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The Application of Spatial Analysis and Time Series in Modeling the Frequency of Earthquake Events in Bengkulu Province

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Abstract – This study provides an overview in combining spatial analysis and time series analysis to model the frequency of earthquake. The aim of this research is to apply the spatial statistical analysis and time series analysis in estimating semivariogram parameters for the next four steps. The data in this study is secondary data that has been validated based on sources that publish parameters of earthquake events. Looking at the characteristics of the earthquake frequency frequency data, there are spatial and time elements. The method used in this research is interpolation kriging and Autoregressive Moving Average (ARMA) model. The semivariogram models used in kriging interpolation are: Spherical, Exponential, Gaussian, and Linear. The parameters of the semivariogram model are modeled using ARMA time series analysis adjusted to the model diagnostic results. To measure of fit model is used Mean Square Error (MSE). The result of research is a suitable semivariogram model to be applied in the modeling of earthquake events is the Spherical model. While each parameter is estimated using ARMA model (2,2) with different coefficient estimation value.

Keywords: frequency, earthquake, spatial analysis, time series analysis, MSE

Introduction

Forecasting of earthquake events is very interesting to be studied. Until now there are many researchers study this project, not only partially, but also simultaneously. The forecasting of tsunami after earthquake is a unity that can not be separated by earthquake forecasting. The forecasting of earthquake events is still based on assumptions, the earthquake occurred above 6.1 SR, the depth of the quake <10 km and the location of the incident was in the sea (bmkg.go.id).

The trend of the earthquake frequency data can be categorized as time series data. So the analysis of data used to determine the relationship between previous events with current events even the future can use a time series analysis. Based on the result of research by Irwanto *et al.* (2014), the trend of tectonic earthquake in Sumatra region has a high frequency of occurrence with an average value close to 5 SR. From the point of view of spatial statistics, Fachri *et al* (2014) examines the relationship between earthquake events between points of occurrence, where the results are statistically closely related to the occurrence of earthquakes between points of location. This has been previously investigated by Kannan (2011), the occurrence of earthquakes can be predicted by using Poisson distribution based on distance and cesarean zone. Another approach in earthquake forecasting is done by Fong and Nannan (2011) with Time series Analysis method, namely ARIMA adaptive model. Based on these results, it is possible to estimate the distance and occurrence of occurrences in forecasting earthquakes. In addition to the above weaknesses, generally the results of the analysis is still limited to a number. Yulian *et al.* (2012), managed to provide an alternative in describing the simulation results using geographic information system (GIS).

Studies conducted, spatial analysis applications and time series, are still done partially, such as Carr *et. al* (1986) implemented Disjuntive Kriging to estimate theearthquake ground motion. Furthermore Carr *et. al* (1989), continued his research by comparing between Universal Kriging and Ordinary Kriging in the case of earthquake ground motion. Then Sugai *et al.* (2015) introduced a practical method to estimate the special distributions of ground motion, based on Ordinary Kriging analysis. Cakmak *et al.* (1985), modeled the earthquake ground motion in California using the parametric time series methods. Lin (2014), conducted a time series modeling of earthquake ground motion using ARMA-GARCH models.

Based on the above description, the research objective can be formulated to provide an overview in applying spatial statistical analysis and time series analysis to estimate semivariogram parameters for the next 4 steps. The method used is applying semivariogram models such as: Spherical, Exponential, Gaussian, and Linear. Then the parameters of the semivariogram model are modeled using ARIMA time series analysis adapted to the model diagnostic results.

Materials and Methods

Mechanism of data collecting and processing

Data used in this paper are the number of earthquake events which have magnitude above 5 Mw that occurred in Bengkulu Province within the period of 2000-2016. The data are obtained from the website www.usgs.com with the amount of data as much as 534. Variables of data are the coordinate position of the center of the earthquake, latitude, longitude, depth and magnitude. Based on the longitude position, earthquake events in the data range from 99.00^{O}E to 106.00^{O}E, while based on the latitude position, the minimum data is at 7.00^{O}S and the maximum is at 1.00S. distribution of earthquake data of Bengkulu Province is presented in Figure 1

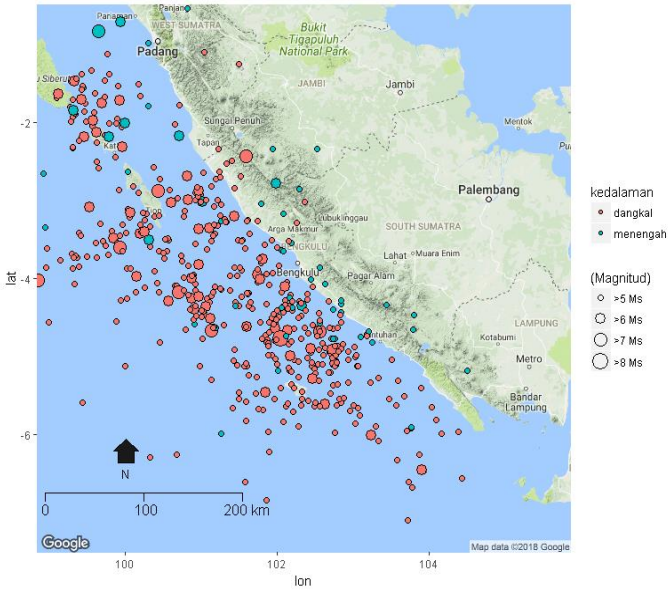


Figure 1. Distribution Map of Earthquake Events with The magnitude of ≥5Ms in 2000-2016

After collecting data, the next step is processing. Firstly, data are grouped into two parts. The first part (A) as much as 85% is used in the formation of semivariogram model and while the second part (B) as much as 15% is used in the model validation/conformity stage. The first part (A) consists of the earthquake events in 2000-2014, while the second one is earthquake events in 2015-2016.

Data in the first part (A) are processed in two steps. The first step is grouping data.In each year, data are grouped into two group, the first semester in January-June and the second semester in July-December. So that for this step, 30 groups of data are obtained. Furthermore, each group of data is determined the value of parameters of the semivariogram model such as Variance Nugget, Sill, and Range. There are four models of semivariogram used in this research, namely Spherical model, Exponential, Gaussian, and linear. The output generated at this steps consists of four semivariogram models along with the value of each parameter for each group.

The second step is further analysing the output resulted in the first step. In this step, the analysis is done for parameter values produced in first step by time series analysis approach. The time series analyses used is ARMA model and characteristics of ACF and PACF. The outputs resulted in this step are total of twelve time series models consisting of four semivarogram models (Spherical, Exponential, Gaussian and Linear Model). Each semivariogram model consists of three-time series models of parameter values (Nugget Variance, Sill and Range). The final process involves the data in the second part (B) of data. Based on this data. The best semivariogram model is selected based on MSE. The output resulted in this step is one of the best semivariogram models, with 3 time series models of the model parameters.

Spatial Statistics Analysis

Spatial data being in the form of data point location coordinates of earthquakes in the first part (A)areprocessed by semivariogram model. The semivariogram is a statistical tool for describing, modeling, and explaining spatial correlations between observations. The semivariogram is defined as follows (Wackernagel, 2003):

$$2\gamma(h) = Var[Z(s+h) - Z(h)] = E[Z(s+h) - Z(h)]^2 \quad (1)$$

where γ(*h*) is a semivariogram.The above semivariogram is also called theoretical semivariogram. There are two types of semivariogram: isotropic semivariogram (γ(*h*) depends only on distance h) and anisotropic semivariogram (γ (h) depends on distance h and direction).

An experimental semivariogram is a semivariogram obtained from known data:

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|}\sum_{i=1}^{N(h)}[z(s_i + h) - z(s_i)]^2 \quad (2)$$

with s_i is location of sample (coordinate), $Z(s_i)$ data value in location s_i

$|N(h)|$: #pairs ($s_i, s_i + h$) with distance$h$.

In the semivariogram prediction, the theoretical semivariogram model is fitted in the experimental semivariogram $\hat{\gamma}(h)$. There are four theoretical semivariogram models that are used:

- Spherical Model : $\gamma(h) = \begin{cases} C_0 + C\left[\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right] & ,0 < h \le a \\ C_0 + C & ,h > a \end{cases}$ (3)
- Exponential Model :$\gamma(h) = \begin{cases} C_0 + C\left[1 - exp\left(-\frac{h}{a}\right)\right]; & h > 0 \\ 0, h = 0 \end{cases}$ (4)
- GaussianModel : $\gamma(h) = \begin{cases} C_0 + C\left[1 - exp\left(-\frac{h^2}{a^2}\right)\right]; & h > 0 \\ 0, h = 0 \end{cases}$ (5)

C_0: nugget variance, $C_0 + C$: sill, and$a$: range

- Linear Model : $\gamma(h) = \alpha h$, α = gradient(Amstrong, 1998).

Ordinary Kriging

The Ordinary Kriging Method (OK) is a method of estimating a random variable at a given point by observing similar data in another location with the mean data assumed to be constant but not known in value. In the ordinary kriging method, the known sample values are used as linear combinations to estimate the points around the sample's location.In other words, to estimate any non-sampled point(s_0)can use a linear combination of random$Z(s_i)$and kriging weight values respectively, mathematically can be written by:

$$\hat{Z}(s_0) = \sum_{i=1}^{n} \lambda_i Z(s_i) \quad (6)$$

where$\hat{Z}(s_0)$is the value of the random variable estimation at the points_0, and$Z(s_i)$is the value of the random variable$Z(s)$at the point i, and$\lambda_i$is the kriging weightat the point i (Pfeiffer & Robinson, 2008). The variance of the estimated error (kriging variance) can be expressed by

$$\sigma_{OK}^2(s_0) = E\left(\sum_{i=1}^{n}\lambda_i Z(s_i) - Z(s_0)\right)^2 = \sum_{i=1}^{n}\lambda_i\gamma(s_0 - s_i) + m \quad (7)$$

where *m* is the Lagrange multiplier factor.

Autoregressive with p-order and Moving Average with q-order (ARMA (p, q))

In the second step, parameter values produced by semivariogram models are analyzed using time series model. There are several formulations used in spatial model parameter modeling by time series analysis approach, one of them is ARMA models. Initial stage of modeling is establishment of Autocorelation Function (ACF) and Partial Autocorelation Function (PACF). Characteristics of ACF and PACF can be used in determining the diagnosis of ARMA models and their order. The autocorrelation function (ACF) is denoted by(ρ_k), is the correlation or relationship between the observed data *n* of a time series dataX_t. The value of $\rho_k$can be assumed by the formula:

$$\rho_k = \frac{\sum_{t=1}^{n-k}(X_t - \bar{X})(X_{t+k} - \bar{X})}{\sum_{t=1}^{n}(X_t - \bar{X})^2} \quad (8)$$

While partial autocorrelation coefficient or Partial Autocorrelation Function (PACF) is a measure of the relationship between variables$X_t$withX_{t+k}. The value of the autocorrelation function$(\rho_{kk})$is formulated as follows:

$$\rho_{kk} = \frac{\rho_k - \sum_{j=1}^{k-1}(\rho_{k-1,j})(\rho_{k-j})}{1 - \sum_{j=1}^{k-1}(\rho_{k-1,j})(\rho_j)} \quad (9)$$

Time series model Autoregressive Moving Average p-q order (ARMA (p, q)) is a combined model of Autoregressive model with order (p) and Moving Average model with order (q).The following formulation of the ARMA model (p, q)

$$X_t = \left(\emptyset_1 X_{t-1} + \cdots + \emptyset_p X_{t-p}\right) + a_t - \left(\theta_1 a_{t-1} + \cdots + a_{t-q}\right) \quad (10)$$

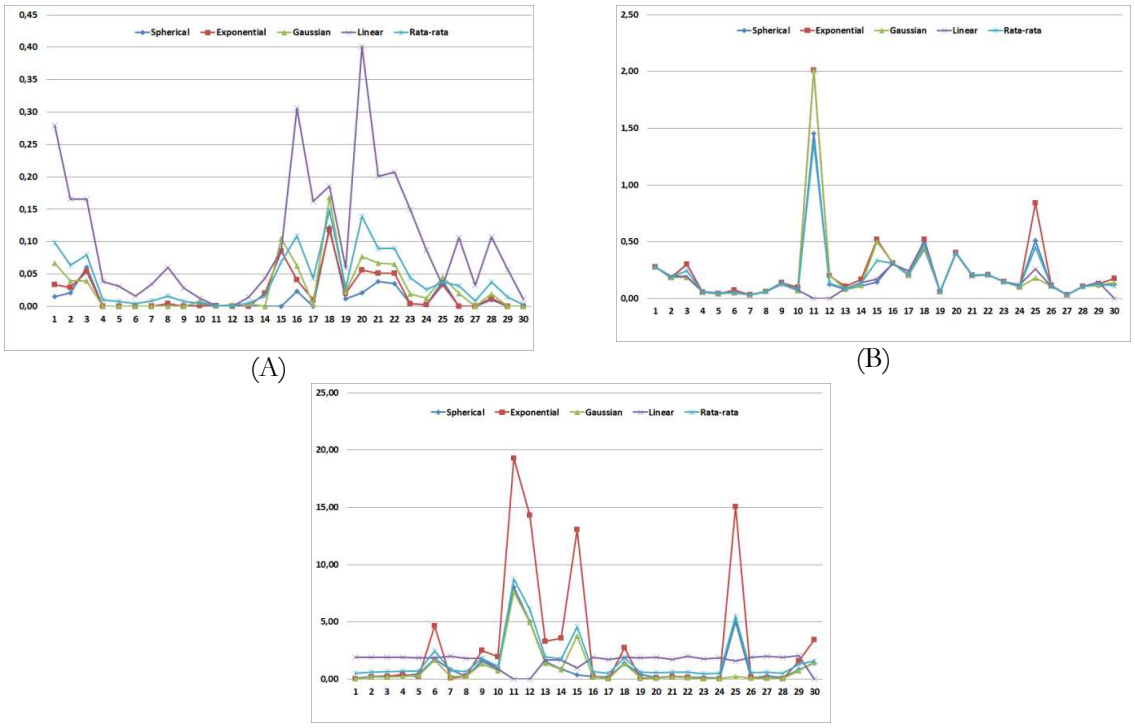
where, $\emptyset_p$denotesthe *p* th autoregressive paramete, $\theta_q$states the moving average parameter to q, and$a_t$denotes a random noise (white noise) during period *t*. In practice, the ARMA model is not unique. Therefore, it takes a measure in choosing the best model. In this paper the criteria used are Mean Square Error (MSE), which is expressed by the formula:

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(X_t - \hat{X}_t)^2 \quad (11)$$

Results and Discussions

Figure 2 presents three displayed images characterize the parameter value fluctuation of the semivariogram model. Figure 2A is a graph for the Variance Nugget parameters. Globally, the linear model parameter value (the colored line) is predominantly above the parameter values of other models and even above the average rating. In Figure 2B, the Gaussian model sill parameter values are mostly larger than other semivariogram models. While in Figure 2C, the parameter values of the exponential model range globally are larger than other semivariogram models.

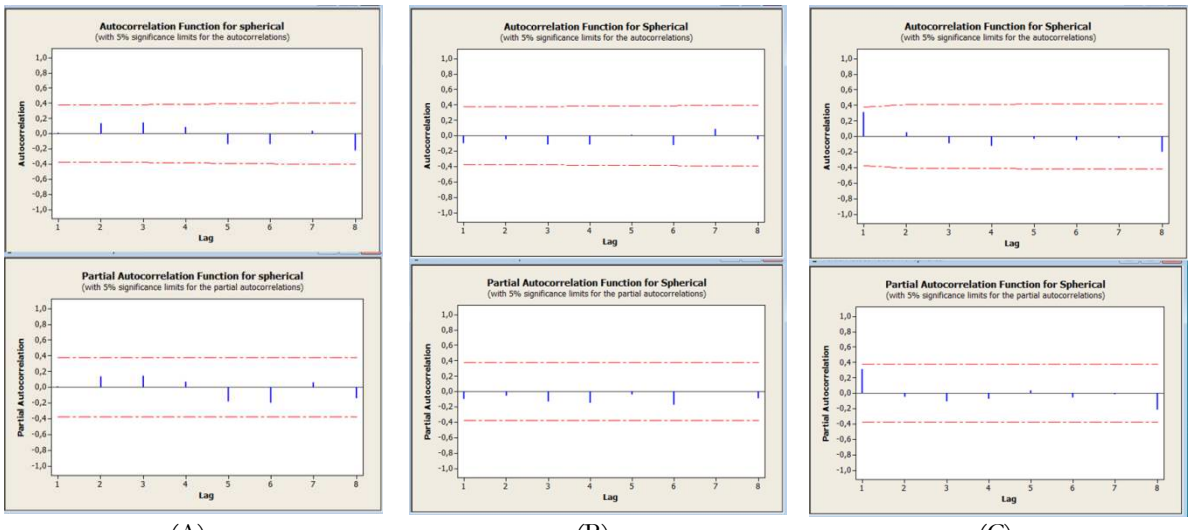
The second step generates ACF and PACF graphs of parameter values from four semivariogram models. Figure 3 shows the ACF and PACF for each of the Spherical model parameters. Those are within the upper and lower limits of the correlation value (red dashed lines), it indicates that data from the Spherical model parameters are statsioner. For Nugget Variance parameters, ACF plot on the third lag has the greater correlation value than the previous lag, as well as the fourth lag until the seventh lag. As for PACF plot, its characteristics are almost same as ACF plot. Of the two characteristics, the possible model is ARMA (2,2). Both in Sill and Range parameters, the characteristics of ACF and PACF are identical (only different marked), which there is a larger correlation value in the first lag and the next lag is relatively small. Sill and Range parameters are expected to have ARMA (1.1) model.



(A) (B)

(C)

Figure 2. Parameter Values of Spherical, Exponential, Gaussian, and Linear model; (A) Nugget Variance, (B) Sill, and (C) Range



(A) (B) (C)

Figure 3. ACF and PACF Characteristics of the Spherical Model for parameters (A) Nugget Variance, (B) Sill, and (C) Range

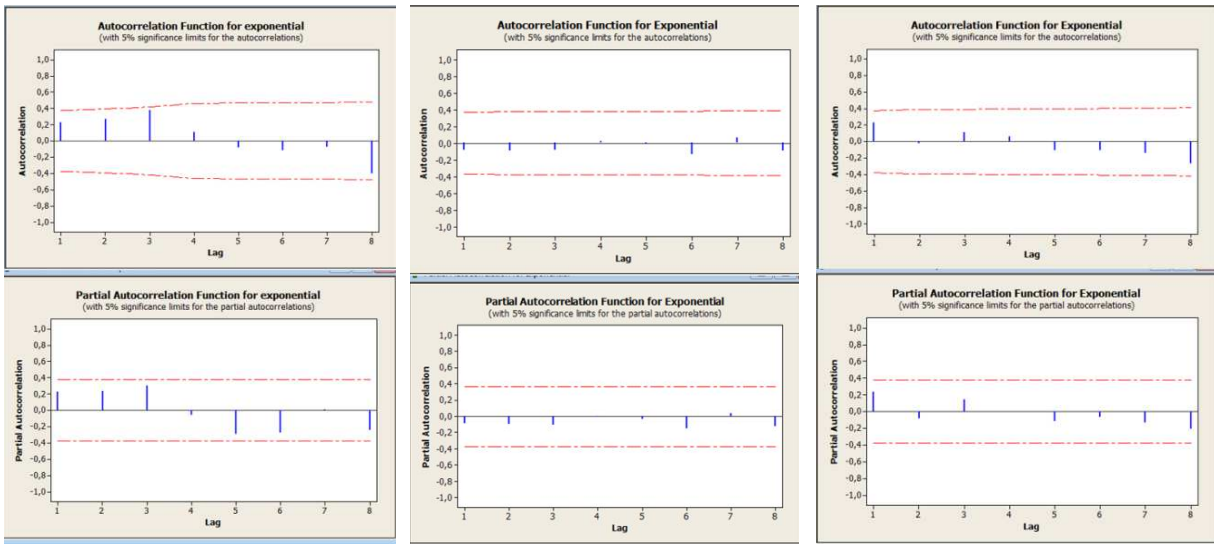
Table 1. Mean Square Error (MSE) value of ARMA model For Spherical Model parameters

No	Spherical Model Parameters	AR Models(p)		MA Model(q)		ARMA Models(p,q)			
		(1)	(2)	(1)	(2)	(1,1)	(1,2)	(2,1)	(2,2)
1.	Nugget Variance$(x\ 10^{-4})$	6. 786	6.908	6.786	6.921	6.979	7.258	7.082	6.453*)
2.	Sill $(x\ 10^{-2})$	7.263	7.507	7.250	7.428	5.939*)	5.994	6.771	5.989
3.	Range	3.024	3.130	3.040	3.122	3.132	2.726	3.250	2.672*)
4.	Average	1.032	1.069	1.038	1.066	1.064	0.929	1.106	0.911*)

*) the smallest value

Regarding to ACF and PACF graphs in figure 3, four ARMA model are choosen Next step ischoosing the best time series model. Model selection measurement is using the smallest MSE value criteria presented in Table 1. From Table 1, for the Nugget Variance parameter, the ARMA Model (2,2) results the smallest MSE value,$6.453\ x\ 10^{-4}$. While the Sill parameter of ARMA model (1,1) generates the smallest MSE value,$5.939\ x\ 10^{-2}$ and for the Range parameters, the smallest MSE value is 2.672 forARMA (2,2)model. ARMA model (2,2) for sill parameter is choosed for the simplification of model, because the difference of MSE value between ARMA (1,1) and ARMA (2,2) is relatively small and for ARMA (2,2) has minimum average value. It can be inferred that for the three parameters of Spherical semivariogram model has ARMA model (2,2), but coefficient values are different.

By doing the same method, the time series model is diagnosed for all three parameters of the Exponential semivariogram model. Figure 4 describes the ACF and PACF of exponential semivariogram model. Based on Figures 4A, 4B, and 4C, the characteristics of stationary data and possible order of ARMA are 1, 2, and 3, however it can be tried to get the minimum order. In order to determine the best model, MSE of ARMA model are calculated and MSE of ARMA(1,1), ARMA (1,2), ARMA (2,1) and ARMA (2,2) are presented in Table 2.



(A) (B) (C)

Figure 4. ACF and PACF Characteristics of Exponential Model for parameters (A) Nugget Variance, (B) Sill, and (C) Range

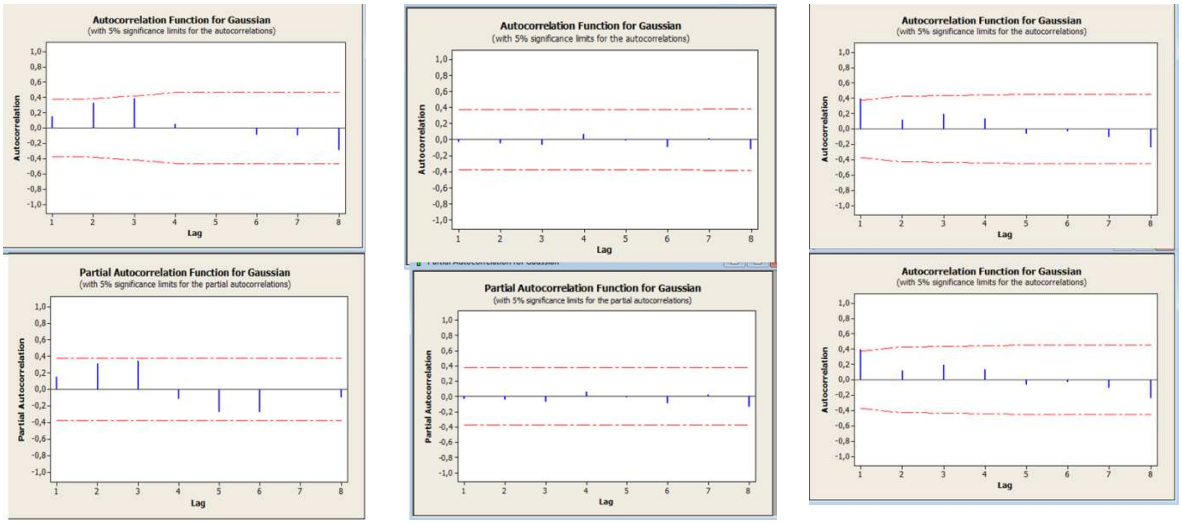
Table 2. Mean Square Error (MSE) Value of ARMA Model For Exponential Model Parameters

No	Exponential Model Parameters	AR Models(p)		MA Models(q)		ARMA Models(p,q)			
		(1)	(2)	(1)	(2)	(1,1)	(1,2)	(2,1)	(2,2)
1.	Nugget Variance$(x\ 10^{-4})$	8.479	8.279	8.617	8.685	8.235	9.126	8.253	6.563 *)
2.	Sill $(x\ 10^{-1})$	1.443	1.482	1.440	1.476	1.213	1.197*)	1.250	1.226
3.	Range $(x\ 10^{1})$	2.681	2.764	2.653*)	2,742	2,739	2.845	2.845	2.954
4.	Average	8.985	9.263	8.892*)	9.189	9.171	9.524	9.525	9.888

*) the smallest value

In Table 2, it can be seen that for the Nugget Variance parameters, the ARMA Model (2,2) has the smallest MSE value with $6.563\ x\ 10^{-4}$. While, Sill parameter of the ARMA model (1,1) has the smallest MSE value,$1.197\ x\ 10^{-1}$ and for the Range parameter, the smallest MSE value is 26.53 for the MA model (1). For this case, the model can not be simplified, because the three parameters have different models and the average values of the three models are not different significantly. So for the three parameters of the Exponential semivariogram model, Nugget Variance, Sill, and Range have ARMA (2,2), ARMA (1,2), and MA (1) respectively.

For the Gaussian model parameters, ACF and PACF plots as shown in Figure 5reveal stationary conditions. It can be seen from each plot of ACF and PACF in 5A, 5B and 5C, specifically for the nugget variance parameter in 5A, that the possible value of the code is more than one.



(A) (B) (C)

Figure 5. ACF and PACF Characteristics of the Gaussian Model for parameters (A) Nugget Variance , (B) Sill, and (C) Range

Table 3. Mean Square Error (MSE) Value of ARMA Model For Gaussian Model Parameters

No	Gaussian Model Parameters	AR Models(p)		MA Models(q)		ARMA Models(p,q)			
		(1)	(2)	(1)	(2)	(1,1)	(1,2)	(2,1)	(2,2)
1.	Nugget Variance $(x\ 10^{-3})$	1.572	1.454	1.587	1.268	1.510	1.117*)	1.432	1.157
2.	Sill $(x\ 10^{-1})$	1.236	1.371	1.326	1.371	1.173*)	1.186	1.419	1.245
3.	Range	2.467	2.553	2.451*)	2.537	2.,535	2.620	2.655	2.700
4.	Average	0.864	0.897	0.862*)	0.892	0.885	0.913	0.933	0.942

*) the smallest value

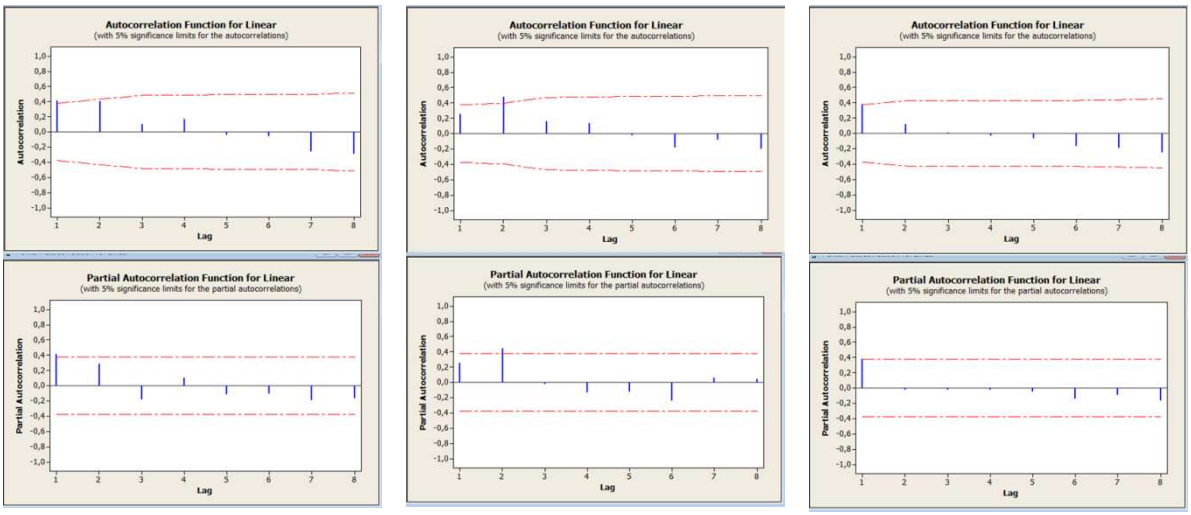
Table gives the information of the smallest MSE value in ARMA models for the gaussian semivariogram models. In Table 3, for the Nugget Variance parameter, ARMA (1,2)model has the smallest MSE value,$1.117\ x\ 10^{-3}$. WhileSill parameter of ARMA (1,1)model has the smallest MSE value,$1.173\ x\ 10^{-1}$andfor the Range parameterof MA (1)model, the smallest MSE value is 2.451. Similar to the case of the Exponential model, in the Gaussian model, the model can not be simplified, because the three parameters have different models and the mean values of the three models are not different significantly. So for the three parameters of the Gaussian semivariogram model, NuggetVariance, Sill, and Range have ARMA (1,2), ARMA (1,1), and MA (1) respectively.

The last semivariogram model used is Linear model. The parameters of the model, especially the Sill Model tend not to be stationary, as seen from the ACF and PACF values in the second lag above the upper limit. While other parameters are stationary (see Figure 6).

Table 4. Mean Square Error (MSE) Values of ARMA Model For Linear Model Parameters

No	Linear Model Parameters	AR Models (p)		MA Models (q)		ARMA Models (p,q)			
		(1)	(2)	(1)	(2)	(1,1)	(1,2)	(2,1)	(2,2)
1.	Nugget Variance $(x\ 10^{-3})$	8.296	7.719	9.330	7.757	8.176	9.126	7.461*)	7.740
2.	Sill $(x\ 10^{-2})$	1.375	1.,098	1.429	1.193	1.295	1.295	1.140	0.982*)
3.	Range $(x\ 10^{-1})$	3.001	3.030	2.951	3.034	3.031	2.475*)	3.147	2.934
4.	Average	0.107	0.107	0.106	0.108	0.108	0.090*)	0.111	0.104

*) the smallest value



(A) (B) (C)

Figure 6. ACF and PACF Characteristics of the Linear Model for parameters (A) Nugget Variance, (B) Sill, and (C) Range

As shown in Table 4, it is selected for the three parameters of the Gaussian semivariogram model which has the smallest MSE values. For Nugget Variance, Sill, and Range, the smallest MSE values are for ARMA (2,1), ARMA (2,2), and ARMA (1,2) respectively.

Table 5. The Comparison of Mean Square Error (MSE) values for ARMA model (Spherical, Exponential, Gaussian, and Linear parameters)

Semivariogram Model	Parameters for ARIMA Model						
	Nugget Variance	MSE	Sill	MSE	Range	MSE	Average
Spherical	ARMA (2,2)	1.109	ARMA (2,2)	1.699*)	ARMA (2,2)	6.810	3.206*)
Exponential	ARMA (2,2)	64.808	ARMA (1,2)	12.493	MA (1)	111.152	62.818
Gaussian	ARMA (1,2)	0.925*)	ARMA (1,1)	53.653	MA (1)	19.841	24.806
Linear	ARMA (2,1)	39.157	ARMA (2,2)	2.677	ARMA (1,2)	0.818*)	14.217

*) the smallest value

The final step is searching the MSE value of each model that has been generated. The value of this MSE is obtained from the difference of the square of the forecast value to the original data of part B (15%). Table 5 represents MSE values of the twelve selected models applied to the data of part B. As shown in Table 5, the average value of the three model parameters having the smallest MSE value of 3.206 is the Spherical model. Here are the ARMA models for each of the Spherical model parameters. For the Nugget Variance,Sill, and Range Parameters following the ARMA model (2.2), and the model can be written as follows:

$$X_t = 0.0097 + 0.9663X_{t-1} - 0.6799\,X_{t-2} + e_t - 1.0460\,e_{t-1} + 0.9732\,e_{t-2}$$

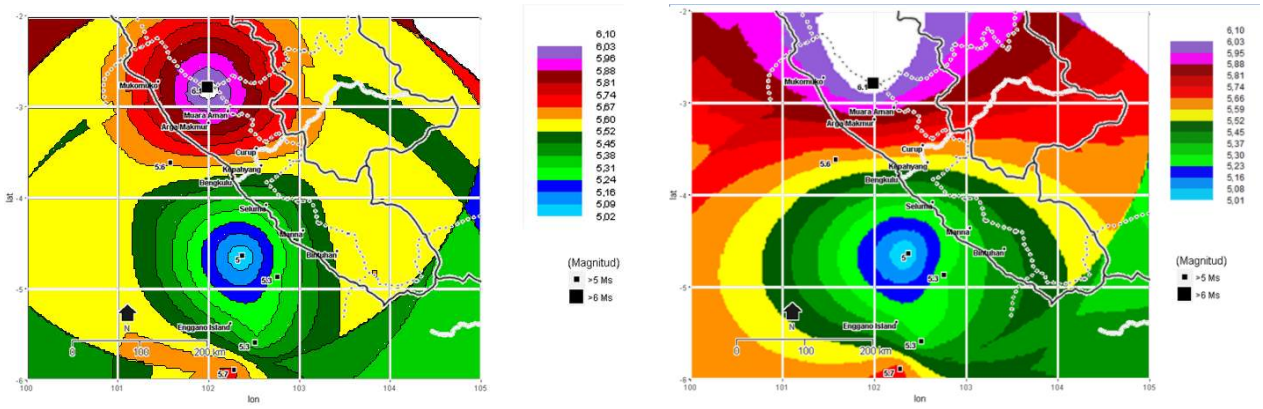
$$Y_t = 0.1403 - 0.2500Y_{t-1} + 0.6139\,Y_{t-2} + e_t - 0.0882\,e_{t-1} - 1.0457\,e_{t-2} \quad (12)$$

$$Z_t = 0.1457 + 1.1823Z_{t-1} - 0.3106\,Z_{t-2} + e_t - 0.9852\,e_{t-1} - 0.1389e_{t-2}$$

The semivariogram models and the estimation contours of the strength of earthquake events based on ARMA model sare described as follows:

1. Spherical Model:

$$\gamma(h)=\begin{cases}0{,}0081+0{,}266\left(\dfrac{3h}{2.1{,}5785}-\dfrac{1}{2}\left(\dfrac{h}{1{,}5785}\right)^3\right) & ,0<h\le 1{,}5785\\ 0{,}2741 & ,h>1{,}5785\end{cases} \quad (13)$$



(a) (b)

Figure 6. Contour The Number Of Earthquake Events For One Step (2015-I): (a) Based on ARMA Model, (b) Based on Testing Data.

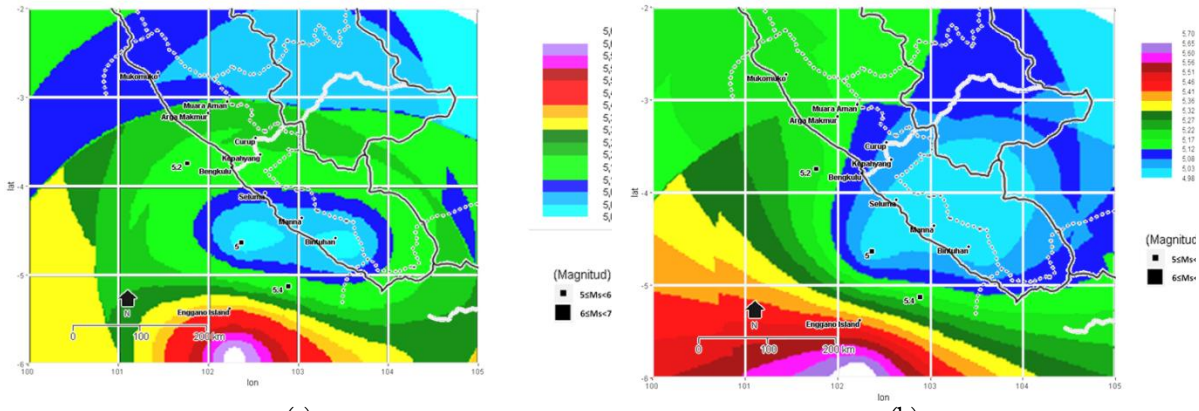
Figure 6(a) presents the contour formed from the spherical parameters of the model in the Equation 13. This contour describes the estimation of areas with magnitude of earthquakes in 6 months (January-June 2015). The earthquake strength estimation is obtained by the value of the Nugget Variance, Sill, and Range and colored according to its strength. In figure 6(a) the magnitude estimation of earthquake occurred in the range of 5.02-6.10 Ms. The blue gradation colored area is concentrated in around101.90-102.70^{O}E and 6.80-5.70^{O}S and it shows the possibility of earthquake area with magnitude of 5.02-5.24 Ms, while the earthquake with magnitude of more than 6 Ms is in the vicinity of white areas. Surrounding the blue area is green gradation color with a range of 5.23-5.45 Ms. Figure 6(a) is dominated by yellow that indicates the strength of earthquake range 5.52-5.60 Ms.

In the testing data, there are 6 earthquakes occurred during January-June 2015. The five events of them are in the color contours according to the estimates. Earthquake with the strength of 5.6Ms that occurred at the center 3.62^{O}S and 101.58^{O}E is in an orange color with a range of 5.59-5.66Ms and an earthquake with a strength of 6.1 Ms with the center 2.79^{O}S and 101.99^{O}Eis in a white area with a range of 6.03 and 6.10 Msas well as for the other three images that can be seen in Figure 6(a). One of the earthquakes that is not suitable to the contour color is the event occurred in 5.5^{O}S and 102.51^{O}E with magnitude 5.3 Ms that is in the green color with the range 5.31-5.38.

Figure 6(a) can be compared to figure 6(b) which shows the contour map using real data in testing data. Both of the contour have the similar pattern and nearly same range. Therefore, in 6(b) the red colors dominate the contour map and the violet color areas in 6(b) is larger than the figure 6(a). In other words, figure 6 (b) gives the probability of earthquake events greater than 5.59 more likely to occur.

2. Spherical Model:

$$\gamma(h)=\begin{cases}0{,}0139+0{,}2884\left(\dfrac{3h}{2.1{,}588}-\dfrac{1}{2}\left(\dfrac{h}{1{,}588}\right)^3\right) & ,0<h\le 1{,}588\\ 0{,}3023 & ,h>1{,}588\end{cases} \quad (14)$$



(a) (b)

Figure 7. Contour of earthquake occurrence for the second step (2015-II): (a) based on ARMA model, (b) based on testing data.

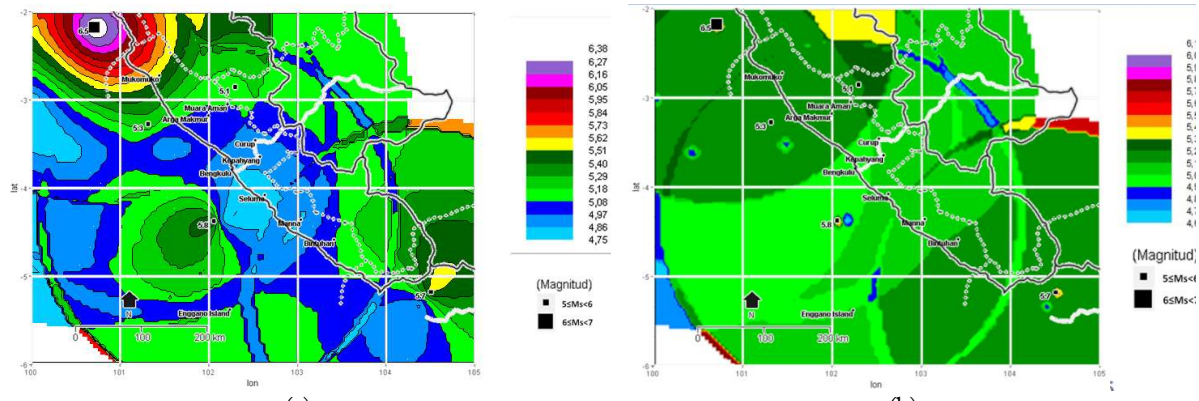
Figure 7(a) shows the contour formed from the spherical parameters of the model in the Equation 14. It describes the estimation of areas with magnitude of earthquakes in 6 months (July-December 2015). In figure 7(a) the magnitude estimation of earthquake occurred in the range of 5.00-5.68 Ms. The blue gradation colored area in two locations, around 102-104^{O}E and 5-4^{O}S and around 100-105^{O}E and 4-2^{O}S. Yellow, orange, red violet and white areas are in 100-104^{O}N and 6-4^{O}S and present the magnitude more than 5.32 Ms, although it remains green color area.

In testing data, there are 3 earthquakes occurred during July-December2015. The two events of them are in the color contours according to the estimates. Earthquake with the strength of 5 Ms that occurred at the center 4.64^{O}S and 102.37^{O}E is in a blue color with a range of 5.00-5.05Ms and an earthquake with a strength of 5.2 Ms with the center 3.75^{O}S and 101.77^{O}E is in a green color with a range of 5.14-5.23 Ms. The earthquake occurred in 5.13^{O}S and 102.89^{O}E is not suitable to the contour color. With the magnitude 5.4 Ms, it is in the green area, while it should be in red area.

If they are compared, Figure 7(a) and figure 7(b) are relatively seen a bit like. In figure 7(b), color gradations are more concentrated in one location. The blue gradation colored area is in around 102-105^{O}E and 5-2^{O}S and it shows the possibility of earthquake area with magnitude of 4.98-5.12 Ms, while the earthquake with magnitude of 5.12-5.32 Ms is in the vicinity of green areas. Yellow, orange, red violet and white areas are in 100-104^{O}E and 6-4^{O}S and present the magnitude more than 5.32 Ms.

3. Spherical Model :

$$\gamma(h) = \begin{cases} 0{,}0176 + 0{,}2154\left(\dfrac{3h}{2.1{,}533} - \dfrac{1}{2}\left(\dfrac{h}{1{,}533}\right)^3\right) & ,0 < h \le 1{,}533 \\ 0{,}233 & ,h > 1{,}533 \end{cases} \quad (15)$$



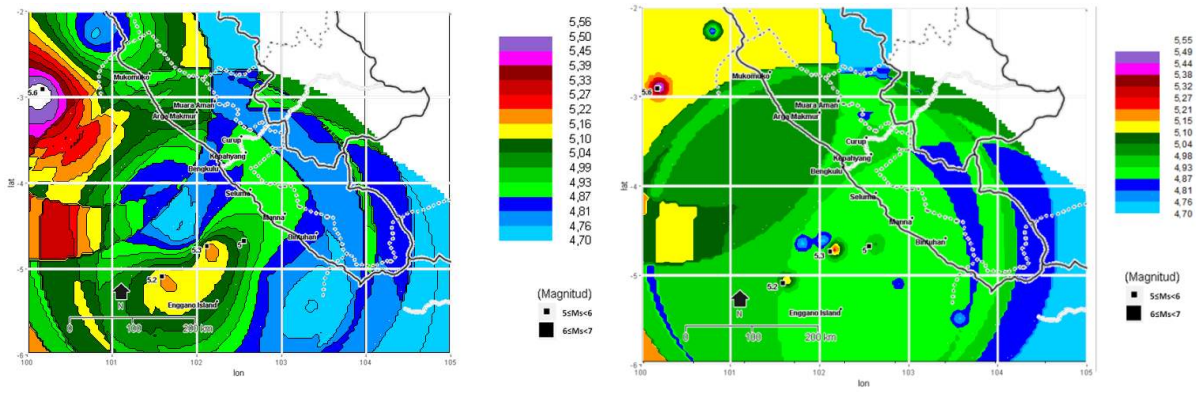
(a) (b)

Figure 8. Contour of earthquake occurrence for the third step (2016-I): (a) based on ARMA model, (b) based on testing data.

Figure 8(a) describes the estimation area from the spherical parameters of the model in the Equation 15 and figure 8(b) shows the contour map using real data in testing data. Figure 8(a) shows the probability of earthquake range 4.75-6.38 Ms. the green gradation area is almost as large as the blue area and they are dominated contour map. Only one of four earthquake events is in the color contours according to the estimates, that is the earthquake occurred in 2.86^{O}S and 102.3^{O}E with the magnitude 5.1 Ms. In figure 8(b) green areas are dominated the contour map and indicate the areas of probability of earthquake with magnitude 4.97- 5.35. it is larger than others area and there is a small area that has estimation magnitude more than 5.35 Ms. There is no earthquake event that occurred in suitable color in this figure.

4. Spherical Model :

$$\gamma(h) = \begin{cases} 0{,}0173 + 0{,}2503\left(\dfrac{3h}{2.1{,}4649} - \dfrac{1}{2}\left(\dfrac{h}{1{,}4649}\right)^3\right) & ,0 < h \le 1{,}4649 \\ 0{,}2676 & ,h > 1{,}4649 \end{cases} \quad (16)$$



(a) (b)

Figure 9. Contour The Number of Earthquake Occurrences For The Fourth Step (2016-II): (a) based on ARMA model, (b) based on testing data.

Figure 9(a) and 9(b) have the same pattern, while they show different color. Figure 9(a) produces the contour which has distributed colors and estimation range 4.70-5.56 Ms. On the other hands, figure (b) results the contour with the majority color is green. In figure 9(b), Only one of four earthquake events is in the color contours according to the estimation. Meanwhile There is no earthquake event that occurred in suitable color in this figure.

Conclusion

Earthquakes in the province of Bengkulu for the period 2000-2016 can be modeled by combining two methods, namely kriging interpolation and time series analysis. Based on the modeling step using 85% of historical data and the validation phase using 15% of historical data, Spherical model is selected model with minimum MSE value of 3.206 and time series model for each parameter ARMA (2,2) having different coefficient estimate values.

From the four contours map based on the ARMA models, contours resulted on the estimation of the first step parameters are most similar to the contours produced by testing data in January-June 2015. While on other contours, the contours of the ARMA model estimation parameter show the contours that have similar pattern to the contours on data testing. However, the contours of the ARMA model show the color and area of the earthquake estimation is more varied. Based on the earthquake occurrence, the contours of the ARMA model and data testing do not show that the earthquakes that occur are all located on the contour of the appropriate estimation area.

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References

Amstrong, M. 1998. Basic Linear Geostatistics, Springer-Verlag Berlin Heidelberg, Germany.

Box, G.E.P and Jenkins, G.M. 1994. Time Series Analysis: Forecasting and Control. Third Edition. Prentice Hall: New Jersey

Cakmak,A.S., Sherif, R. I.,and Ellis, G. 1985. Modelling Earthquake Ground Motions In CaliforniaUsing Parametric Time Series Methods. International Journal of Soil Dynamics and Earthquake Engineering, 4(3): 125-131.

Carr, J.R., Deng, E.D., and Glass, C.E. 1986. An Application of Disjunctive Kriging forEarthquake Ground Motion Estimation. Mathematical Geology, 18(2): 197-213.

Carr, J.R., and Roberts, K.P. 1989. Application of Universal Kriging for Estimation of Earthquake Ground Motion: Statistical Significance of Results. Mathematical Geology, 21(2): 255-265

Faisal, F. 2014. Aplikasi Program Linier dalam Menentukan Nilai Parameter Model Semivariogram Teoritis. Prosiding Semirata IPB Bogor, 2014. Bogor

Fauzi, Y., Suwarsono, and Rizal, J.2012. PenataanRuang Wilayah Pesisir Berbasis Mitigasi Bencana Sebagai Upaya Meminimalisir Dampak Resiko Bencana Tsunami Bagi Masyarakat Kota Bengkulu. Laporan Penelitian Unggulan Universitas Bengkulu. Bengkulu.

Fong, S and Nannan, Z. 2011. Toward an Adaptive Forecasting of Earthquake Time Series from Decomposable and Salient Characterictics. The Third International Conferences on Pervasive Patterns and Applications, ISBN:978-1-61208-158-8

Hardle, W. 2007.Applied Multivariate Statistical Analysis. Humbolt-Universitat zu Berlin. Jerman

Irwanto, A., Nugroho, S., and Rizal, J. 2014. Penggunaan Model AFRIMA untuk Peramalan Gempa Tektonik di Wilayah Sumatera, Skripsi, Tidak Dipublikasikan.

Kannan, S. 2014. Innovative Mathematical Model for Earthquake Prediction. Engineering Failure Analysis,41: 89-85.

Lin, J.H.2014. Time Series Modeling of Earthquake Ground Motions UsingARMA-GARCH Models. Applied Mechanics and Materials, 470: 240-243

Makridakis, S., S.C Wheelwright dan V. McGee. Alih bahasa Ir. Hari Suminto. 1999. Metode dan Aplikasi Peramalan.Edisi revisi. Binarupa Aksara: Jakarta.

Pfeiffer, D.U., and Robinson,T. P. 2008.Spatial Analysis in Epidemiology. Oxford University.

Sugai, M., Mori, Y., and Ogawa, K. 2015. Application ofKriging Method Into Practical Estimations OfEarthquake Ground Motion Hazards. J. Struct. Constr. Eng., AIJ, 80(707): 39-46.

Wei, W.W.S. 2006. Time Series Analysis: Univariate and Multivariate Methods.Addison-Wesley Publishing Company: New York.