Minimizing the Total Cost in the Out Patient Department (OPD) of a Multispecialty Hospital

A. Nellai Murugan, S.Vijayakumari Saratha

Abstract— Waiting lines are experienced in out daily activities. Waiting in line or queue causes inconvenience to individuals (Patients) and economic costs to firms and organizations. Providing too much service capacity to operate a system involves excessive cost. But not providing enough service capacity results in excessive waiting time and cost. In this study the queuing characteristics at the specialist clinic was analysed using a multiserver queuing model and the waiting and service costs determined with a view to determining the optimal service level. This model can also be used by decision and other policy makers to solve other multi-server queuing problems.

Index Terms— Patients, multi-server queuing.

I. INTRODUCTION

A common situation that occurs in everyday life is that of queuing or waiting in line. Queues are usually seen at bus stops, hospitals, bank counters and so on. One of the major uses of operational research in health care is in the form of queuing theory. Queuing theory was first analyzed by A.K. Erlang in 1913 in the context of telephone facilities. Queuing theory application is an attempt to minimize the cost through minimization of inefficiencies and delays. In hospital, queuing theory can be applied to assess a multitude of factors such as registration fill-time, patient waiting time, patient counselling time and receptionists and technician staffing levels.

Now a days, out-patient Department (OPD) services of majority of the hospitals are having queuing and waiting time problems. Patient waiting time refers to the time from the registration of the patient for appointment with the doctor till they enter the consulting room. Waiting time in out-patient department is a problem throughout India. One consistent feature of patient dissatisfaction has been expressed with the lengths of waiting time in the outpatient department.

II. MATERIALS AND METHODS

The data for this paper was obtained from the Multispeciality Hospital. The data was collected from Monday to Friday. The mean arrival rates and the mean service rates were calculated from the data collected and their result was used to measure the performance of the entire system. The queue discipline is first come, First served (FCFS) basis by any of the servers.

There is no priority classification for any arrival. Servers here represent only doctors and the service providers are working at their full capacity.

The outpatient clinic is open from 9.30 am till 5.30 pm giving a total of 8 hours per day. It is important to note that after 5.30 pm there is still an emergency out-patient clinic but for the purposes of this study that aspect was not included. Using the information collected the average patient arrival rate is given by 320 patients per day divided by 8 hours per day resulting 40 patients per hour. One doctor averagely uses 5 minutes to complete the care of one patient. Hence the doctor can see 12 patients per hour.

Parameters in Queuing Models (M/M/S : FCFS / ∞ / ∞)

n - Number of total patients in the system

r - Number of parallel servers

λ - Mean arrival rate

μ - Mean service rate

cμ - Service Rate when c > 1 in a system

ρ - System intensity or load

\[ \frac{\lambda}{c\mu} \] - Utilization factor

Assuming the system is in steady - state condition

(i) \( P_0 \) - Steady state probability of all idle servers in the system

\[ P_0 = \left[ \sum_{n=0}^{c-1} \left( \frac{\lambda}{\mu} \right)^n \frac{1}{n!} + \left( \frac{\lambda}{\mu} \right)^c \frac{1}{c!(1-\rho)} \right]^{-1} \]

where \( \rho = \frac{\lambda}{c\mu} \)

(ii) \( P_n \) - Steady state probability of exactly n patients in the system

\[ P_n = \left( P_0 \right) \frac{\lambda^n}{c!\mu^{n-c}} \mu^n, \ n > c \]

(iii) \( L_q \) - Average number of patients the queue

\[ L_q = \left( \frac{\lambda}{\mu} \right)^2 \mu \times P_0 \]

(iv) \( L_s \) - The expected number of patient in the system

\[ L_s = L_q + \frac{\lambda}{\mu} \]

A. Nellai Murugan, Head & Associate Professor in Mathematics, V.O.Chipadamaram college,Tuticorin-8.

S. Vijayakumari Saratha, Assistant Professor, Womens’ Christian College, Nagercoil.
Flow chart for OPD

Patient Arrives at OPD

Registration

Queue Starts

Nurse Checks Vitals

Consulting Room

Server 1

Server 2

Server 3

Server 4

Server 5

Laboratory

Radiology

To ward if Admitted

Entry of Prescriptions

Dispensary

Home
(v) $W_q$ - Average waiting time a patient spends waiting in line
$$W_q = \frac{L_q}{\lambda}$$

(vi) $W_s$ - The expected time a customer spends in the system
$$W_s = W_q + \frac{1}{\mu}$$

**Various costs involved in this model**

To determine the optimum number of servers in the system, we have to choose two costs namely service costs and waiting time costs of customers. Service cost is the cost of providing service (eg) salaries paid to servers. Waiting time cost is the cost which include loss of business as some patients might not be willing to wait for service and may decide to go to the competing organizations. Customer satisfaction is improved by predicting and reducing waiting time and adjusting staffing.

Expected service cost = $C_s$

Expected waiting cost = $(\lambda W_s) C_w$

Where $C$ is the number of servers, $C_s$ is the service cost of each server, $\lambda$ is the number of arrivals $W_s$ is an average time an arrival spends in the system and $C_w$ is the waiting cost of customers

$\therefore$ Expected Total cost = Expected Service Cost + Expected Waiting Cost = $C C_s + (\lambda W_s) C_w$

### III. DATA ANALYSIS

Arrival rate $\lambda = 40$ patients / hour
Service rate $\mu = 12$ patients / hour
Number of servers $C = 5$
Server cost $C_s = 100$ per hour
Waiting cost $C_w = 200$ per hour

<table>
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<th>Servers</th>
<th>$\lambda$</th>
<th>$\mu$</th>
<th>Avg. Service utilization</th>
<th>Avg. no. in the queue ($L_q$)</th>
<th>Avg. no. in system ($L_s$)</th>
<th>Avg. time in queue ($W_q$)</th>
<th>Avg. time in system ($W_s$)</th>
<th>Total cost</th>
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**Graph : Multi Server queuing model of 5 Servers**
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IV. DISCUSSION
The graph shows that optimal server level at the clinic is achieved when the number of servers is 9 with a minimum total cost of Rs. 1692 per hour against the present server level of 5 doctors at the clinic which have high total cost of Rs. 2623.2 per hour. Also the average waiting time for the customer is less at this optimal server level.

V. CONCLUSION
Providing patients with timely access to appropriate medical care is an important element of health care and increases patient satisfaction. The result of the analysis showed the average queue length, waiting time of patients could be reduced when the service capacity level of doctors at the clinic is increased from 5 to 9 at a minimum total cost which include waiting and service cost.

REFERENCES

Dr. A. Nellai Murugan is Head and Associate Professor in PG and Research Department of Mathematics. His research area is Graph Theory and Operations Research. He has published more than 130 research papers in reputed national and international journals. Currently five candidates are pursuing their research.