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Analysis of Stiffening Methods and Effects on Irregular Single-layer Lattice Shell Structures

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Abstract: Local stiffening is often a good solution to mechanical property problems of irregular single-layer lattice shell structures. The effects of three local stiffening methods, namely section enlargement stiffening, planar truss stiffening, and space truss stiffening, on their structural stiffness, strength, and overall stability were analyzed in this study. A practical engineering example showed that these three stiffening methods could effectively reduce the deformation of the lattice shell under vertical and lateral loads, reduce the comprehensive stress ratio, and increase the buckling eigenvalue and ultimate bearing capacity factor. The local space truss stiffening method had the best comprehensive effect. The same stiffening methods were applied to a regular lattice shell and the analysis showed that the stiffening effect on a regular shell is quite different from that on an irregular lattice shell. The three stiffening methods could not reduce its deformation under vertical loading but could reinforce the strength and overall stability of the structure effectively. Proper suggestions are proposed according to the preceding analysis in case a single-layer lattice shell structure cannot meet the demands of the design code.

Keywords: *irregular shell; planar truss; single-layer lattice shell; space truss; stiffening.*

1 Introduction

As space structures, single-layer lattice shells can span great distances and have graceful shapes. Such curved grid structures allow architects sufficient freedom of creation in many respects such as architectural plane, external form, and shape [1]. Therefore, single-layer lattice shell structures have been welcomed by public building designers for years [2-5]. With more intense designing market competition and diversification of personal aesthetics, regular curved shells, such as hemispherical, cylindrical, conical, elliptical, hyperbolic, and parabolic shells, can no longer meet the demands of architects and owners. As a result, the irregular lattice shell [6], or free-curved lattice shell, has emerged. Figure 1 shows some examples of irregular lattice shells. There are some differences between the mechanical properties (including strength, stiffness, and stability) of irregular and regular lattice shells. Such problems can be solved by two

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methods: using double-layer lattice shells [1], and stiffening of the single-lattice shell [7].

The former method requires too many additional members and may lead to a poor indoor visual effect. The latter only requires the addition of a small number of members and can achieve a good balance between the indoor visual effect and the mechanical properties of the structure. The roof of the Ordos Museum with a stiffened single-layer lattice shell is an example [7]. Its stiffening members are planar trusses with a height of 1 m, arranged in specific areas in specific directions with specific spacing, solving the problem of large deformation and poor stability. A further example is Tianjin Water World, which has a large-span single-layer elliptical shell [8]. Three structure schemes were compared, namely a profile steel arch, a double-layer shell, and an arch-stiffened single-layer shell. The third scheme had a positive effect.

Shanghai Chenshan Botanical Garden

Qingdao Oriental Cinema

Germany Mannheim Exhibition Building

Chengdu Tianfu International Airport

Figure 1 Examples of irregular lattice shell structure.

The preceding literature yields good solutions for specific projects of irregular single-layer lattice shells. In this paper, various local stiffening methods for single-layer lattice shells are classified more systematically. Furthermore, the stiffening effects of various methods are discussed using practical irregular shells as examples. In addition, an example of a regular single-layer shell stiffened using the above methods is presented and the stiffening effects are compared with that of an irregular shell.

2 Three Methods to Stiffen Irregular Single-Layer Lattice Shells

2.1 Stiffening Method 1: Local Enlargement of Member Section

This stiffening method involves the enlargement of a section of a member in certain areas. The section can be widened, increased in height, or both. For a rectangular tube shell member, the cross-section of the shell prior to stiffening is shown in Figure 2 and the stiffening method is shown in Figure 3.

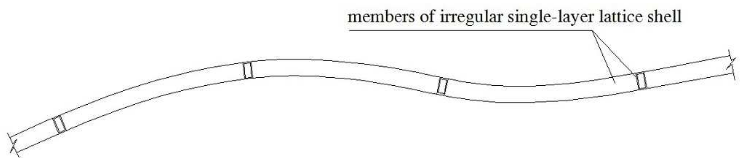


Figure 2 Schematic view of local section of irregular single-layer lattice shell.

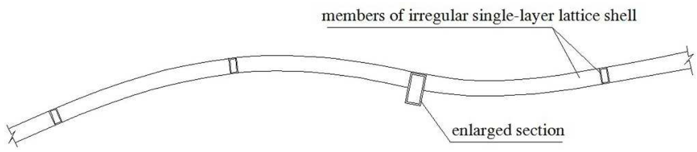


Figure 3 Schematic view of stiffening method 1.

Stiffening method 1 has the least influence on the interior visual effect and the stiffening position can be chosen from the inflection points of the shell curve or other areas with large stress variation.

2.2 Stiffening Method 2: Planar Truss Stiffening

At the stiffening position, the original shell member is taken as the upper chord of the planar truss. The bottom chord and web are added to constitute a planar truss, as shown in Figure 4.

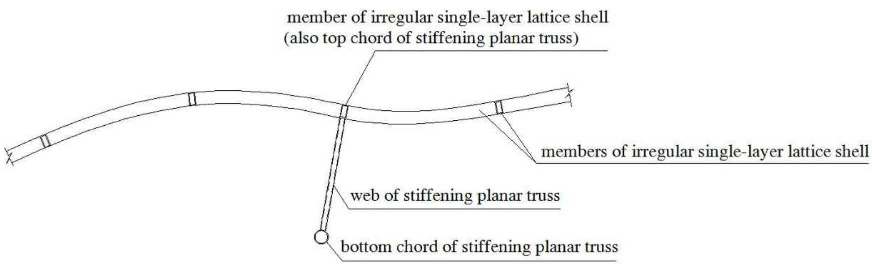
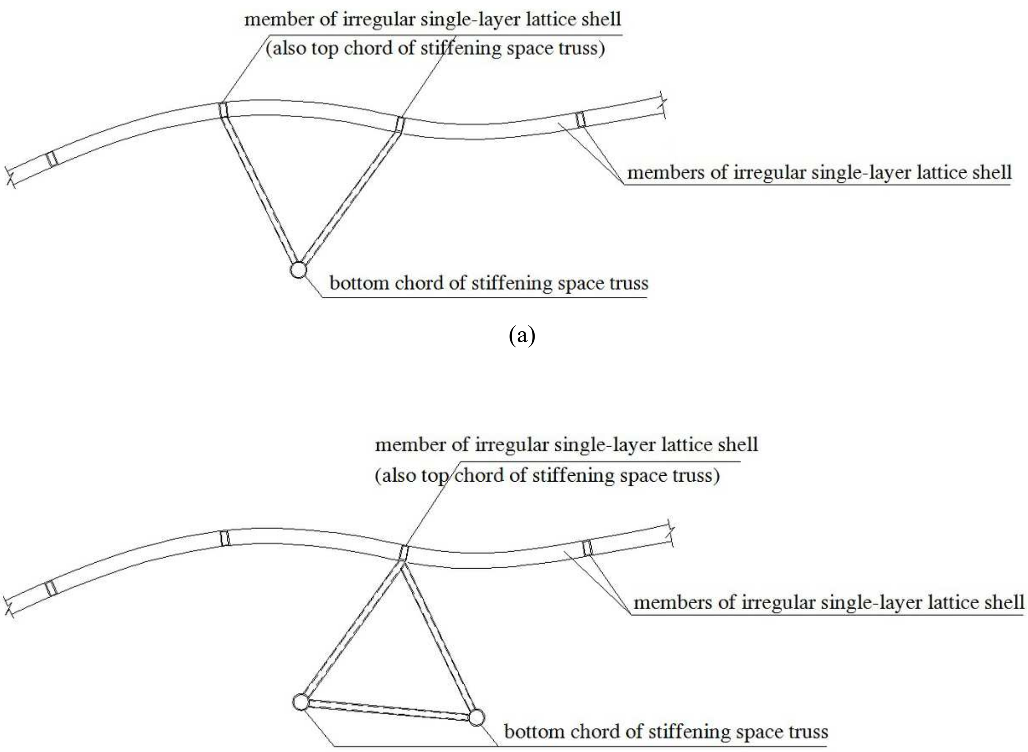


Figure 4 Schematic view of stiffening method 2.

Since the planar truss adopts the original shell member as the upper chord, the height of the structure is smaller than when a separate planar truss is introduced. Moreover, the structure is relatively simple. A small number of members are added; thus, the influence on the interior visual effect is small.

2.3 Stiffening Method 3: Space Truss Stiffening

At the stiffening position, two rows or one row of original shell members is chosen as the upper chord. The bottom chord and web are added to constitute a space truss, as shown in Figure 5. Stiffening method 3 also adopts the original shell member as the upper chord of the stiffening space truss; thus, the structure height is less smaller than when a separate space truss is introduced. In terms of the interior visual effect, stiffening method 3 has more influence than that of methods 1 and 2.



(b)

Figure 5 Schematic view of stiffening method 3.

3 Comparison of Effects of Various Stiffening Methods for Irregular Single-Layer Lattice Shells

A practical project was used as a calculation example to make a comprehensive comparison between the three stiffening methods with respect to strength, stiffness, and stability.

3.1 General Project Description

The project is a greenhouse in a botanical garden. The whole building is shaped like a ‘puffer fish’ with a height, projected area, span, and length of 25 m, 3800 m^2, 45 m, and 94 m, respectively. A single-layer lattice shell is the main structural system. The member section is a rectangle tube and the grid length ranges from 1.5 to 2.5 m. The shell is covered by a glass curtain wall and roof. The plane and an axonometric projection of the structural configuration are shown in Figure 6.

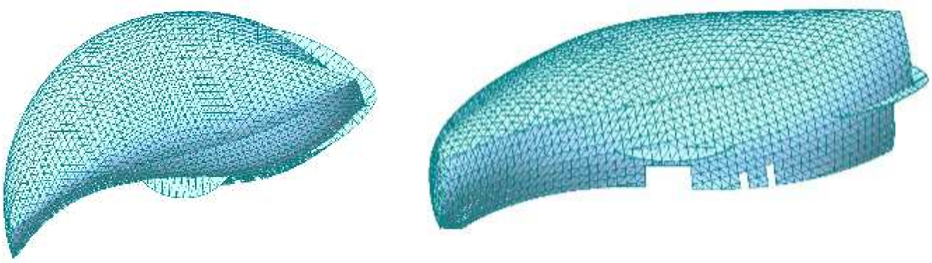


Figure 6 Plan and axonometric drawing of structure outline.

3.2 Structural System

Owing to its complicated shape, the shell cannot be expressed by a single surface equation. Besides, several sharp angles exist at the head and tail of the ‘puffer fish’. There are both positive and negative Gauss curvature surfaces in the roof and the wall [9]; thus, it is a typical irregular single-layer lattice shell structure. During the design process, the three stiffening methods discussed above were considered. An axonometric projection of the overall structure is shown in Figure 7. The stiffening members are located at the following positions:

1. Two transversal stiffening areas at the middle of the structure;
2. Stiffening areas at the head and tail of the structure;
3. Circular stiffening area at the border between the roof and the wall.

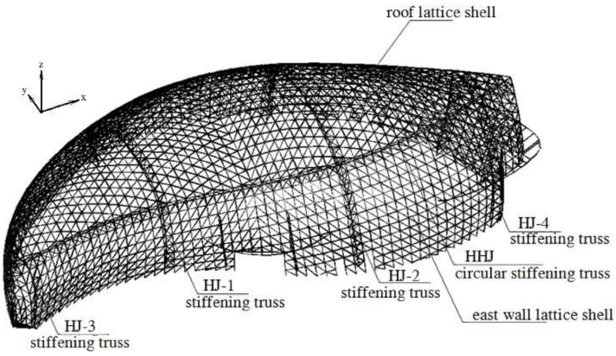


Figure 7 Axonometric drawing of the overall structure for stiffening method 3.

Midas was adopted as the analysis software and all members are beam elements. All joints are rigid, with the exception of the bottom joints, which are pinned to the structure underneath. The strength grade of the steel is Q235B. The yield strength is 235 N/mm^2, the elastic modulus is 2.06×10^5 N/mm^2, and the Poisson ratio is 0.3. The section type of all shell members in the roof and the wall is a rectangular tube, with a size range of 200×80×8 to 200×80×14. The section sizes of the stiffening member of the three stiffening methods are listed in Table 1.

Table 1 Section sizes of stiffening members.

Stiffening method	Section sizes of stiffening members
1	Original shell member section is enlarged to a box section of 800×200×12×20
2	A stiffening planar truss is introduced with a bottom chord of circular tube 203×6–273×16 and a web section of circular tube 89×5–140×6
3	A stiffening space truss is introduced with a bottom chord of circular tube 203×6–273×16 and a web section of circular tube 89×5–140×6

The following load cases were considered: dead load; live load; +x direction wind load; -x direction wind load; +y direction wind load; -y direction wind load; temperature rise effect; temperature fall effect; and earthquake action in x, y, z direction.

For such a large span structure, non-linear analysis is needed. The following analyses of the lattice shell are all geometrically non-linear analyses.

3.3 Calculation Result of Structural Deformation

The load-vertical deformation curve is shown in Figure 8. The load here refers to the normal service load, i.e. dead load plus live load. The maximum deformation occurs at the middle of the roof. The horizontal deformation is shown in Figure 9. In addition, the load-vertical curve of the original (unstiffened) lattice shell is also illustrated in the figures. The iteration method in Figures 8 and 9 is the Newton-Raphson iteration method; the convergence tolerance was set to 0.001.

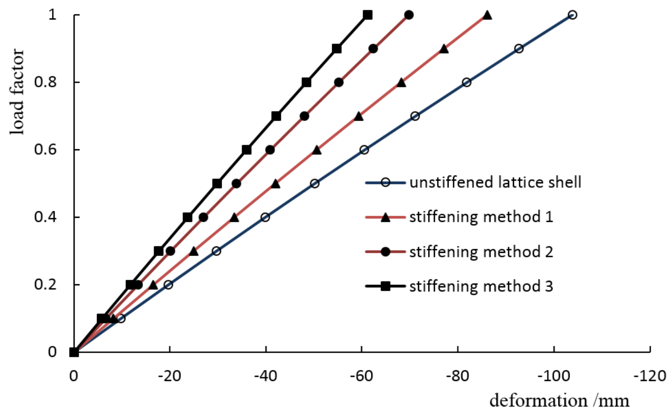


Figure 8 Load-vertical deformation curve.

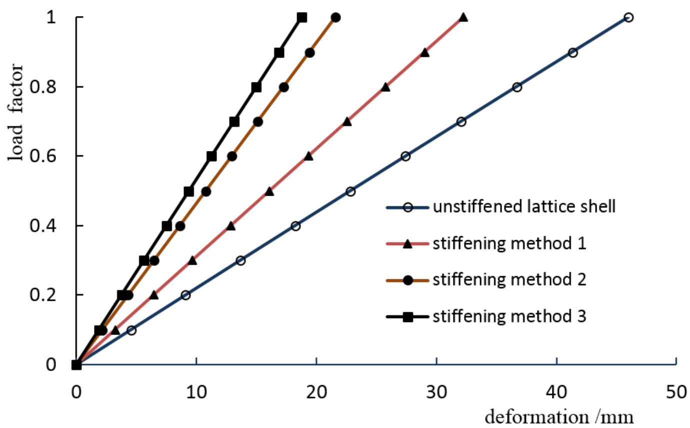


Figure 9 Load-lateral deformation curve.

Figures 8 and 9 show that all three stiffening methods could increase the stiffness of the single-layer lattice shell. The vertical deformation decreases of methods 1, 2, and 3 were 21.9%, 35.6%, and 44.4%, respectively. The lateral deformation decreases of methods 1, 2, and 3 were 29.1%, 52.3%, and 58.7%, respectively. Stiffening method 3 had the most significant effect on the stiffness of the single-layer lattice shell.

3.4 Calculation Results of Member Force and Stress

According to the internal force results, although there is some difference between the internal force distribution of various stiffening methods, the following things are clear:

1. The internal force distribution is relatively homogeneous. On the whole, the forces in the upper field are relatively small and those in the bottom field are relatively large.
2. The main internal forces of the shell members are axial forces, whereas the member moments are relatively small. This shows that the overall mechanical performance of the members closely approximates that of an ordinary lattice shell, whereas the main function of the stiffening members is to stiffen the lattice shell locally.
3. For stiffening method 3, the internal force differences in the stiffening truss members are relatively large and some members have very large internal forces. Stiffening method 2 exhibits similar results.

The maximum comprehensive stress ratios of shell members of various stiffening members are shown in Figure 10, where value 0 of the abscissa indicates no stiffening, values 1 to 3 indicate stiffening method 1 to 3. It was shown that all three stiffening methods could reduce the maximum stress ratio of the shell members. With a stress decrease rate of 33.1%, stiffening method 3 had a significant effect on improving the strength of the lattice shell. The stress decrease was 16.1% and 3.6% for stiffening methods 1 and 2, respectively; these did not have a pronounced effect.

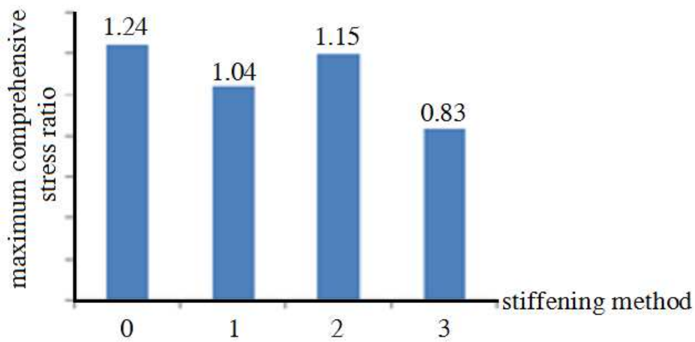


Figure 10 Maximum comprehensive stress ratios of structural members.

3.5 Modal Analysis

Eigenvalue analysis yielded the first three natural frequencies, which are shown in Table 2. The mass of the structure is taken under dead load plus half of the live load.

Table 2 Top three natural vibration frequencies.

Stiffening method	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
no stiffening	1.35	1.60	1.91
1	1.72	1.90	1.97
2	1.92	1.99	2.07
3	1.94	2.06	2.14

All frequencies of the stiffened lattice shell were higher than those of the unstiffened shell. The extent of the increase in the results of stiffening methods 1 to 3 is gradual.

3.6 Overall Stability Analysis of the Lattice Shell

The buckling modes and the corresponding eigenvalues were obtained through elastic buckling analysis. The eigenvalue of the first buckling mode is shown in Figure 11. Figure 11 shows that the eigenvalues for all three stiffening methods are similar and significantly higher than those of the unstiffened lattice shell.

Then, a nonlinear elastic full-process analysis was carried out. By using the first buckling mode as the initial imperfection and adjusting the maximum imperfection to 1/300 of the lattice shell span [11-12], i.e. 150 mm, the ultimate load force was obtained by full-process analysis, as shown in Figure 12. Both the displacement convergence criterion and the force convergence criterion were adopted here, and the convergence tolerances were both set to 0.001.

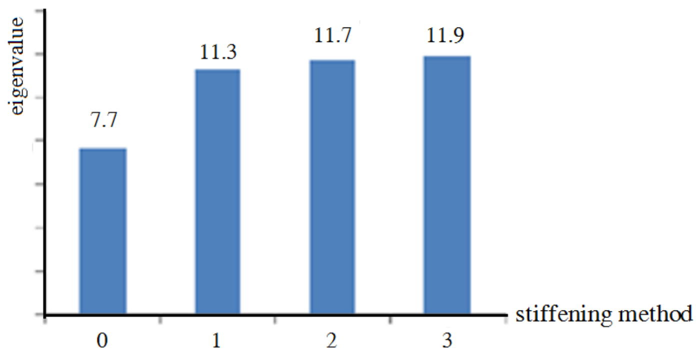


Figure 11 Eigenvalue of 1st mode for all stiffing methods.

Figure 12 shows that the ultimate load factors for all three stiffening methods are greater than those of the unstiffened lattice shell; however, there is a considerable difference between the increase extent and the regularity is not obvious.

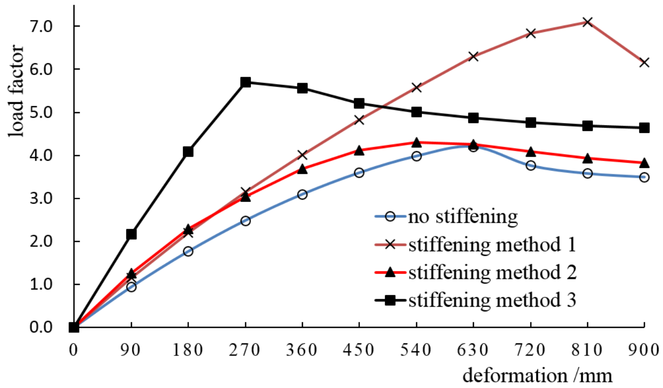


Figure 12 Load-deformation curve of full-process analysis.

To discuss the post-buckling stage, taking stiffening method 3 as an example, Figure 13 gives the load-deformation curve of the stiffened lattice shell by setting the maximum deformation to 3800 mm. It can be seen that the snap-through phenomenon occurs in the post-buckling stage and the equilibrium state changes from ultimate point A to point B. Though the load factor of point B is greater than that of point A, the deformation at point B is as much as ten times that at point A. This shows the snap-through is not a stable state and there is a sudden drop in the load factor after point B.

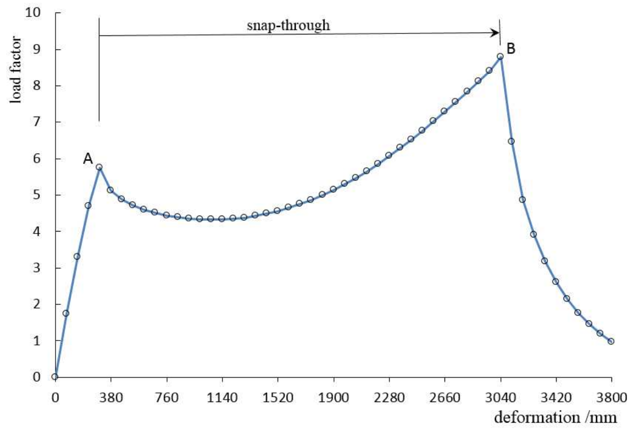


Figure 13 Snap-through in the post-buckling stage of the lattice shell.

3.7 Comprehensive Effect of the Three Stiffening Methods

The stiffening effects of the three stiffening methods on overall stiffness, strength, and overall stability according to the above results are listed in Table 3. The number of stars represent the stiffening effect, where one, two, and three stars means *small*, *obvious*, and *significant*, respectively.

Table 3 Comprehensive comparison of stiffening effects.

Stiffening method	Stiffening effect on overall stiffness	Stiffening effect on strength	Stiffening effect on overall stability
1	★	★	★★
2	★★	★	★
3	★★★	★★	★★

It is obvious that stiffening method 3, i.e. the space truss stiffening method, has the greatest stiffening effect on the irregular single-layer lattice shell.

4 Stiffening Effect Comparison for Regular Single-Layer Lattice Shells

4.1 Calculation Example and Results

Further analysis was carried out to investigate the stiffness characteristics of irregular single-layer lattice shells by comparing the stiffening effects on regular and irregular lattice shells.

Take a spherical single-layer lattice shell of 40 m span as an example. The axonometric projections of the original and stiffened lattice shells are shown in Figure 14. Only stiffening method 3 is illustrated; the stiffening positions of the other methods are the same as those of method 3.

Regular shell（unstiffened） Regular shell（stiffened）

Figure 14 Axonometric drawing of regular grid lattice shell.

The histogram of the maximum vertical deformation for the unstiffened regular lattice shell and the stiffened regular shell using the three stiffening methods is shown in Figure 15. The deformation value was taken under dead load plus live load. The maximum comprehensive stress ratio of shell members under various load combinations, the buckling eigenvalues, and the ultimate load factor obtained from the full-process analysis are shown in Figures 16 to 18.

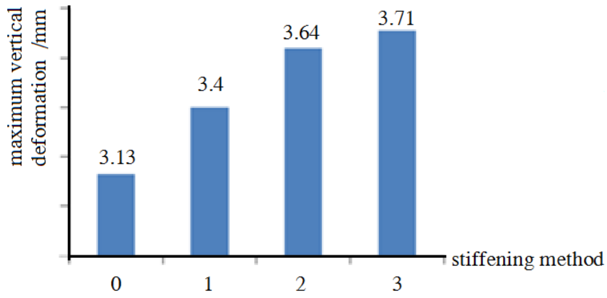


Figure 15 Maximum vertical deformation of all stiffening methods.

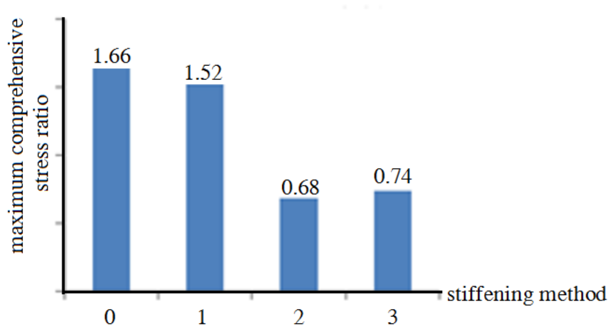


Figure 16 Maximum comprehensive stress ratio of structural members.

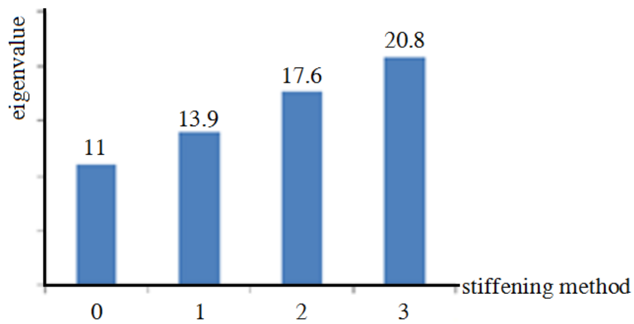


Figure 17 Eigenvalue of 1st mode for all stiffening methods.

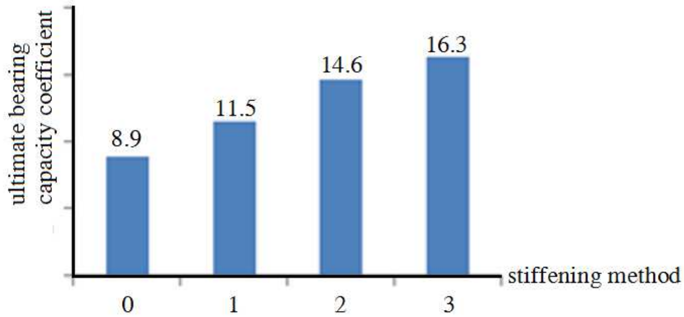


Figure 18 Ultimate bearing capacity coefficient.

Interestingly, Figure 15 shows that all three stiffening methods did not reduce but increased the maximum deformation, which indicates that local stiffening cannot increase the overall stiffness of regular single-layer lattice shells. Figure 16 shows that all three stiffening methods decreased the maximum stress ratio of the members; the effects of stiffening methods 2 and 3 are significant. Figures 17 and 18 show that all three stiffening methods increased the stability of regular lattice shells, where the stiffening extent increases from method 1 to method 3.

4.2 Analysis of Stiffening Effects

The stiffening effects of the three stiffening methods on overall stiffness, strength, and overall stability are listed in Table 4 according to the above results. The meaning of the stars is the same as in Table 3, while × indicates *no effect*.

Table 4 Comparison of stiffening (for regular lattice shells).

Stiffening method	Stiffening effect on overall stiffness	Stiffening effect on strength	Stiffening effect on overall stability
1	×	★	★
2	×	★★★	★★
3	×	★★★	★★

Comparison of Tables 3 and 4 indicates the following:

1. The effect of local stiffening on single-layer lattice shells has a close relation to the regularity of the lattice shell.

2. For the example above, all three stiffening methods increase the strength, stiffness, and stability of irregular single-layer lattice shells, but cannot increase the stiffness of regular single-layer lattice shells.
3. For regular single-layer lattice shells, the stiffening method of enlarging member sections locally has small effects on structural strength and overall stability. The stiffening methods of planar and space truss stiffening have significant effects on the structural strength and overall stability.

These differences between regular and irregular single-layer lattice shells, and the stiffening effects of local stiffening on structural mechanical properties are caused by the mechanical characteristics of the lattice shell structure. With regard to stiffness, since the mechanical characteristics of regular lattice shells more closely approximate a 'perfect shell'; nearly all members are compressive. Since the axial stiffness of the members is far greater than the flexural stiffness, the vertical deformation of unstiffened regular lattice shells is very small. After local stiffening, the members become compression-bending members, which leads to greater vertical deformation, especially in unstiffened positions.

With regard to overall stability, the single-layer lattice shell is a deflection-sensitive structure [9]. As the stress states of many members change from compressive to compressive-flexural by local stiffening, the structural sensitivity to deflection decreases; thus, the overall stability is improved. Such regulation applies to both regular and irregular lattice shells.

With regard to strength, the stiffening members of both regular and irregular lattice shells share part of the load; thus, they decrease shell member stresses. For this reason, local stiffening can improve the strength of single-layer lattice shells. Since the bearing capacity of local stiffening space trusses is higher than that of solid section members, stiffening method 3 has better stiffening improvement than method 1. The effect of stiffening method 2 is between that of method 1 and method 3.

5 Conclusions

Based on the analysis of locally stiffened irregular single-layer lattice shells and a comparative analysis of a locally stiffened regular and an irregular lattice shell, the following conclusions can be drawn:

1. The effect of local stiffening on single-layer lattice shells is not only related to the stiffening method but also to the regularity of the lattice shell. The three stiffening methods have different effects. The effect of the locally stiffened irregular single-layer lattice shell is also quite different from that of the locally stiffened regular lattice shell.

2. All three stiffening methods are effective in improving the overall stiffness, mechanical properties and overall stability of irregular single-layer lattice shells. The space truss stiffening method has the best comprehensive effect.
3. All three stiffening methods can improve the strength and overall stability of regular single-layer lattice shells, but none of the stiffening methods has an effect on the overall stiffness of regular lattice shells. Therefore, in case of a poor overall stiffness of a regular lattice shell, local stiffening is not a good choice to solve the stiffness problem.

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