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Utilizing Modern Systems Approach to Realize Higher Order Dynamic Analogue Circuits

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***Abstract*- Modern system approaches will be used to realize a 5th order dynamic differential equation as an analogue circuit using resistors, capacitors, and operational amplifiers. The resulting circuit will be tested using Matlab® and those results will be verified using Multisim. The state space approach will be used to convert the 5th order differential equation to five first order differential equations. Using the Multisim simulator, various input types were tested across the input terminals of the analog computer and the results were recorded.**

***Index Terms*—Analog computer, state space, operational amplifier, resistor, capacitor, modern system approach.**

I. INTRODUCTION

The goal of this initiative is to build a 5th order electronic circuit to solve and simulate a 5th order differential equation with any input.

Consider the generic differential equation to be solved

$$a\frac{d^5}{dt^5}y(t)+b\frac{d^4}{dt^4}y(t)+c\frac{d^3}{dt^3}y(t)+d\frac{d^2}{dt^2}y(t)+e\frac{d}{dt}y(t)$$

$$+f\,y(t)=g\,x(t) \tag{1}$$

x(t) is the forcing function (the input to the system represented by this differential equation) and y(t) is the solution (the output of the same system). The variables a, b, c, d, e and f are some real constant numbers. [1]

In the last equation, (assuming zero initial conditions) let

$$\begin{cases} y_1(t)=y(t) \\ y_2(t)=\frac{d}{dt}y(t) \\ y_3(t)=\frac{d^2}{dt^2}y(t) \\ y_4(t)=\frac{d^3}{dt^3}y(t) \\ y_5(t)=\frac{d^4}{dt^4}y(t) \end{cases}$$

Thus we have the set of four first order differential equations

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$$\begin{cases} \frac{d}{dt}y_1(t)=y_2(t) \\ \frac{d}{dt}y_2(t)=y_3(t) \\ \frac{d}{dt}y_3(t)=y_4(t) \\ \frac{d}{dt}y_4(t)=y_5(t) \\ \frac{d}{dt}y_5(t)=-\frac{f}{a}y_1(t)-\frac{e}{a}y_2(t)-\frac{d}{a}y_3(t) \\ \quad -\frac{c}{a}y_4(t)-\frac{b}{a}y_5(t)+\frac{g}{a}x(t) \end{cases} \tag{2}$$

II. METHODOLOGY

Consider the Operational amplifier circuit shown in Figure 1. The input-output relationship is given as

$$y(t)=-A\frac{1}{RC}\int x_1(t)dt-B\frac{1}{RC}\int x_2(t)dt \tag{3}$$

In Figure 1, the output y(t) is the integral of the input arriving at the negative terminal of the Operational Amplifier. Thus the negative of the derivative of y(t) is located at the negative terminal of the Operational Amplifier. [2,3,4]

If we set RC=1 in equation (3) we will have

$$y(t)=-A\int x_1(t)dt-B\int x_2(t)dt \tag{4}$$

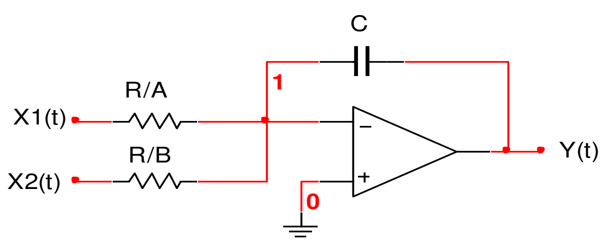

Fig. 1. Operational Amplifier Circuit

One final step before we attempt to implement Equation (4), the solution of a generic 1st order linear constant coefficient differential equation. Consider the circuit given in Figure 2. The input-output relationship is

$$y(t)=-\frac{R_f}{R}x(t) \tag{5}$$

You also can see that if R_f = R then we have pure inversion (unity gain). The circuit containing an inverter and an integrator connected in series can solve the differential equation given in (6). Figure 3 is a typical example of such a circuit.

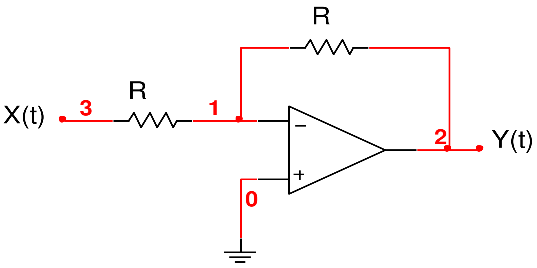

Fig. 2. Inverter

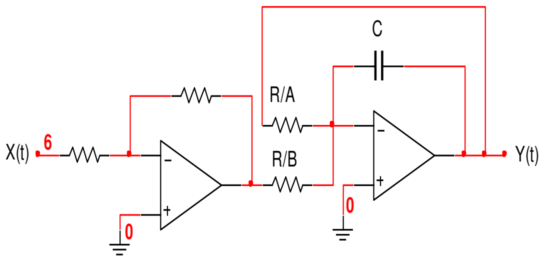

Fig. 3. A Circuit to Solve Equation 6

The circuit in Figure 3 would solve any first order differential equation of the form

$$y'(t) + Ay(t) = Bx(t) \qquad (6)$$

Knowing how to solve equation (6) is helpful in solving the set of the four coupled equations in (2) with y(t) being the output in the fourth-order differential equation given in (1).

In building a circuit to solve the given differential equation in (1) we will use the set of equations in (2). We have tried step input, impulse input, and sinusoidal input. All worked nicely. Next we present the step response for different real coefficients and consider over damped, under damped, critically damped and oscillatory cases.

CASE 1

For the over damped case we selected a=1, b=5, c=8.75, d=6.25, e=1.5, f=0.0002 and g=0.002. The differential equation is then

$$\frac{d^5}{dt^5}y(t)+5\frac{d^4}{dt^4}y(t)+8.75\frac{d^3}{dt^3}y(t)+6.25\frac{d^2}{dt^2}y(t)+1.5\frac{d}{dt}y(t)$$

$$+0.0002\,f\ y(t)=0.0002\ u(t)$$

where u(t) is the step unit signal. The eigenvalues are then at

-2.0000
-1.5000
-1.0000
-0.5000
-0.0001

Using Matlab®, the solution is plotted as seen in Figure 4.

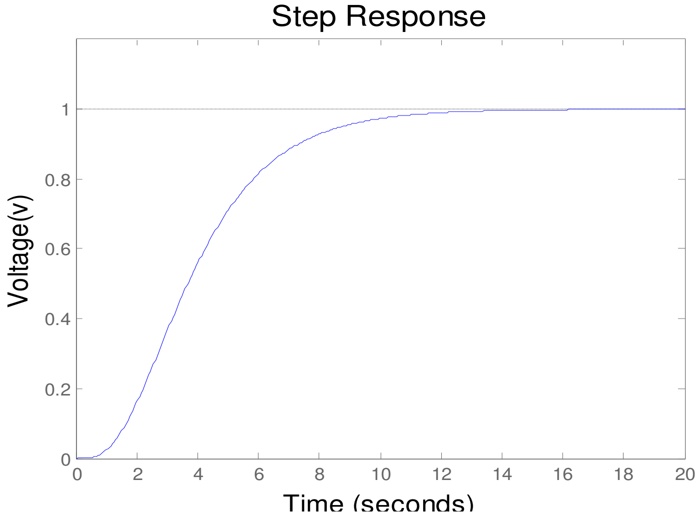

Fig. 4. Over Damped Case: Output from Matlab

The circuit to solve the fifth-order differential equation with the given constant values and with the input u(t) is shown in Figure 5. For the input we used a pulse signal with pulse width 0f 20sec and period of 40sec. To make RC=1, we used R=1kΩ and C=1mF. We simulated for the first 20 second. The simulated output is shown in Figure 6.

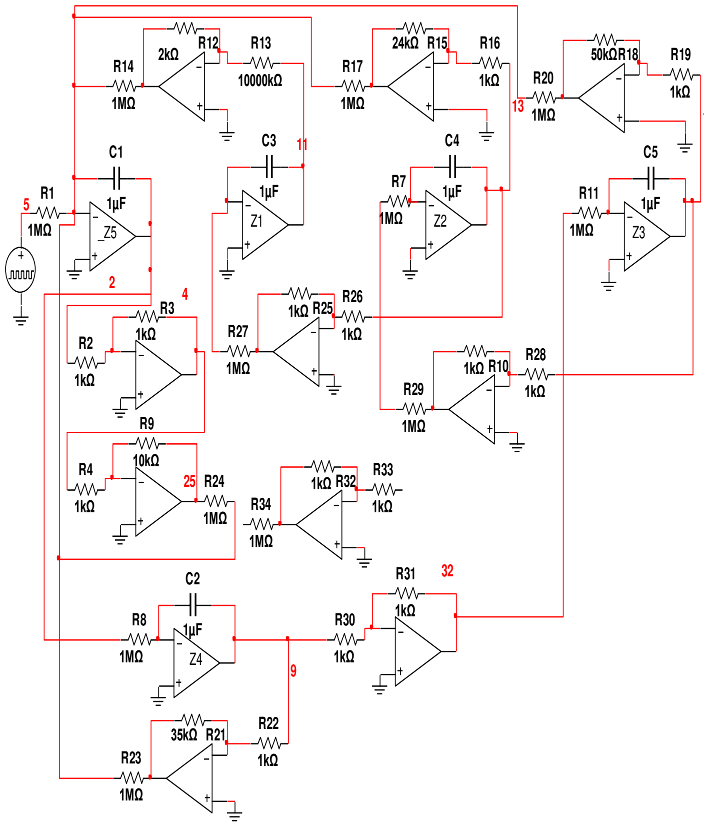

Fig. 5. Circuit to Solve the Differential Equation

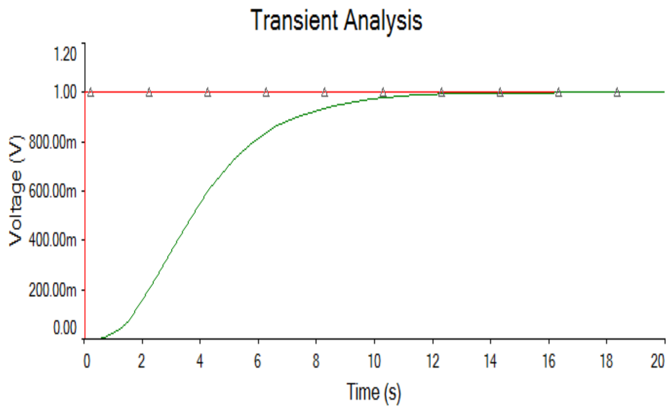

Fig. 6. Over Damped Case: from Circuit Simulation

CASE 2

For the critically damped case we selected a=1, b=6, c=13, d=12, e=4 and f=0.0004. The differential equation is then

$$\frac{d^5}{dt^5}y(t)+6\frac{d^4}{dt^4}y(t)+13\frac{d^3}{dt^3}y(t)+12\frac{d^2}{dt^2}y(t)+4\frac{d}{dt}y(t)$$

$$+0.0004f\ y(t)=0.0004\ u(t)$$

1.0000 6.0001 13.0006 12.0013 4.0012 0.0004
The eigenvalues are then at

-1.0000
-1.0000
-2.0000
-2.0000
-0.0001

Using Matlab, the solution is plotted as seen in Figure 7. For the circuit we used a pulse signal with pulse width 0f 20sec and period of 40sec. To make RC=1, we used R=1kΩ and C=1mF, the simulation done for the first 20 second and it is shown in Figure 8.

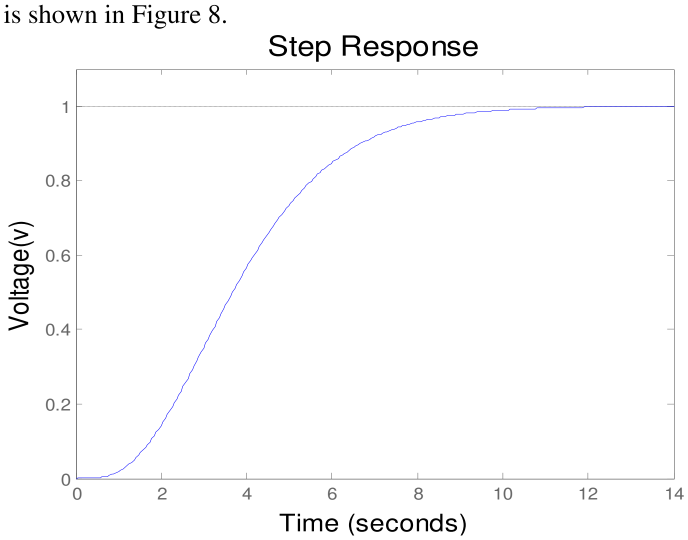

Fig. 7. Critically Damped Case: Output from Matlab

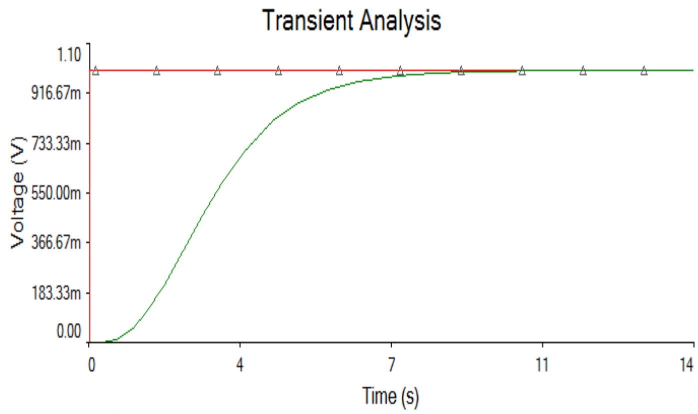

Fig. 8. Critically Damped Case: from Circuit Simulation

CASE 3

For the oscillatory case we selected a=1, b=3, c=3, d=3, e=2 and f=0.0001. The differential equation is then

1.0000 3.0001 3.0003 3.0003 2.0003 0.0002

$$\frac{d^5}{dt^5}y(t)+3\frac{d^4}{dt^4}y(t)+3\frac{d^3}{dt^3}y(t)+3\frac{d^2}{dt^2}y(t)+2\frac{d}{dt}y(t)$$

$$+0.0001f\ y(t)=0.0001\ u(t)$$

The eigenvalues are then at

-2.0000
-0.0000 + 1.0000i
-0.0000 - 1.0000i
-1.0000
-0.0001

Using Matlab, the solution is plotted as seen in Figure 9. For the circuit we used a pulse signal with pulse width 0f 20sec and period of 40sec. To make RC=1, we used R=1kΩ and C=1mF, the simulation is done for the first 20 second and it is shown in Figure 10.

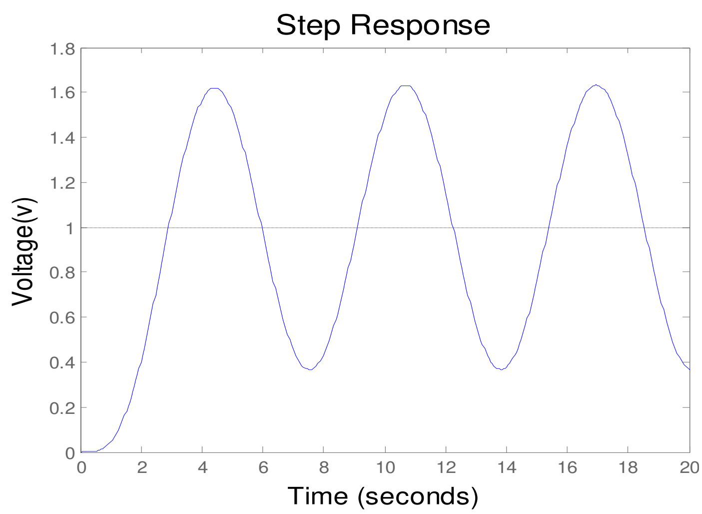

Fig. 9. Oscillatory Case: Output from Matlab

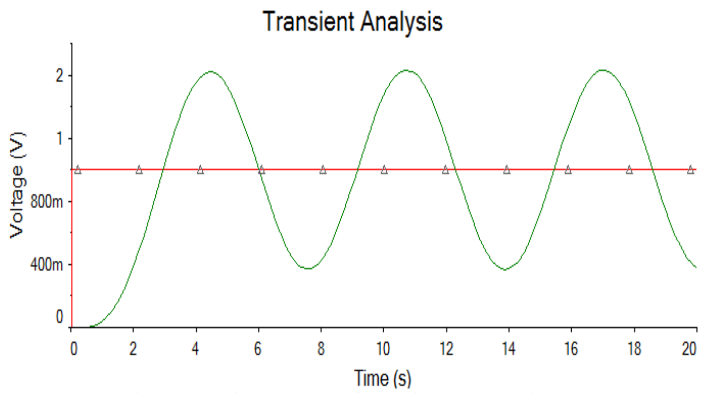

Fig. 10. Oscillatory Case: from Circuit Simulation

CASE 4

For the under damped case we selected a=1, b=4, c=6, d=9, e= 5 and f=5. The differential equation is then

$$\frac{d^5}{dt^5}y(t)+4\frac{d^4}{dt^4}y(t)+6\frac{d^3}{dt^3}y(t)+9\frac{d^2}{dt^2}y(t)+5\frac{d}{dt}y(t)$$

$$+0.0005f\ y(t)=0.0005\ u(t)$$

1.0000 4.0000 6.0003 9.0008 5.0010 0.0005

The eigenvalues are then at

-2.7720

-0.2138 + 1.4860i

-0.2138 - 1.4860i

-0.8003

-0.0001

Using Matlab, the solution is plotted as seen in Figure 11. For the circuit we used a pulse signal with pulse width 0f 25sec and period of 50sec. To make RC=1, we used R=1kΩ and C=1mF, the simulation is done for the first 25 second and it is shown in Figure 12.

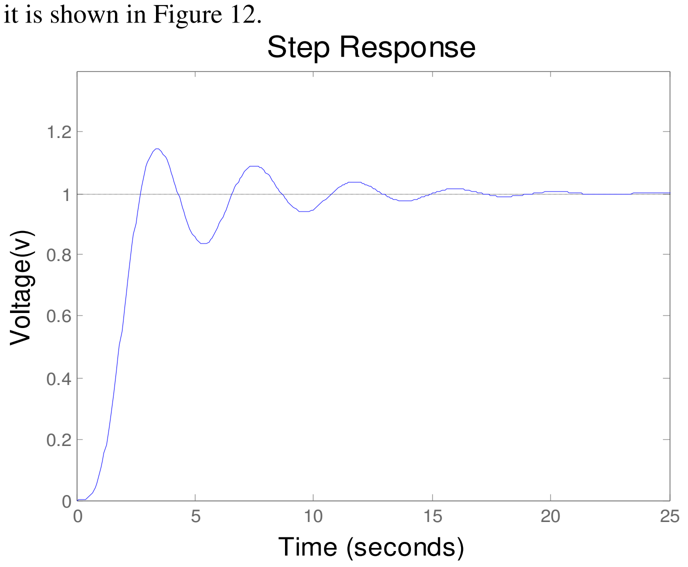

Fig. 11. Under damped Case: Output from Matlab

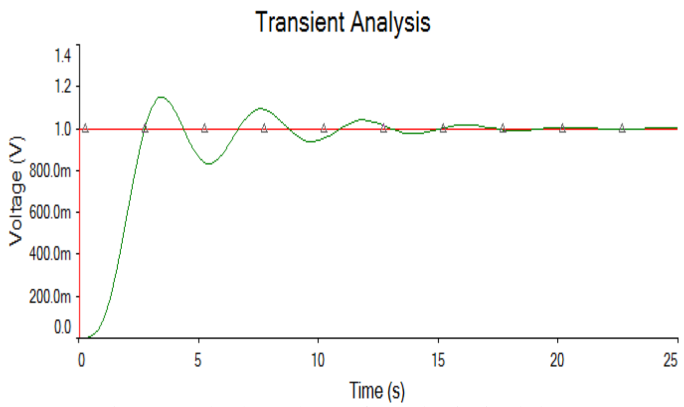

Fig. 12. Under damped Case: from Circuit Simulation

In the process of simulating the circuit we had to adjust the resistor values to accommodate the change in the coefficients.

The input x(t) has to be adjusted too to accommodate the necessary time required to show enough time for the transients to settle.

III. CONCLUSION

It would be discovered by looking at the graphs and also by comparing these results with what was derived analytically that the circuits worked as desired. The differential equation was solved and its outputs were verified. The analog computer can be used to solve the two-point boundary-value problem for a fifth optimal control problem. It can also be applied to the study of micro-economic inventory system, for investigation a stock control system where the supply is discontinuous. [6] The analog computer also was found helpful in solving problems like simulation of a sampled data system, simulation of forecasting methods, locating a ware house or a distribution center, and controlling and resetting policies for process subject to trend. [7] In the future we will attempt to solve higher order differential equations. Practically, to solve any sixth order differential equation with any arbitrary coefficients requires a huge set of resistive values. However, since the constant values of a, b, c, d and e can be translated to ratios of resistor values that makes things easier. Issues related to amplifier saturation should also be studied. [5]

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