

Path Relatively Prime Cordial Graph

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Abstract— Let $G = (V, E)$ be a graph with p vertices and q edges. A Analytic Mean Cordial Labeling of a Graph G with vertex set is a bijection from $v = \{1, 2, \dots, p\}$. The induced edge labelling are defined by $f(u, v) = 0$ if either $f(u)$ divides $f(v)$ (or) $f(v)$ divides $f(u)$ one otherwise and if any one of the vertex label is 1, the induced edge label is 0.

The graph that admits a Relatively Prime Cordial Graph is called Relatively Prime Cordial Graph. In this paper, we proved that path related graphs $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$ are Relatively Prime Cordial Graph.

Index Terms— Relatively Prime Cordial Graph, Relatively Prime Cordial Labeling.

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{u, v\}$ of vertices in E is called an edge or a line of G in which e is said to join u and v . We write $e = uv$ and say that u and v are adjacent vertices (sometimes denoted as $u \text{ adj } v$); vertex u and the edge e are incident with each other, as are v and e . If two distinct edges e_1 and e_2 are incident with a common vertex, then they are called *adjacent edges*. A graph with p vertices and q edges is called (p, q) -graph. In this paper, we proved that path related graphs $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$ are Relatively Prime Cordial Graph. For graph theory terminology, we follow [2].

II. PRELIMINARIES

Let $G = (V, E)$ be a graph with p vertices and q edges. A Analytic Mean Cordial Labeling of a Graph G with vertex set is a bijection from $v = \{1, 2, \dots, p\}$. The induced edge labelling are defined by $f(u, v) = 0$ if either $f(u)$ divides $f(v)$ (or) $f(v)$ divides $f(u)$ one otherwise and if any one of the vertex label is 1, the induced edge label is 0.

The graph that admits a Relatively Prime Cordial Graph is called Relatively Prime Cordial Graph. In this paper, we proved that path related

graphs $P_n, P_n \odot K_1, P_n^2, P_2: S_n, P_n + K_1$ are Relatively Prime Cordial Graph.

Definition: 2.1 P_n is a path of length $n - 1$.

Definition: 2.2 The Corona $= G_1 \odot G_2$ of two graph G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2 . The graph $P_n \odot K_1$ is called a *comb*.

Definition: 2.3

(P_n^2) is a path of length $n-1$ of twice.

Definition: 2.4

Star of length one is joined with every vertex of a path P_n through an edge. It is denoted by $P_2: S_n$

Definition: 2.5

The join of G_1 and G_2 is the graph $G = G_1 + G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$. The graph $P_n + K_1$ is called a Fan.

III. MAIN RESULTS

THEOREM: 3.1

Path P_n is Relatively Prime Cordial Graph.

Proof:

Let G be P_n

Let $V(G) = \{u_i : 1 \leq i \leq n\}$

Let $E(G) = \{(u_i u_{i+1}) : 1 \leq i \leq n-1\}$

Define $f : V(G) \rightarrow \{1, 2, \dots, p\}$

The vertex labeling are

When $n = \text{even}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n}{2}$$

When $n = \text{odd}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n+1}{2}$$

The induced edge labeling are,

When $n = \text{even}$

$$f^*(u_{n-1} u_n) = 0$$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n}{2} \leq i \leq n-2$$

When $n = \text{odd}$

$$f^*(u_{n-1} u_n) = 0$$

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$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n-1}{2} \leq i \leq n-2$$

Here, When $n = 2m, m > 1$

$$e_f(0) = m$$

$$e_f(1) = m - 1$$

$$n = 2m + 1, m > 1$$

$$e_f(0) = m = e_f(1)$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence, Path P_n is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of P_5 are shown in the figure

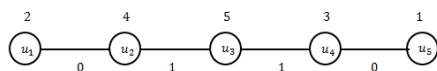


figure 3.1

THEOREM: 3.2

$P_n \odot K_1$ is a Relatively Prime Cordial Graph.

Proof:

Let G be $P_n \odot K_1$

Let $V(G) = \{ (u_i, v_i) : 1 \leq i \leq n \}$

Let

$$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i v_i) : 1 \leq i \leq n] \}$$

Define $f : V(G) \rightarrow \{1, 2, \dots, p\}$

The vertex labeling are,

$$f(u_i) = 2i \quad 1 \leq i \leq n$$

$$f(v_i) = 2i - 1 \quad 1 \leq i \leq n$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq n-1$$

$$f^*(u_1 v_1) = 0$$

$$f^*(u_i v_i) = 1 \quad 2 \leq i \leq n$$

Here, When $n = m, m > 1$

$$e_f(0) = m$$

$$e_f(1) = m - 1$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence, $P_n \odot K_1$ is Relatively Prime Cordial Graph.

For example, Relatively Prime Cordial Graph,

$P_5 \odot K_1$ are shown in the fig

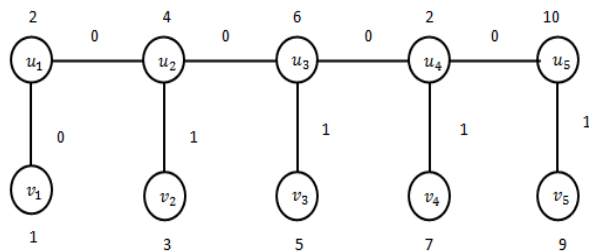


figure 3.2

THEOREM: 3.3

Graph Pn^2 is a Relatively Prime Cordial Graph.



Proof:

Let G be Pn^2 Graph

Let $V(G) = \{ u_i : 1 \leq i \leq n \}$

Let

$E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i u_{i+2}) : 1 \leq i \leq n-2] \}$

Define $f : V(G) \rightarrow \{1, 2, \dots, p\}$

Vertex Labeling:

When $n = \text{even}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n}{2}$$

$$f(u_i) = 2i - 1 \quad \frac{n+2}{2} \leq i \leq n$$

When $n = \text{odd}$

$$f(u_i) = 2i \quad 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_{n+1-i}) = 2i - 1 \quad 1 \leq i \leq \frac{n+1}{2}$$

Edge Labeling:

When $n = \text{even}$

$$f^*(u_n u_{n-1}) = 0$$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-2}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n}{2} \leq i \leq n-2$$

$$f^*(u_{n-2} u_n) = 0$$

$$f^*(u_i u_{i+2}) = 0 \quad 1 \leq i \leq \frac{n-4}{2}$$

$$f^*(u_i u_{i+2}) = 1 \quad \frac{n-2}{2} \leq i \leq n-2$$

When $n = \text{odd}$

$$f^*(u_i u_{i+1}) = 0 \quad 1 \leq i \leq \frac{n-3}{2}$$

$$f^*(u_i u_{i+1}) = 1 \quad \frac{n-1}{2} \leq i \leq n-2$$

$$f^*(u_{n-1} u_n) = 0$$

$$f^*(u_{n-2} u_n) = 0$$

$$f^*(u_i u_{i+2}) = 0 \quad 1 \leq i \leq \frac{n-3}{2}$$

$$f^*(u_i u_{i+2}) = 1 \quad \frac{n-1}{2} \leq i \leq n-3$$

When $n = 2m + 1, m > 1$

$$e_f(0) = 2m - 1$$

$$e_f(1) = 2m$$

Hence, Path $P5^2$ is a Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of $P5^2$ are shown in the figure

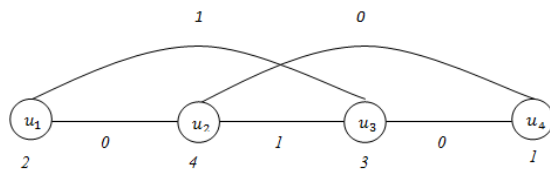


figure 3.3

THEOREM: 3.4

Graph $(P_2; S_n)$ is a Relatively Prime Cordial Graph.

Proof:

Let

$$V(p_2; s_n) = \{u_i : 1 \leq i \leq 4; u_{1i}, u_{2i}, 1 \leq i \leq n\}$$

Let

$$E(p_2; s_n) = \{[u_3 u_4] \cup [u_1 u_4] \cup [u_2 u_3] \cup [u_1 u_{1i} : 1 \leq i \leq n] \cup [u_2 u_{2i} : 1 \leq i \leq n]\}$$

Define $f : V(G) \rightarrow \{1, 2, \dots, p\}$

Vertex Labeling:

$$\begin{aligned} f(u_i) &= i & 1 \leq i \leq 4 \\ f(u_{1i}) &= 2i + 4 & 1 \leq i \leq n \\ f(u_{2i}) &= 2i + 3 & 1 \leq i \leq n \end{aligned}$$

Edge labeling:

$$\begin{aligned} f^*(u_i u_{i+1}) &= 1 & 2 \leq i \leq 3 \\ f^*(u_1 u_4) &= 0 \\ f^*(u_1 u_{1i}) &= 0 & 1 \leq i \leq n \\ f^*(u_2 u_{2i}) &= 1 & 1 \leq i \leq n \end{aligned}$$

Here, When $n = m$

$$e_f(0) = n + 1$$

$$e_f(1) = n + 2$$

$$e_f(0) + 1 = e_f(1)$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence, $P_2; S_n$ is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of

$P_2; S_n$ are shown in the figure

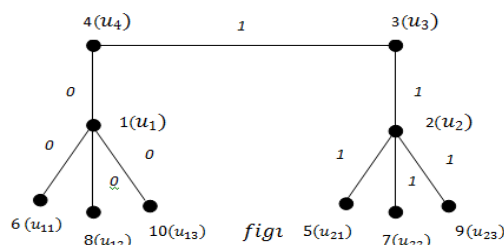


fig 3.4

THEOREM: 3.5

Graph $P_n + K_1$ is Relatively Prime Cordial graph.

Proof:

Let $p_n + k_1, n > 1$

$$V(p_n + k_1) = \{u, u_i : 1 \leq i \leq n\}$$

Let

$$E(p_n + k_1) = \{[uu_i] : 1 \leq i \leq n \cup [u_i u_{i+1}] : 1 \leq i \leq n - 1\}$$

Define $f : V(G) \rightarrow \{1, 2, \dots, p\}$

Vertex Labeling:

$$f(u) = 1$$

$$f(u_i) = i + 1 \quad 1 \leq i \leq n$$

Edge Labeling:1

$$f^*(uu_i) = 1$$

$$f^*(u_i u_{i+1}) = 1 \quad 1 \leq i \leq n - 1$$

Here, When $n = m, m > 1$

$$e_f(0) = m$$

$$e_f(1) = m - 1$$

$$|e_f(1) - e_f(0)| \leq 1$$

Hence, $P_n + K_1$ is Relatively Prime Cordial Graph.

For example, The Relatively Prime Cordial Graph of

$P_n + K_1$ are shown in the figure

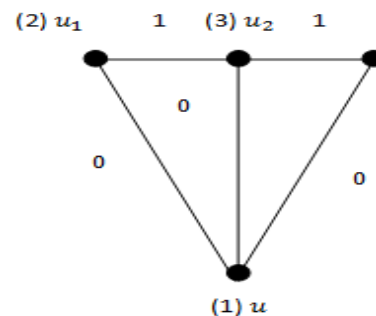


figure 3.5

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