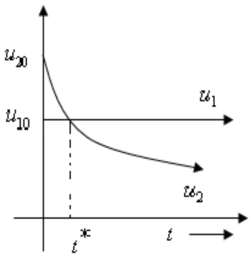


Figure 10



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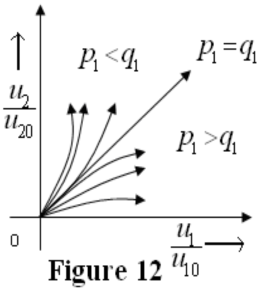

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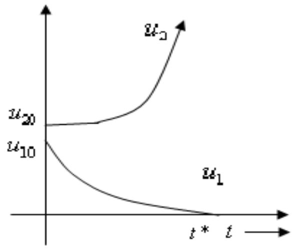


Figure 13

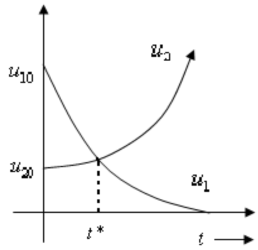


Figure 14

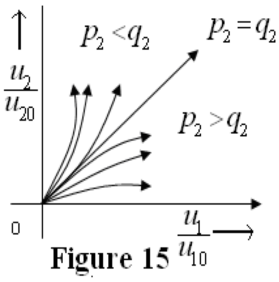


Figure 15

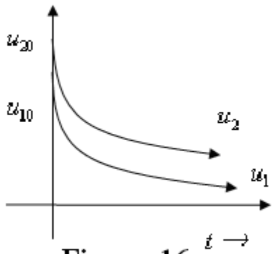
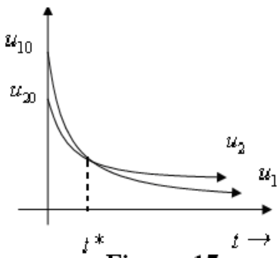


Figure 16



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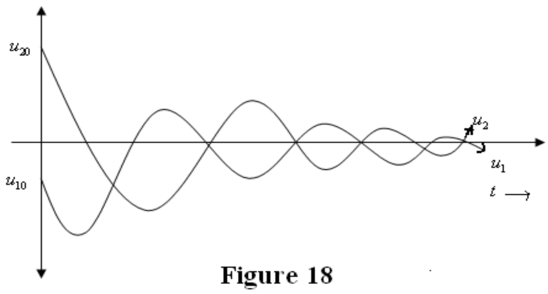

Figure 18

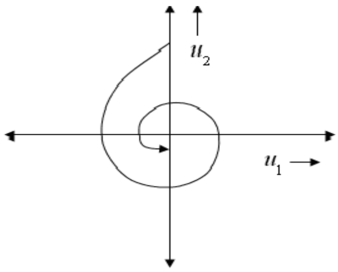

Figure 19

VII. Conclusion:

A two species Prey-Predator model is studied with cover for prey and alternative food for predator. All the four equilibrium points are identified. It is noted that interior equilibrium point is asymptotically stable. Fully extinct state is unstable and other two are conditionally stable Existence of Lypunov's function shows that the system is globally asymptotically stable.

REFERENCES

[1] Michael Olinck : An introduction to Mathematical models in the social and Life Sciences, 1978, Addison Wesley.

[2] Kapur, J. N. : Mathematical models in Biology and Medicine, Affiliated East- West,1985.

[3] Smith, J. M. : Models in Ecology, Cambridge University Press, Cambridge, 1974.

[4] Paul Colinvaux: Ecology, John Wiley and Sons Inc., New York, 1977.

[5] Freedman, H. I : Deterministic Mathematical Models in Population Ecology, Marcel – Decker, New York, 1980.

[6] May, R. M. : Stability and complexity in Model Eco-systems, Princeton University Press, Princeton, 1973.

[7] 7. Varma, V. S. : A note on 'Exact solutions for a special prey-predator or competing species system', Bull. Math. Biol., Vol. 39, 1977, pp 619-622.

[8] Lakshmi Narayan, K. at el. "A Model of Two Mutually Interacting Species with limited resources and Time Delay" Advances in Theoretical and Applied Mathematics, Vol. 5(2),2010

[9] Lakshmi Narayan.K at. el. " Stability Analysis of a Three Species Syn-eco dynamical system with a Limited Alternative Food for all Three Species" Bulletin of Society for Mathematical Services and Standards" Vol.1(1) pp 38-48,2013

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