Numerical Simulation of the Deformation of Some MEMS

Gutiérrez G. S., Bermúdez B., Alanís J. D.

***Abstract*— In this paper we present the numerical simulation of the deformation of two Micro-Electromechanical Systems (MEMS), a trampoline-type one i.e. a rectangular cantilever beam and an accelerometer that consists of a square plate with all edges simply supported. The deformation of these systems is modeled by fourth-order differential equations, ordinary and partial respectively. We find the approximate solutions by using the finite differences method programmed in Matlab, solving the system of linear equations associated with different methods to evaluate the efficiency of these. We obtained very good approximations with small errors compared to other articles that use other approaches.**

***Index Terms*— Accelerometer-type MEMS, differential equations, finite differences, MEMS, trampoline-type MEMS.**

I. INTRODUCTION

The Micro-Electromechanical Systems (MEMS) are devices created by using nanotechnology that combine sensors and actuators that sense and control physical parameters at microscale. Now days there is a great number of MEMS with applications in medicine, engineering, telecommunications, etc. [6].

The trampoline-type MEMS are mainly used as sensors in biology and chemistry, e.g. for the measurement of chemical absorptions in the order of picograms or inbiomolecular measurements [7]. Consider a rectangular cantilever beam subject to certain forces. Such forces cause a deformation in the beam which is the main topic of this article.

There are many different accelerometer-type MEMS that are used, for example, to activate air bags or in electronic devices to detect its orientation. This device consists of a square plate with all edges simply supportedand receive an electric pulse causing a deformation in the z axis [3], [6].

The main goal of these simulations isa saving in production costs, also in time and efforts needed to achieve the good operation of the devices [3].

We will compare the results obtained with an analytical solution for the differential equations (when possible) and also with simulations realized in previous works, like in [1] and [3], in which solutions were obtained by using similar methods.

In section 2 we present the differential equations that governthe devicesbehavior.

Later, in section 3, we give a brief explanation of the finite difference method used to obtain the solution for the differential equations. For this kind of systems, it is more common to use the finite element method [1], [2].

Sergio Gutiérrez Galarza, Departamento de matemáticas y física, Universidad Autónoma de Aguascalientes, Aguascalientes Ags., México,
Blanca Bermúdez Juárez, Facultad de Ciencias de la Computación, Benemérita Universidad Autónoma de Puebla, Puebla, México,
José David Alanís Urquieta, Carrera de Tecnologías de la Información y Comunicación, Universidad Tecnológica de Puebla, Puebla, México.

II. MATHEMATICAL MODELS

For both situations a physical analysis of the behavior leads to a fourth-order differential equation. These equations are presented below.

A. *Model 1: Trampoline-type MEMS*

Suppose that we have the cantilever beam shown in figure 1 and there is an area stressed out*s*,uniformly distributed on the upper surface. It is possible to model this as a uniformly distributed axial stress *sw*along the beam's neutral axis and a uniformly distributed bending moment *m* along the beam as showing un figure 2[4].

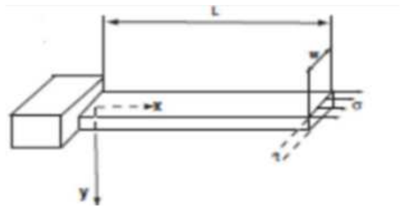

Figure 1: Characteristics of the cantilever beam[4]

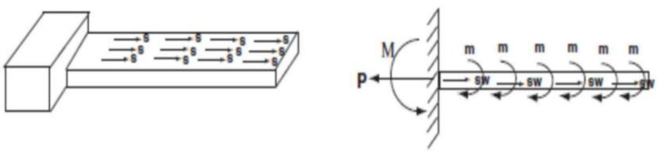

Figure 2: Forces applied to the beam [4]

To avoid dimensional problems, we formulate the equation in terms of the dimensionless variables below [1], [2]:

$$\xi = \frac{x}{L} \quad , \qquad Y = \frac{y}{L}$$

The equation that describes the behavior of the beam is [2], [4]:

$$Y'''' - (\beta L)^2(1-\xi)Y'' + (\beta L)^2 Y' = 0 \qquad (1)$$

Here$\beta = (swL)/E^*I$, where I is the area moment of inertia andE^* is the biaxial modulus, defined as$E^* = E/(1-v)$ with$E$ the Young's modulusand v the Poisson ratio.

The boundary conditions for (1) are the following ones [2], [4]:

$$Y(0) = 0, \quad Y'(0) = 0, \quad Y''(1) = 0$$

$$Y'''(1) + \frac{\alpha(\beta L)^2}{2} = 0$$

where$\alpha = t/L$.

Because the variable ξ appears multiplying the second derivative in equation (1) it is very difficult to obtain an analytical solution. For this reason, is necessary to use a numerical approximation.

B. *Model 2:Accelerometer-type MEMS*

For the accelerometer described in the introduction, the deformation of the plate is modeled by a Bi-Laplacian type equation [3], [5]:

$$\frac{\partial^4 \omega}{\partial x^4} + 2\frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = q \qquad (2)$$

Where ω is the deformation of the plate, and$q$isthe force magnitude applied to the plate given by the following equation:

$$q = \frac{q_0}{D}\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$

Considering the length and width of the plate as$a$ and$b$ respectively. Here q_0 is the force applied in the middle of the plateand$D$is the material rigidity. In turn, such rigidity depends on the material Young's modulus (E), the inertial moment (I) and the Poisson ratio (v) that is given by:

$$D = \frac{EI}{1 - v^2}$$

The boundary conditions for this model are the following ones:

$$\omega = 0, \ \frac{\partial^2 \omega}{\partial x^2} = 0 \text{ para} x = 0, \ \ x = a$$

$$\omega = 0, \ \frac{\partial^2 \omega}{\partial y^2} = 0 \text{ para} y = 0, \ \ y = b$$

For simplicity we suppose that$a = b = 1$ therefore the equation (2) has the following analytical solution [3], [5]:

$$\omega = \frac{1}{4\pi^4}\frac{q_0}{D}\sin(\pi x)\sin(\pi y)$$

III. NUMERICAL SOLUTION

The finite difference method that we use to obtain the approximate solutions for equations (1) and (2),consists of a domain discretization obtaining a mesh with certain step length h. In each node we approximate the derivativesthrough formulas that relate the function value in the node and its neighbors. In this way, after the discretization we get a system of linear equationswhose solution is the approximate value of the solution of equations (1) or (2) at each node.

A. *Model 1*

After applying this procedure to equation (1) we obtained the following equations for interior nodes, i.e. nodes away from the boundary:

$$\begin{aligned} Y_{n-2} - \frac{1}{2}\left[8 + h^2(\beta L)^2[2(1-\xi) + h]\right]Y_{n-1} \\ + [6 + 2h^2(\beta L)^2(1-\xi)]Y_n \\ - \frac{1}{2}\left[8 + h^2(\beta L)^2[2(1-\xi) - h]\right]Y_{n+1} \\ + Y_{n+2} = 0 \end{aligned}$$

Applying the boundary conditions, we obtained similar equations for nodes near the boundary.

The relative error is a number that represents a measurement (or a percentage) of how far is the result from the real solution:

$$E = \frac{\left\| y_{real} - y_{approx} \right\|}{\left\| y_{real} \right\|}$$

Since we do not have an exact solution for this model (in 1D), a mesh size independence study was performed, that is, we are considering results obtained with a finer mesh as the exact solution, and the ones with a coarser mesh as the approximation to the solution, and we calculate the relative error.

For this model we take $(\beta L)^2 = 0.1$ and $\alpha = 0.05$ obtaining the graph showed in figure 3 for the approximation for the deformation.

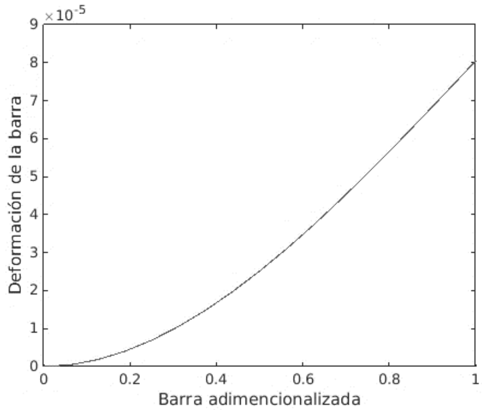

Figure 3: Approximate solution for model 1 with h=1/16

The relative error computing with different size of grids is shown in table 1.

Mesh	Error relative
h=1/5 vs h=1/10	0.0339
h=1/5 vs h=1/20	0.0807
h=1/5 vs h=1/40	0.1705
h=1/10 vs h=1/20	0.0453
h=1/10 vs h=1/40	0.1321
h=1/20 vs h=1/40	0.0831

Table 1: Relative error for model 1

We observed that the error is about 17% (in case h=1/5 vs h=1/40) and in the case h=1/20 vs h=1/40 the error was reduced to half of its value which means a good approximation is obtained.

In [1] similar results are obtained using the Finite Element Method getting an error of about 18%.

B. Model 2

For this model we have the following equations for the interior nodes

$$20\omega_{ij} - 8[\omega_{i-1j} + \omega_{i+1j} + \omega_{ij-1} + \omega_{ij+1}] + 2[\omega_{i-1j-1} + \omega_{i-1j+1} + \omega_{i+1j-1} + \omega_{i+1j+1}] + \omega_{ij-2} + \omega_{ij+2} + \omega_{i+2j} + \omega_{i-2j} = q_{ij} h^4$$

Again with similar equation for nodes near to the bound applying the boundary conditions.

We used two different methods for solving the system of linear equations, LU factorization and SOR (Successive Over-Relaxation). LU factorizationis a direct method to obtain the exact solution for the system,this represents a decrease in the error of the approximation but requires a high computation time.

In this model we consider the silicon qualities to obtain the parameters of equation (2):

$$\frac{q_0}{D} = 7.0544 \times 10^{-5}$$

In figure 4 is shown the approximation taking 64 nodes in each axis.

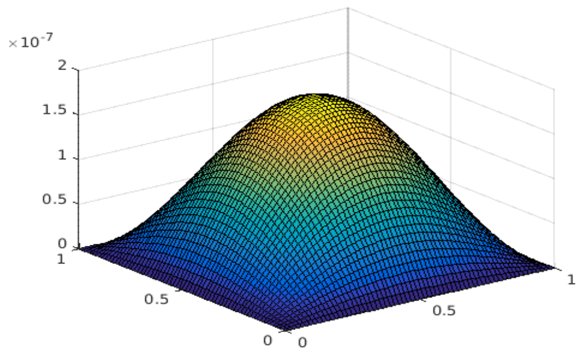

Figure 4: Approximate solution using LU factorization for model 2 with h=1/64

As we mentioned above we solve the system of linear equations using two different methods. The tables 2 and 3 show the errors and CPU times for the two methods and different values for *h*

.

h	Relative error	Time in seconds
1/16	0.0064	0.3
1/32	0.0016	2.6
1/64	0.0004	56
1/100	0.00016	552

Table 2: Time and error for LU factorization

H	Tolerance	Relative error	Time in seconds
1/16	10^{-11}	0.0462	0.57
1/16	10^{-15}	0.0065	1.49
1/32	10^{-12}	0.1272	188
1/32	10^{-15}	0.0015	1288

Table 3: Time and error for SOR

One can observe the increased accuracy and efficiency of LU factorization. With an error of 0.016% for 100 nodes with an error of 0.15% for 32 nodes with SOR.

For the solution with SOR it is necessary to give a very small error tolerance and a large number of iterations.With less than 10^{-12} and 5000 the method seems no to converge.

V. Conclusions

For Model 1, the results are similar to those presented in [1], where the finite element method was used, which as mentioned in the introduction is the usual method used for this kind of models, However, there is a small error reduction.

On the other hand, the error was very small with model 2, the difference between direct and iterative model to solve the system of linear equations lies in the associated matrixwhich is tri-diagonal by blocks and the LU factorization algorithm will help to avoid unnecessary operations. However, for the SOR the samenumber of operations are performed at each iteration and because the dimensions of the variables the convergence is very slow.

These problems can be solved using different packages of software such as FEMLab, ANSYS and its derviates, COMSOL, even a matlab toolbox, but the idea is to have a own software and no to depend of commercial software.

These results can be used in future articles to perform simulations of other types of MEMS, such as gyroscopes, comb engines, etc.

In addition, it is intended to perform simulations by using our own techniques of visualization and translate the method to be implemented by using free software.

References

[1] AlanísUrquieta J. D., BermúdezJuárez B., Hernández Rebollar J. L.: *Simulation of type trampoline's MEMS by Finite Element Method using Java. 1st.* ICIAS (2010), SOMI CCADET UNAM.

[2] Cruzado L., González A.L, Alanís J.D:,Bermúdez B., Hernández J.L.: *Numerical Simulation of Trampoline-Type MEMS*, ICCME (2015), Kerala, Kochi, India.

[3] Alanís José D ., Bermúdez B., Hernández José L.: *Solución de Ecuaciones Diferenciales tipo Bilaplaciano para la Simulación de MEMS tipo acelerómetro*1° CIICA, CCADET UNAM (2013)

[4] Yin Zhang, QuanRen, Ya-pu Zhao: *Modelling analysis of surface stress on a rectangular cantilever beam* J. Phys. D: Appl. Phys. 37 (2004) 2140-2145.

[5] Timoshenko, S, Woinowsky Krieger: *Theory of Plates and Shells*, McGraw hill International Book Company, USA (1984).

[6] Maluf, Nadim: *An Introduction to Micro- electromechanical Systems Engineering*, Artech House Publishers, USA (2000).

[7] Zhao, Yue: *Surface stress detection and mechanism study with microcantilever based sensor for biomolecular monolayers*, Graduate Theses and Dissertations, Paper 13690, USA (2014).

[8] Burden, R. L. and Faires, J. D.: *Numerical Analysis*, 9th edn. Brooks/Cole Cengage Learning (2011)

Sergio Gutiérrez Galarza Student of appliedmathematics in Universidad Autónoma de Aguascalientes México. AMC summer of scientific's research 2016.

Blanca BermúdezJuárez Professor and researcher Titular B FCC BUAP, PhD. in applied mathematics, member of National System of Researchers (SNI).

José David AlanísUrquieta Full time professor and researcher associated C, TICS UTP. PhD. in technologies of information and decisions' analyze, member of the Program for the Professional Development of teachers (PRODEP).