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An Analysis of Monetary Policies in an Uncertain Economic Environment in a Ugandan Perspective

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***Abstract*— In recent years, progress in macroeconomic modeling was done to give new perspectives on the impact of uncertainty on the evolution of monetary policy. As, uncertainty is dominant, it is important to understand how alternative policies can be selected when the central bank cannot accurately observe the macroeconomic variables.. Therefore, the central bank of developing countries , must have at their disposal the tools for modeling the most accurate. For this reason, in this paper we try to determine the robust optimal monetary policy in an uncertain economic environment: First, we present the main types of uncertainties; we discuss the models used and their derivation. Then we show the techniques used, and how to introduce uncertainty. Finally, we present the optimal robust responses to different forms of uncertainty considered. Our approach is to estimate a dynamic macroeconomic model with three representations of the dynamics of inflation, each with microeconomic foundations for the data of the economy with Bayesian techniques. Our results show that the uncertainty of the structural parameters affect the dynamic solutions for the economy, but also on the objective functions of various agents, particularly that of the central bank. Indeed, the central bank will be more careful in certain circumstances. Increased caution with the weights carried by the interest rate in the loss function. Thus, our results show the effectiveness of the simple robust simple rule of Levin and Williams.**

***Index Terms*— economic environment, monetary policies, uncertainty.**

I. INTRODUCTION

In order to make decisions, central banks use models for forecastingthe implications of their actions. Although there are many availablemodels, economists tend to study principally issues about monetarypolicy in a Neo-Keynesian framework. Current research on monetarypolicy business cycles introduces sticky prices and inefficientmarket equilibrium as a source of monetary non-neutrality. The maincharacteristics of Neo-Keynesian models, known as the Neo-KeynesianPhillips (NKP) curve, is the introduction of optimizing behaviour,rational expectations and as the resulting inflation specification,the dependence of current inflation and a measure of output gap (e.g.Claridа*et al*.,1999).

$$\pi_t = E_t \pi_{t+1} + \alpha y_t \qquad (1)$$

The Equation (1) depends only on the future behaviour of the drivingvariable. However, the implications of the NKP curve are in contradictionwith empirical studies, regarding the impact of disturbances(Mankiw and Reis, 2002). The inconsistencies between purely forward-lookingmodels and the data, led many researchers (e.g. Clarida*et al*., 1999) to use "Hybrid' NKP curve which includes both backward- andforward-looking elements.So the Equation (1) can be rewritten as follows:

$$\pi_t = (1-\phi) E_t \pi_{t+1} + \phi \pi_{t+1} + \delta y_t \qquad (2)$$

The motivation for including inertia is largely empirical and justified theoretically with an assumption that a fixed proportion of firms has backward-looking price setting behaviour. Empirically, the adequacy of this model, which nests the pure forward-lookingsticky price model and inherits the good properties of backward-looking models, to data is very controversial. Indeed, empirical work about weights on Backward- Looking (BL) behaviour, don't lead to a set of consensual values. In general, studies based on maximum likelihood estimation suggest that the estimated weight on backward-looking behaviour is the highest, while works using full information maximum likelihood techniques show that inflation dynamics depend only on Forward-Looking (FL) behaviour. To be more precise, on the one hand, Clarida*et al*., (1999) argue that purely forward-looking curve provides a good approximation of the dynamics of inflation, in particular the assumption , and find that the degree of backward-looking behaviour is highly significant in the "hybrid' curve. On the other hand, Fuhrer (1997), Roberts (2001), find that BL behaviour seems more important than FL behaviour, when using the full information method instead, respectively. Estimates are not clear and fluctuate significantly.

In this situation, monetary authorities face inertial degree uncertainty. Moreover, we know that this parameter affects dramatically the effectiveness of monetary policy. We know also that properties of optimal monetary policy depend on degree of inflation persistence (Lansing and Trehan, 2003 and McCallum and Nelson, 2004). Various models have been proposed in the literature but none has yet provided an uncontroversial description of the transmission process. McCallum (1999) sums up this situation in the following manner:

"It is not just that the economics profession does not have a well-tested quantitative model of the quarter to quarter dynamics, the situation is much worse than that; we do not even have any basic agreement about the qualitative nature of the mechanism. This point can be made by mentioning some of the leading theoretical categories, which includes: real business cycle (RBC)models, monetary misperceptions models, semi-classical price adjustment models, models with overlapping contracts of the Taylor or the Fischer variety,

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models with nominal contracts set as in Fuhrer and Moore (1995), NAIRU [1] models, Lucas (1982)supply function models, MPS-style markup pricing models, and soon. Thus there are dozens or perhaps hundreds of competing specifications regarding the precise nature of the connection between monetary policy actions and their real-short term consequences".

When monetary authorities examine the implications of uncertainty with one model, they underestimate largely the actual amount of uncertainty, since each model in itself constitutes a simplification which abstracts from relevant aspects of reality. As a result, they can't afford to rely on a single reference model of the economy but need to have a number of alternative modeling tools available. In practice, central banks avail themselves of alternative quantitative models of the economy as opposed to relying on a single all-encompassing one and make 'average' choices (Blinder, 2000). So, it would seem wise to develop robust rules in order to face uncertainty adequately. A number of studies have therefore started expressing model uncertainty in the form of a variety of alternatives models (Levin and Williams, 2003).

The rest of the paper is organized as follows: the second section describes three types of uncertainty: shocks, parameters and models suggested in the literature. In section three, we study in particular the last type of uncertainty, using different nominal rigidities modeling strategies. Then (section four), after having estimated the model, we propose to find a robust simple rule across the range of models selected. A final section concludes.

II. TYPES OF UNCERTAINTY FACED BY THE CENTRAL BANKS

In the literature, we can distinguish 3 kinds of uncertainty. The first one consists in studying shocks robustness. The second one takes into account uncertainty parameters within a model. The last one refers to model uncertainty. In this paper, we describe succinctly these 3 approaches.

A. Robustness to alternative types of disturbances

Initially, the literature has questioned the way the monetary surprises could be incorporated in expectations to generate short-term increase in production. A growing number of central banks have abandoned the system of intermediate target variables such as exchange rate or money growth and lead to direct policies around the stability of inflation in a discretionary manner is ie, subject to the influences of the economy. However, Kydland and Prescott (1977) showed that, besides being associated with an inflation bias, discretionary policy suffers from a bias stabilization and is much lower than with the political commitment in terms of Welfare. Woodford (2003) attempts to adopt an intermediate position based on the principles of commitment while presenting the advantages of discretionary framework. He advocates the use of optimal targeting rules while offering the possibility to change in ways that provide no incentive to deviate from the objectives declared by the central banker. In this context, the transparency of the policy pursued by the authorities determines its credibility.

[1]Non-Accelerating Inflation Rate of Unemployment (NAIRU)

The idea of a Timeless perspective means it suffices that the equilibrium be optimal subject to certain constraints on the economy's initial evolution to prevent the policymaker from exploiting the existing expectations at the time that the policy commitment is chosen. In choosing its commitment for the future, it takes into account the consequences of its commitments for the private-sector expectations at earlier dates. The rational expectation equilibrium expected to prevail from time $t = t_0$, given a commitment to the policy rule at that date, needn't minimize expected losses from that date conditional upon the economy's state at that time. That behavior allows the policymaker's behavior to be consistent over time the advantages that allows for the credibility to the private sector of the central bank's commitments and for the likelihood that the private sector can learn to predict future policy. The choice of a rule of conduct from such a perspective eliminates the problem of the time inconsistency of optimal policy in the sense that the same reasoning that is used to support the choice of the optimal rule at one date can be used to support for the same rule at any later date. Formally, Woodford (2003) defines optimality of a rule from a timeless perspective as follows:

1) The non-predetermined endogenous variables z_t can be expressed as a time-invariant function of a vector of predetermined variables $\bar{z}$ and a vector of exogenous variables $\bar{s}$ represented by Equation (3):

$$Z_t = f_0 + f_Z \bar{Z} + f_S \bar{S},\ \forall t \geq 0 \tag{3}$$

2) The equilibrium evolution of the endogenous variables y_t, for all dates $t \geq t_0$, minimizes the discounted quadratic loss function subject to the constraints implied by the economy's initial state z_{t_0}, the structural equations and a set of additional constraints of the form: $\tilde{E}_{Z_t 0} = \bar{e} \equiv \tilde{E}[f_0 + f_z \bar{Z}_{t_0} + f_s \bar{S}_{t_0}]$ on the initial behavior of the predetermined endogenous variables. The equilibrium dynamics resulting from commitment to a policy that is optimal from a timeless perspective involve the same responses to unanticipated shocks in all periods $t > t_0$. This commitment to an optimal rule dominates both the average inflation bias and the sub-optimal dynamic responses to shocks, sometimes with discretionary policy. Woodford (2003) describe these rules as robustly optimal. Its are optimal in that the rule supports the equilibrium consistent with an optimal commitment policy when evaluated from the timeless perspective. And its are robust in that the coefficients in the policy rule are independent of the parameters that characterize the behavior of the exogenous, stochastic disturbances. Thus, the policymaker implementing such a rule does not need to know whether disturbances are highly persistent or transitory or whether demand shocks are more volatile than cost shocks. This form of robustness is not exhaustive. There is also considerable uncertainty about the correct specification of the non-stochastic terms in the model equations.

B. Uncertainty within a model

1) Robust policy faced with parameters uncertainty

Söderström (2002) assume that the policymaker wants to determine the robust optimal rule faced with parameters uncertainty into the following model:

$$y_{t+1} = \alpha_{t+1} y_t - \beta_{t+1}(i_t - \pi_t) + \varepsilon_{t+1}^{y} \quad (4)$$

$$\pi_{t+1} = \delta_{t+1}\pi_t + \gamma_{t+1} y_t + \varepsilon_{t+1}^{\pi} \quad (5)$$

Where y_t and π_t are respectively the output and the inflation (in log), i_t represents the nominal interest rate, ε_{t+1}^{π} and ε_{t+1}^{y} are iid(supply and demand) shocks *i.i.d.* with mean zero. Equation (4) is the IS curve. Output gap depends positively on its own past value; negatively on the Δi.Equation (5) is the Phillips curve. The change in inflation depends positively on the lag of *y,* and a shock (ε_{t+1}^{π}).At each time t, the main parameters are assumed independent and to be random variables with means: $\mathbb{E}_t(\alpha_{t+1}) = \alpha$, $\mathbb{E}_t(\gamma_{t+1}) = \gamma$, $\mathbb{E}_t(\delta_{t+1}) = \delta$, and variances σ_{α}^2, σ_{β}^2, σ_{γ}^2 ,et σ_{δ}^2 .

When the Central Bank sets its interest rate at time t, it isassumed to know all realizations of the parameters up to andincluding period t but it does not know their future realizationsand thus cannot be certain about the effects of policy on theeconomy. The Central Bank's optimization problem consists in minimizing aquadratic loss function subject to the model equations. Toillustrate the effects of including multiplicative uncertainty intothe model, he rewrites the problem as a control problem with valuefunction, using a state-space formulation. The resolution becomes more complicated than the certain case. Theoptimal rule is in particular a function of a matrix given byiterating on the Riccati equation.When the parameters are non-stochastic, the variance-covariancematrix of the state vector coincides with the variance matrix of thedisturbance vector. Thus it is independent of the instrument rule.The optimal policy rule is independent of the degree of uncertainty,that is to say, is certainly equivalent. In contrast, when the parameters are uncertain in a multiplicativeform, the variance-covariance matrix depends on the state of theeconomy, the instrument and the variances of the parameters as wellas those of the additive disturbances. Thus, certainty equivalenceno longer holds. Due to uncertainty's multiplicative feature, Söderström(2002) resolutionbecomes interesting.

2) Uncertainty a la Hansen and Sargent

Hansen and Sargent (2003) introduce another type of uncertainty betweenparameters uncertainty and model uncertainty. They study the robustoptimal policy solving a robust Stackelberg[2] problem within thefollowing forward looking model is given by Equation (6):

$$y_{t+1} = Ay_t + BU_t \quad (6)$$

Where y_t a vector of state variables and "jump variables" is, U_t is a decision rule. The decision rule is chosen, as theprevious section, by minimizing a quadratic loss function $y_t' Q y_t + U_t' R U_t$, subject to the model represented by the Equation (6).We should take into consideration that Hansen and Sargent (2003) assumethat all agents share the same approximating model and their doubtsare common knowledge. The starting point consists in considering the model represented by Equation (6) as an approximation of the following true model is given by Equation (7):

$$y_{t+1} = Ay_t + BU_t + CW_{t+1} \quad (7)$$

Where W_{t+1} represent a vector of unknown specification errors finite, around the approximating model used by the leader where $W_{t+1} = 0$. Its can feed back, possibly nonlinear on the history y_t (Its are history dependent meaning that W_{t+1} depend on lags of z_t , the state variables.

Hansen and Sargent (2003) impose a constraint upon the variances of the misspecifications what simplifies the problem. They investigate the determination of the decision rules conducting worst-cases analyses. All the agent share the same approximating model and their errors are common knowledge. The policymaker considers the model as the reference model, which represents the most likely description of the economic structure. However, he knows this model could be subject to a wide range of distortions. Under robust control, the resulting policy rule performs sufficiently well even if the underlying economic structure does not coincide with the policymaker's reference model.

Hence, the equilibrium is the outcome of a 2-person game. The central Bank wants to minimize the maximum welfare loss due to model misspecifications by specifying an approximate policy.The "*evil agent*" shares the same reference model that the centralbank and the same objective function but he wants to maximize ratherthan minimize the loss contrary to the policymaker. They use a three step algorithm for solving a multiplier version ofthe robust Stackelberg problem. The first step is similar to theprevious section (2.2.1). It uses Bellman equation and yields to a Riccati equation which determines implicitly the optimal rule.Steps 2 and 3 use the lagrangian problem and the first orderconditions in order to obtain a relation between the lagrangian multipliers and y_t . Then we can obtain a robust optimal rule inwhich current state variables are function of the past statevariables.

C. Model uncertainty

Levin and Williams (2003) show optimal rules for a given model havea representation that is invariant to know changes in the shockprocess. Thus, these rules are not robust to varying model. Theywonder whether any simple rule can provide robust performance acrossa range of alternatives representations of the economy. They showthat a robust outcome is possible only in cases where the objectivefunction

[2]For more details on the Stackelberg problem see for instance "Market Structure and Equilibrium".

places substantial weight on stabilizing both output andinflation.

Each of 3 models they use, provide a plausible representation of thedynamic behaviour of the US economy and represent differentperspectives about expectations formation and other structuralcharacteristics of the economy, that is to say, a New Keynesian Benchmark (NKB) model(forward looking model), the Rudebuschand Svensson (1999),(RS) model (VAR model) and the Fuhrer (2000) Habit Persistence (FHP) model in which the inflation rateresponds to a combination of forward looking and backward lookingterms.

They assume that the policymaker's loss function $\mathfrak{L}$ has theform given by Equation (8):

$$\mathfrak{L} = \mu \, Var(\pi_t) + \lambda Var(y_t) + \phi Var(i_t). \quad (8)$$

Where, μ , λ and ϕ denote weights attached to the stabilization of inflation, output gap and interest rate, respectively. Identifying the source of the lack of robustness of each optimalrule is difficult because of their complicated formulation in termsof leads and /or lags of the target variables.Thus, Levin and Williams (2003) use the following class of simple three parameters (α, β, ρ)rules are given by Equation (9):

$$i_t = \rho i_{t-1} + (1-\rho)\bar{\pi}_t + \alpha \bar{\pi}_t + \beta y_t \quad (9)$$

When the model is not the one used to define the control parametersthe authors compare the value obtained with the optimal value. In fact, the Equation (10) below interpreted their results using a "fault tolerance"approach of each model with respect to deviations from optimalpolicy. For example, if the true model is (RS) and the policymaker choose NKB or FHP so the consequences in terms of loss couldgenerate indetermination or bad results.

$$\%\Delta\mathfrak{L} = 100 * (\mathfrak{L}_{\text{rule generated}} - \mathfrak{L}_{\text{optimal rule}}) / \mathfrak{L}_{\text{rule generated}} \quad (10)$$

These rules can generate dynamic instability. This method leads to robust results for a large range of macro-models.In general, the results naturally depend on the preferences of the policy (λ and ϕ), However, they also find large differences depending on the chosen reference model.

As Levin and Williams (2003), we study the robustness of the models by the Bayesian approach to obtain a robust rule. This approach represented by Equation (11) is to minimize the objective function using the control variables (α , β , ρ) under constraints of the three (RS, NKB, FHP) models.

$$\mathfrak{L}^B = \omega_{NKB}\mathfrak{L}_{NKB} + \omega_{FHP}\mathfrak{L}_{FHP} + \omega_{RS}\mathfrak{L}_{RS} \quad (11)$$

Where $\sum \omega = 1$. For simplicity we assume that the coefficients in Equation (11) are equal.

III. PRICING SCHEME

In this robustness analysis, we consider three non-nested alternativesrepresentations of the inflation dynamics, each one having formalmicroeconomic foundations. These 3 models have the same IS curvein common, in order to be focused on uncertainty about the type ofnominal rigidity, ie:

- **Calvo rigidity**: where in each period, each firm is able to reviseits price with probability $1-\alpha$. Conversely, it must keep its price unchanged from one period to the next with probability α .
- **Taylor rigidity**: The Taylor model is based on the idea that there are rigiditiesconnected to the existence of contracts in the economy. Taylor (1980) uses 2-period contracts. The idea is that the firm can only choose its price every second period. Contrary to Calvo rigidity, this modelization assumes that firms know they cannot change theirprice in the following period. However, they can do it with theprobability 1 within 2 periods. A time t, there are 2 types ofcontracts in the economy: those which are defined at time t andthose are defined at time t-1.
- **Sticky information**: Contrary to the two previous models, firms adjusts their prices at each time bearing in mind that the information set does not develop at the same time.The arrival of information update opportunity is similar to Calvomodel. Every period only a random fraction α of firms receivenew information about the state of the economy.

Of course, the Phillips curves we obtain are fundamentally different.

A. *Common framework*

Consider an economy consists of a continuum of household (mass 1) and a continuum of firms (mass 1).). Indeed, a common framework is a mean to obtain comparable New Keynesian Phillips curves and to explain the main different responses observed across each specification essentially by the nature of nominal rigidities.

1) *Firms*

At each time t , each firm (j) produces one differentiated good (j) using a linear technology: $y_t(j) = a_t l_t(j)$, where y_t the output gap and a_t is the labor productivity and $l_t(j)$ is the labor demand within a monopolistic competition framework on the goods market *a la Dixit-Stiglitz*(1977). The Equation (12) represents the price index and the inflation are written respectively:

$$P_t = \left(\int_0^1 p_t(j)^{1-\theta} dj \right)^{1/1-\theta} \quad (12)$$

and

$$\pi_t = \ln(P_t / P_{t-1})$$

2) *Households*

The workers are assumed mobiles across sectors and the labor supply l_t is assumed independent of the household. In equilibrium:

$$l_t = \left(\int_0^1 l_t(j)dj \right) \quad (13)$$

Each household maximizes the utility[3] function at time t:

$$E\left(\sum_{t=0}^{\infty} \beta^t u(C_t, l_t) \right) \quad (14)$$

With the consumption index:

$$C_t = \left(\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}} \quad (15)$$

Where $c(j)$ is the j firm's consumption of good z, θ represents the elasticity of substitution between varieties of the consumption good.

Consider the following instantaneous CES Utility function:

$$u(C_t, l_t) = \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \frac{1}{1+\eta} l_t^{1+\eta} \quad (16)$$

Where η is the intertemporal labor supply elasticity and σ^{-1}, is the intertemporal consumption elasticity. Moreover, at each period, the representative household faces a budget constraint represented by Equation (17) as follows:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + w_t l_t + \Pi_t \quad (17)$$

Where w_t is the real wage, Π_t is the firms profit in time t and B_t represents the amount of riskless assets holding by the household at time t.

3.1.3 IS CURVE

The maximization of the utility[4] function subject to constraint budget gives the first order condition of the problem. The log-linearization of the Euler condition leads to the optimizing IS curve:

$$\hat{y}_t = \mathbb{E}_t(\hat{y}_{t+1}) - \sigma \mathbb{E}_t(\hat{R}_t - \hat{\pi}_{t+1}) \quad (18)$$

Considering $\hat{x}_t = \hat{y}_t - \hat{y}_t^{(n)}$, and $\hat{i}_t$ the deviation of the interest rate from its level when the prices are flexible, we can rewrite the IS curve as follows:

$$\hat{x}_t = \mathbb{E}_t(\hat{x}_{t+1}) - \sigma(\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1})) \quad (19)$$

3.2.Framework price setting

3.2.1. Calvo pricing

[3]We do not include real money balances (M/P) into our utility function. Because DSGE models assume nominal short-term interest rate as the monetary policy instrument, so that money supply is considered as endogenous; see for instance, Woordford (2003).

[4]Call $\widehat{u_t}$ the proportional deviation of u_t around its steady state value $\bar{u}$: $(\widehat{u_t} = \log(u_t / \bar{u}))$.

In each period, a fraction $1-\alpha$ of firms, drawn at random, are allowed to reset their price optimally. The others are constraints to keep their prevailing price and do not re-optimize.

As a result, the likelihood of any one firm to be able to change its price is independent from the past and all firms who change their price choose the same price.

Soit p_t^* le prix fixé par les firmes qui peuvent ajuster leur prix.

The dynamics of the price index is given by Equation (20):

$$P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)(p_t^*)^{1-\theta} \quad (20)$$

and the log-linearization of price index leads to Equation (21):

$$\hat{P}_t = \alpha \hat{P}_{t-1} + (1-\alpha)\hat{p}_t^* \quad (21)$$

An each time t, the firm must set its price taking account of the risk that it will not be allowed to change its price in the future. Let p_t^* be the optimal nominal price which maximize the escompted sum of expected profits, multiplied by the probability to not modify the price. The log-linearization gives:

$$\hat{p}_t^* = (1-\beta\alpha)\mathbb{E}_t\left(\sum_{s=t}^{+\infty} (\beta\alpha)^{s-t}(\hat{\phi}_s + \hat{P}_s) \right) \quad (22)$$

Where $\phi_s = \frac{w_s}{a_s p_s}$ is the real marginal cost at time s. It is a function of current and future expected nominal marginal costs.Combining Equation (22) with the one including $\hat{P}_t$, $\hat{P}_{t-1}$ and $\hat{p}_t^*$, we obtain Equation (23):

$$\hat{\pi}_t = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \widehat{MC}_t + \beta \mathbb{E}_t(\hat{\pi}_{t+1}) \quad (23)$$

Optimal reset prices are given by marginal costs, as firms maximize profits in an imperfectly competitive environment. Moreover, the determination of the labor supply and the price chosen by firms gives the Equation (24):

$$(\eta + \frac{1}{\sigma})(\hat{y}_t - \hat{y}_t^n) = \hat{\phi}_t \quad (24)$$

Thus, noting $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$ and the Equations (23) and (24), the Phillips curve can be written by Equation (25):

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t(\hat{\pi}_{t+1}) \quad (25)$$

With, $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\eta + \frac{1}{\sigma})$ .We assume at each period, an exogenous shock $\epsilon_t = \begin{pmatrix} \epsilon_t^{\pi} \\ \epsilon_t^{x} \end{pmatrix}$ affects the economy structure. Finally, the IS and Phillips curve[5]can be written by Equation (26) as follows:

[5] See Equations (16) and (22).

$$\begin{cases} \hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t(\hat{\pi}_{t+1}) + \epsilon_t^{\pi} \\ \hat{x}_t = \mathbb{E}_t(\hat{x}_{t+1}) - \sigma(\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1})) + \epsilon_t^{x} \end{cases} \tag{26}$$

3.2.2. Taylor contracts

Following Taylor (1980), firms are followed to reset their contract price every 2 periods. Firms are otherwise symmetric in every other respect. At any period, 2 overlapping contracts are in force.

At time t, consider $p_t(t-1)$, the price defined by the firm which has updated at period t-1. The program of the firm which is allowed to reset its price contracts, at time t, is:

$$\max_{p_t(t)} \mathbb{E}_t \left[\sum_{i=0}^{1} \left(\frac{p_t(t)}{P_{t+i}}\right)^{-\theta} C_{t+i} \beta^i \left[\frac{p_t(t)}{P_{t+i}} - \phi_{t+i} \right] P_{t+i} \right] \tag{28}$$

And the first order condition gives the Equation (29):

$$p_t(t) = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t\left[C_t \phi_t P_t^{\theta+1} + \beta C_{t+1} \phi_{t+1} P_{t+1}^{\theta+1}\right]}{\mathbb{E}_t\left[C_t P_t^{\theta} + \beta C_{t+1} P_{t+1}^{\theta}\right]} \tag{29}$$

Moreover, the price index P_t is a function of the prices fixed at time t-1 and those at time t. In a symmetric 2-period setup, the log of the aggregate price level is given by Equation (30):

$$\hat{P}_t = \frac{\hat{p}_{t-1}(t-1) + \hat{p}_t(t)}{2} \tag{30}$$

Combining Equations (25) and (26), one obtains around the steady state: $\bar{\phi} = \frac{\theta - 1}{\theta}$ .Thus,

$$\frac{p_t(t)}{\bar{P}} = \frac{\mathbb{E}_t\left[\frac{C_t \phi_t}{\bar{C}\bar{\phi}} \left(\frac{P_t}{\bar{P}}\right)^{\theta+1} + \beta \frac{C_{t+1}\phi_{t+1}}{\bar{C}\bar{\phi}} \left(\frac{P_{t+1}}{\bar{P}}\right)^{\theta+1} \right]}{\mathbb{E}_t\left[\frac{C_t}{\bar{C}} \left(\frac{P_t}{\bar{P}}\right)^{\theta} + \beta \frac{C_{t+1}}{\bar{C}} \left(\frac{P_{t+1}}{\bar{P}}\right)^{\theta} \right]} \tag{31}$$

Using a fist-order Taylor approximation of the kind $\frac{x_t}{\bar{x}} = 1 + \hat{x}_t$, to the equation (30), we find:

$$\hat{P}_t = \left(\frac{1}{2}\right) \frac{1}{1+\beta} \begin{bmatrix} \xi \hat{x}_t + \hat{P}_t + \beta \xi \mathbb{E}_t(\hat{x}_{t+1}) + \beta \mathbb{E}_t(\hat{P}_{t+1}) \\ + \xi \hat{x}_{t-1} + \hat{P}_{t-1} + \beta \xi \mathbb{E}_{t-1}(\hat{x}_t) + \beta \mathbb{E}_{t-1}(\hat{P}_t) \end{bmatrix} \tag{32}$$

However, as $\hat{\phi}_t = \xi \hat{x}_t$ with $\xi = \lambda + \frac{1}{\sigma}$ and $\hat{x}_t$ the output-gap, the inflation $\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1}$,is given by the Phillips Curve:

$$\pi_t = \frac{1}{1+\beta} \begin{pmatrix} \xi(x_t + x_{t-1} + \beta(\mathbb{E}_t(x_{t+1}) + \mathbb{E}_{t-1}(x_t)) \\ + \beta(\mathbb{E}_t(\pi_{t+1}) + \mathbb{E}_{t-1}(\pi_t)) \end{pmatrix} \tag{33}$$

Finally, we obtain the following system of equation, adding demand and supply shocks:

$$\begin{cases} \hat{x}_t = \mathbb{E}_t(\hat{x}_{t+1}) - \sigma(\hat{i}_t - \mathbb{E}_t(\hat{\pi}_{t+1})) + \epsilon_t^{x} \\ \hat{\pi}_t = \frac{1}{1+\beta}\left(\xi(\hat{x}_t + \hat{x}_{t-1} + \beta(\mathbb{E}_t(\hat{x}_{t+1}) + \mathbb{E}_{t-1}(\hat{x}_t))) + \beta(\mathbb{E}_t(\hat{\pi}_{t+1}) + \mathbb{E}_{t-1}(\hat{\pi}_t))\right) + \epsilon_t^{\pi} \end{cases} \tag{34}$$

3.2.3. Sticky Information framework price setting

Mankiw and Reis (2002) assume that information diffuses slowly throughout price setters. That means that when a firm sets its price, it has not full information about the state of the economy. Nevertheless prices are flexible in the sense that firms can change their price every period, but they do not have necessarily the information about the actual state of the economy. The fraction of firms that did not receive new information set prices according to their older information set. Therefore prices fixed based on different information sets coexist in the economy.

Formally, at time t, a firm f that up dated its information, j period ago, set its price $x_t(f)$ such as:

$$x_t(f) = \mathbb{E}_{t-j}(p_t^*(f)) \tag{35}$$

Where $p_t^*(f)$ is the f could choose if it receives information at time t. In that case, the firm would choose $p_t^*(f)$ which maximizes its profit given by Equation (36):

$$p_t^*(f) = \arg\max_{p_t(f)} \left(\left(p_t(f) - \frac{w_t}{a_t} \right) \left(\frac{p_t(f)}{P_t} \right)^{-\theta} C_t \right) \tag{36}$$

Where P_t is the overall price level at time t .Finally, the Equation (37) yields the expression of optimal price:

$$p_t^*(f) = \frac{\theta}{\theta - 1} P_t \phi_t \equiv p_t^* \tag{37}$$

The log-linearization of the overall price level is given by Equation (38):

$$\hat{P}_t = \alpha \sum_{j=0}^{+\infty} (1-\alpha)^j \mathbb{E}_{t-j}(\hat{p}_t^*) \tag{38}$$

Subtracting price index (38) at time $t-1$ and using the first order condition (37), the inflation equation can be written as follows:

$$\hat{P}_t - \hat{P}_{t-1} = \alpha \sum_{j=0}^{+\infty} (1-\alpha)^j \left(\mathbb{E}_{t-j}(\hat{P}_t + \hat{\phi}_t) - \hat{P}_{t-1} \right) \tag{39}$$

Rearranging the terms in the Equation (39) above, adding in particular the equation relative to the overall price level and supply and demand shocks, we obtain the Equation (40):

$$\begin{cases} \hat{x}_t = \mathbb{E}_t(\hat{x}_{t+1} - \sigma_{t+1}(\hat{i}_t - \hat{\pi}_{t+1}) + \epsilon_t^x) + \epsilon_t^x \\ (1-\alpha)\hat{\pi}_t = \alpha\xi\hat{x}_t + \alpha \sum_{j=1}^{+\infty} (1-\alpha)^j \mathbb{E}_{t-j}\left(\hat{\pi}_t + \xi(\hat{x}_t - \hat{x}_{t-1}) \right) + \epsilon_t^\pi \end{cases} \tag{40}$$

IV. ESTIMATION

In this section, we use Bayesian estimation (Geweke (1999), DeJong*et al*. (2000)) of the Calvo model. The results of this estimation will be also used for the others models in the next sections.

A. *Explanation of prior's choice*

The reasons of the priors' choice refer to Haider and Drissi (2009); Rabanal and Rubio-Ramirez (2005) and Smets and Wouters (2003) who estimate a Calvo model with sticky prices and wagesusing European time data; Ben Aïssa and Rebei (2009) . All the variances of the shocks are assumed to be distributed as an inverted Gamma distribution which guarantees a positive variance with a rather large domain. Certain parameters need the specification of a policy rule. Smets and Wouters (2003) have shown that a Taylor (1993) rule would approximate the behavior of 'synthetic' central bank conduct of policy quite well. We use the following Taylor rule given by the Equation (38):

$$i_t = \rho i_{t-1} + (1-\rho)(\gamma_\pi \pi_t + \gamma_y y_t) + \sigma_i \tag{41}$$

We set the mean of γ_π to 1.5 and that of γ_y to 0.5, which is Taylor's original guess ( ) . The interest ratesmoothing coefficient ρ has a beta prior distribution. The discount factor β is calibrated to be 0.99 (which is quite conventional in the literature). We imposed dogmatic priors over the parameters α , η and ρ because of an identification problem. Since we are interesting in the standard deviation of two main parameters considered as uncertain, we set α to 0.5. The inverse elasticity of labor supply has a normal prior distribution with a mean of 2 and a standard error of 0.75. And the intertemporal elasticity of substitution σ^{-1} has a normal prior distribution with a mean of 1 and a standard deviation of 0.375.

Table-1.Priors and posteriors for model Calvo.

Variable	Distribution of prior	Medium of prior	Mode of posterior	Interval Confidence	of mode	Variance of prior
σ^{-1}	normal	1.000	3.3106	3.2174	3.3588	0.3750
η	normal	2.000	3.7607	2.1302	4.9426	0.7500
ρ	normal	0.800	0.9591	0.9564	0.9620	0.0500
Standard	derviation of schocks:					
π	Inverse gamma	0.501	1.6067	1.3708	1.8062	0.2621
x	Inverse gamma	1.253	0.7253	0.6294	0.8123	0.6551

TheTable 1 shows the results of the Bayesian estimation of the Calvo model using data (1990-2010). The output gap, the inflation and the interest rate have been centered in order to match with variables used in the model. The results suggest that σ is equal to 0.3021 with anestimator standard deviation of 0.00038. η is estimated to 3.7607 with an error of 0.7031. These values give us an idea of parameters uncertainty even if the estimation is not the only source of uncertainty (misspecification...). These figures have been usedin all numerical resolutions which followed (even if were not Calvomodel).

B. Robust optimal policy

It is possible to obtain a robust optimal policy a la Levin and Williams (2003) but this one depends on several variables of each model. However, once the model is solved, the optimal policy depends only on past variables which can be known in a recursive way. Therefore it is possible but not obvious and operational to determine the optimal rule.

That is why we focused on the study of the simple rules during this second step of robustness analysis, such as the following Taylor rule is represents by Equation (42):

$$i_t = \theta_1 i_{t-1} + (1-\theta_1+\theta_2)\pi_t + \theta_3 x_t \qquad (42)$$

This type of rule is of course sub-optimal in comparison with optimal rule but easier to elaborate.The Tables 2 and 3 have been obtained with thefollowing loss function[6] given by Equation (43):

$$L = Var(\pi_t) + Var(y_t) + 0.5Var(i_t) \qquad (43)$$

[6]The values shown in the following table are obtained with a weight in the loss function equal to 1 on the variance of inflation and the output gap andequal to 0.5 in the variance of interest rates.

Table-2. Parameters of the simple rule based on the model chosen.

Models	θ_1	θ_2	θ_3
Calvo	1.34	1.52	2.13
Taylor	1.04	1.43	2.05
Sticky	20	9.5	757
All	0.99	6.67	1.85

The 'all' in Table 2 is refers to the case where the three (Calvo, Taylor, Sticky) models are taken into consideration at the time and with the same probability.Table 2reports the results of each model according to the parameters of the simple rule represents by Equation (14). We can see that the Calvo and Taylor models are quite similar while the Sticky information model behaves differently. This feature shows to which extend it is important to establish a robust optimal rule. The parameters obtained with the Bayesian loss function areidentical to those from the Calvo and Taylor models for θ_1 and θ_3 . But on the contrary, they adapt an intermediateposition for θ_2 .0

Table-3.Comparison of losses in the different combinations.

	$\theta Calvo$	$\theta Taylor$	$\theta Sticky$	θAll
Calvo	2.2057	2.2686	6.5369	2.6125
	(0.0)	**(2.9)**	**(196.4)**	**(18.4)**
Taylor	2.492	2.3563	7.766	2.5681
	(5.8)	**(0.0)**	**(229.6)**	**(9.0)**
Sticky	9.4109	9.163	6.6602	7.2169
	(41.3)	**(37.6)**	**(0.0)**	**(8.4)**

The column "All" in Table 3 is used when we consider the three models at the same time with equal probability. Between parentheses are indicated the percentage of "fault Tolerance".We remind the "fault Tolerance" concept as follows:

$$\Delta L = \frac{L_{\text{another model}} - L_{optimal\ model}}{L_{optimal\ model}} \quad (44)$$

Which means the relative difference between loss when we use the optimal parameters for all model or those obtains with another model.Table 3, just like Table 2, shows the big difference between the model with rigidities according to the sticky information and the two others. Then, using the parameters θ_{Sticky} , while the true model is the one with Calvo rigidity or Taylor rigidity, we obtain losses about 200% higher than the ones obtained choosing the parameters associated.On the contrary, using the Calvo or Taylor parameters leads to a "fault tolerance" from around 30% to 40%.

Using a robust optimal simple rule reduces this gap between the Sticky information and the others. In fact, when we use θ_{All} , we observe a loss close to that one which the optimal parameters for the three models. The "fault tolerance" is therefore very weak in comparison with the worst case and with the mean of the possible "fault tolerance". We should use such arobust optimal rule when we do not know which model represents thebest description of the economic structure. Table 3 suggests that the second parameter is more important for the Sticky information model while the two others one are more important for the Calvo and Taylor models.

In conclusion, a robust rule allows to minimize the most common error and above all to avoid big mistakes. The robust simple rule is quite the same except for the weight of inflation which is slightly higher. That is not very surprising since we knew as we have seen in section 2.2 that the Sticky information model is different because there is a real inertial of the past inflation on the current inflation.

V. CONCLUSION

We studied in this paper the robustness of monetary policy with two types of uncertainty: uncertainty about the parameters and uncertainty between models.

The uncertainty about parameter has been examined within a Calvo model. It has been introduced into the reduced form. However, the way to introduce it is something which should be deepened. One could think mainly of micro-founded uncertainty which could affect the structural parameters. The general and intuitive result is that the uncertainty about the parameter makes the policymaker more careful than in the case under certainty. His caution rises with the weight associated to the interest rate in the loss function. An interesting extension would be to make this weight endogenous, in particular make it dependent on the structure of the economy and the uncertainty about parameters. So it could be of interest to use a loss function more in accord with the micro-foundations of the models.

The results obtained show the efficiency of the robust simple rule of Levin and Williams. It could be interesting to follow the same procedure including uncertainty parameters within each model. In that case, we might obtain a robust rule for two kinds of uncertainty.The resolution of the robust policy with parameter uncertainty can be made according to 2 equivalent approaches: the Bellman equation and the lagrangian equation.The second one is more convenient for the more complex model like the Taylor or Sticky information models. But due to the non-linearity of the constraints in a multiplicative uncertainty framework, a numerical resolution is not obvious.

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