Discrete-time Queuewith Batch Geometric Arrivals and Retention of Reneging Customers

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***Abstract*—This paper considers adiscrete-time queueing system with batch geometric arrivals, in which customers may renege and reneging customers may be retained. The steady state distribution of the number of customers in the system is derived.**

***Index Terms*—Discrete-time queue, geometric service, reneging, retention**

I. INTRODUCTION

Customer reneging is a phenomenon observed commonly in queueing systems, where customers may leave the service system before receiving their service due to the long waiting time. The problem of queueing systems with customer reneging was first analyzed by Palm [1]. A bibliography can be found in Gross et al. [2]. El-Sherbiny [3] considered a truncated heterogeneous two-server M/M/2/N queueing system with reneging and general balk function. Kumar and Sharma [4] considered a queueing system, where reneging customers may be retained for his future service owing to a certain customer retention strategy and analyzed an M/M/1/N queueing system with retention of reneging customers.Kumar and Sharma [5] studied an M/M/c/N queueing system with reneging and retention of reneging customers. Kumar [6] analyzed an M/M/c/N queueing model with balking, reneging, and retention of reneging customers. Kumar and Sharma [7] considered a finite capacity Markovian queueing system with two heterogeneous servers, discouraged arrivals, reneging, and retention of reneging customers. Lee [8] considered a discrete-time Geo/Geo/1/N queueing system with retention of reneging customers.

This paper considers a discrete-time $\mathrm{Geo}^{\mathrm{X}}/\mathrm{D}/1/N$ queueing system with retention of reneging customers. The steady state distribution of the number of customers in the system is derived.

The paper is organized as follows. In Section II, we described the queueing model. In Section III, we formulate the system as a discrete-time Markov chain and find the stationary probability distribution of the number of customers in the system. Conclusion is provided in Section IV.

II. MODEL

In this section, we formulate the queueing model, which is based on the following assumptions:

1. We consider a discrete-time queueing system in which the time axis is divided into fixed-length contiguous intervals, referred to as slots.
2. Customers arrive according to a batch geometric process.
3. The numbers of arrivals during the consecutive slots are assumed to be independent and identically distributed random variables with distribution$\{a_k, k = 0, 1, \ldots\}$.
4. The service of a customer can start only at a slot boundary.
5. The service time of customers is one slot.
6. The system has a buffer of finitecapacity$N$.
7. Customers are served in FCFS order.
8. The reneging times follows a geometric distribution with parameter r.
9. Reneging customers may leave the queue without getting service with probability s.
10. During each slot, reneging occurs before customer arrivals.

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III. STATIONARY DISTRIBUTION

To model this system, we define the random variable N_k as the total number of customers in the system at the end of slot k. Then, the stochastic process $\{N_k, k \geq 0\}$ becomes a discrete-time Markov chain. The state space of this Markov chain is $\{0, 1, 2, \ldots, N\}$.

Define $r_{i,j}$ as the probability that j customers among i waiting customers leave the system due to reneging at a slot. Then, the probability $r_{i,j}$ can be obtained as:

$$r_{i,j} = \begin{cases} 0, & i < j \\ \sum_{k=j}^{i} \binom{i}{k} r^k (1-r)^{i-k} \binom{k}{j} s^j (1-s)^{k-j}, & i \geq j \end{cases} \quad (1)$$

where:

$$\binom{i}{k} r^k (1-r)^{i-k} \quad (2)$$

is the probability that k customers among i waiting customers renege and:

$$\binom{k}{j} s^j (1-s)^{k-j} \quad (3)$$

the probability that j customers among k reneging customers leave the system at a slot, respectively.

The one-step transition probability matrix $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & r_{1,1} & r_{1,0} & 0 & \cdots & 0 & 0 \\ 0 & r_{2,2} & r_{2,1} & r_{2,0} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & r_{N-2,N-2} & r_{N-2,N-3} & r_{N-2,N-4} & \cdots & r_{N-2,0} & 0 \\ 0 & r_{N-1,N-1} & r_{N-1,N-2} & r_{N-1,N-3} & \cdots & r_{N-1,1} & r_{N-1,0} \end{pmatrix} \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} & a_N^+ \\ a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} & a_N^+ \\ 0 & a_0 & a_1 & a_2 & \cdots & a_{N-2} & a_{N-1}^+ \\ 0 & 0 & a_0 & a_1 & \cdots & a_{N-3} & a_{N-2}^+ \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_1 & a_2^+ \\ 0 & 0 & 0 & 0 & \cdots & a_0 & a_1^+ \end{pmatrix} \quad (4)$$

where the probability a_k^+ is given by:

$$a_k^+ \equiv 1 - \sum_{i=0}^{k-1} a_i \quad (5)$$

Let $\boldsymbol{x}$ be the stationary probability vector associated with the discrete-time Markov chain $\{N_k, k \geq 0\}$:

$$\boldsymbol{x} \equiv (x_0, x_1, \ldots, x_N) \quad (6)$$

where:

$$x_i \equiv \lim_{k\to\infty} P\{N_k = i\} \quad (7)$$

Then, the stationary probability $\boldsymbol{x}$ is obtained by solving

$$\boldsymbol{xP} = \boldsymbol{x} \quad (8)$$

$$\boldsymbol{xe} = 1 (9)$$

where$\boldsymbol{e}$is the column vector with all elements1.

IV. NUMERICAL EXAMPLES

Let us illustrate the behavior of the discrete-time $\mathrm{Geo}^{\mathrm{X}}/\mathrm{D}/1/N$ queueing system with impatient customers with the help of some numerical examples. By using MATLAB, we compute the stationary probability distribution $\boldsymbol{x}$ for the number of customers in the system and find its mean value. We have chosen:

$$a_0 = \frac{1}{9},\quad a_1 = \frac{2}{9},\quad a_2 = \frac{3}{9},\quad a_3 = \frac{2}{9},\quad a_4 = \frac{1}{9} \quad (10)$$

$$N = 4 \quad (11)$$

$$r = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, \text{and } 1 \quad (12)$$

$$s = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, \text{and } 1 \quad (13)$$

The effect of parameters, retention probability s and renegingprobability r, on the mean number of customers in the system is illustrated here.

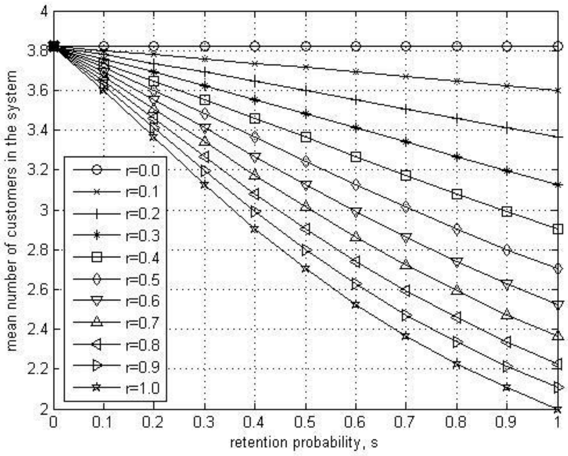

Figure 1. Mean number of customers in system versus retention probability

V. CONCLUSION

In this paper, we have considered adiscrete-time $\mathrm{Geo}^{\mathrm{X}}/\mathrm{D}/1/N$ queueing system with impatient customers, where customers may renege and reneging customers may be retained. The steady state distribution of the number of customers in the system has been derived.

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