

Effect of Radiative Heat Transfer on Cosmic-Ray Transport in a Rotating Cloudy Interstellar Medium

Pekene D.B.J, Ekpe O E

Abstract— Radiative heat transfer on cosmic rays (cr) in a rotating cloudy interstellar medium is modelled by imposing a time dependent perturbation on the cosmic transport in the interstellar medium containing randomly distributed giant diffused molecular clouds the temperature involved are assumed to be large so that radiative heat transfer is significant. This renders the problem very nonlinear even on the assumptions of a differential approximation for the radiative flux in a adiabatic concentration and zero activated energy. When the perturbation is small, the transient flow is tackled by the laplace transform technique, giving solution, for steady state spectrum of cosmic ray in the galaxy temperature up and down stream concentration, also velocity analytical and numerical solutions were obtained for temperature. Velocity of the down and upstream concentration in cases where radiative heat transfer through the rotating medium are taken into account. The incorporation of radiative heat transfer is of particular relevance to cosmic ray transport whose temperature is usually very high which radiate a lot of heat through the media which surround them.

Index Terms— Cosmic Ray, Radiative Heat, laplace transform, interstellar medium.

I. INTRODUCTION

Modeling of cosmic rays(cr)transport in the cloudy interstellar medium containing randomly distributed giant molecular clouds have been considered in the works of(1) cowsik and wilson(1973) dogiel et-al (1987)monfill et-al (1985),(2)osborne et-al (1987) and ptuskin et-al(1990).the latter two papers the theory of diffusion in the cloudy medium which takes into account the cloud's finite transparency for diffusion particles. Due to the high temperature involved its application in astrophysics, meteorology cannot be overemphasised .In all these investigations the problem of radiative heat transfer has been ignored unfortunately high temperature phenomena abound in cosmic ray transport which radiate a lot of heat through the media which surround them. A primary difficulty in thermal radiative heat transfer studies stems from the fact that the radiative flux is governed by an integral expression and one has to handle a nonlinear integro-differential equation. However, under fairly broad realistic assumptions the integral expression may be replaced by a differential approximation for radiation. Thus in one space coordinates z , the radiative flux Q satisfies the nonlinear differential equation cheng (1964) similarly arrienus will be linear zed.

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$$\frac{\partial^2 Q'}{\partial Z'^2} - 3\alpha^2 Q' - 16\sigma\alpha T^3 \frac{\partial T}{\partial Z'} = 0 \quad [1]a$$

$$K_T = \alpha e^{-\frac{E}{RT}} \quad [1]b$$

Where T is the temperature of the CR σ is the Stefan-Boltzmann constant and α is the absorption coefficient which will be assumed constant in the modelling is the chemical reaction. Taking into account the cloud's finite transparency for diffusing particles α and equation 1.1a and 1.1b is approximated by

$$\frac{\partial Q'}{\partial Z'} = 4\sigma\alpha (T^4 - T_\infty^4) \quad [2]a$$

$$K_r = \bar{K} + \frac{Ek}{RT^2} (T - \bar{T}) \quad [2]b$$

in which subscript ∞ will be used denote condition in the undisturbed cloudy interstellar medium. In the subsequent analysis, the mathematical formulation of the problem for finite transparent medium is presented in section 2 for the case in which the temperature of the medium suffers time t' perturbation when this perturbation is small, and the non-linear system tackled by asymptotic approximation so that the first order transient problem is solved by the laplace transform technique the results are obtained in section 3. In section 4 a discussion of the results of the previous section given.

II. NOMENCLATURE

q'_z = radiative heat flux

q = complex velocity

k_β = Boltzmann constant

σ = Stefan-Boltzmann constant.

H_0^2 = constant transverse magnetic field

K_{T_2} = thermo diffusion constant

k_r = constant associated with chemical reaction in the Arrhenius term

P_r = Prandtl number

R = Radiation parameter

D_f = Diffusion parameter

$i = \sqrt{-1}$ complex number

E = rotation parameter

M = magnetic parameter

G_C = free convection parameter due to temperature

D_m = mass diffusivity

S_i = Soret parameter.

S_c = Schmidt number

(u', v') = dimensional velocity component

(X', Y', Z') = dimensional Cartesian co-ordinate

k = thermal conductivity

g = gravitational acceleration

c_p = specific heat at constant pressure

T = dimensional temperature

C = dimensional concentration

T_∞ = reservoir temperature

c_∞ = reservoir concentration

T_ω = constant plate temperature

c_ω = constant plate concentration

T_m = mean temperature

G_c = free convection parameter due to concentration.

μ = is the permeability

α = absorption coefficient

β = coefficient of volume expansion due temperature.

σ_c = electrical conductivity

γ = coefficient of volume expansion due concentration.

ε = is small parameter

ν = kinematic viscosity.

Ω = angular velocity

ρ_∞ = reservoir density.

III. MATHEMATICAL FORMULATIONS.

We consider the cosmic Ray transport (CR) in the interstellar medium containing randomly distributed giant molecular clouds with characteristic velocity U_0 and rotates about the

X-axis with angular velocity Ω . This is maintained at temperature and

concentration. $T_k(1 + \varepsilon)f(t')$ And $C_k(1 + \varepsilon)f(t')$ In

which, $T_k \gg 1$, $f(t')$, $C_k \gg 1$

$f(t')$ is an arbitrary function of time and ε is a parameter is small. Following the arguments in Opara (1990) and employing equation 2,2b. The governing equations for a Ω medium are;

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = \nu \frac{\partial^2 u'}{\partial z'^2} - \sigma_c \frac{\mu^2 H_0^2}{\rho_\infty} u' + g\beta(T - T_k) + \gamma\delta(c - c_k) \quad [3]$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \nu \frac{\partial^2 v'}{\partial z'^2} - \sigma_c \frac{\mu^2 H_0^2}{\rho_\infty} v' \quad [4]$$

$$\rho_\infty \frac{\partial T}{\partial t'} = k_r \frac{\partial^2 T}{\partial z'^2} - 4\sigma\alpha(T^4 - T_\infty^4) \quad [5]$$

$$\rho_\infty \frac{\partial C}{\partial t'} = D_R \frac{\partial^2 C}{\partial z'^2} - K_r(C^4 - c_\infty^4) \quad [6]$$

Where $U = u_0, u' = 0, T = T_k[1 + \varepsilon f(t')]$, and

$C = C_k[1 + \varepsilon f(t')]$ on $Z' = 0, u' = 0$

$v' = 0, v' = 0, T = T_\infty, C = C_\infty$ on $Z' \rightarrow \infty$

[7]

where $(u', v', 0)$ is the velocity component, k_r is the thermal conductivity, g is the gravitational acceleration, σ_c is the electrical conductivity, μ , is the permeability, ν is the kinematic viscosity, α, β are the coefficients of volumetric expansion for concentration and temperature, C_p is the specific heat capacity at constant pressure, C is the up and down stream concentration and D_r is the diffusion coefficient. For simplicity we shall take $f(t') = \mathcal{H}(t')$ the Heaviside step functions.

Introducing the following non-dimensional quantity.

$$(\theta, \theta_k) = \frac{T, T_k}{T_\infty}, (C, C_k) = (C, C_k)/C_\infty, E = \Omega \nu_\infty / U_0^2,$$

$$t = t' U_0^2 / \nu, (u, v) = (u' v') / U_0$$

$$\mathcal{M}^2 = \sigma_c (\mu^2 \mathcal{H}^2 / \rho_\infty U_0^2) \nu_\infty, G_r = g(\beta \nu_\infty / U_0^3) T_\infty,$$

$$G_c = (\gamma \delta \nu_\infty / U_0^3) C_\infty.$$

[8]

$$\mathcal{R}_T = \alpha \nu_\infty / \rho_\infty U_0^2 \left[\frac{4\alpha T_\infty^3}{C_p} + C_\infty^3 \right].$$

Equation (3) to (7) can be written as

$$\frac{\partial q}{\partial t} + 2iEq = \frac{\partial^2 q}{\partial z^2} - \mathcal{M}^2 q + G_r(\theta - 1) + G_c(C - 1)$$

[9]

$$\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - \mathcal{R}_T(\theta^4 - 1)$$

[10]

$$\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} - k_r(C^4 - 1)$$

[11]

such that

$$Z = 0; q = 1, \theta = \theta_k[1 + \varepsilon \mathcal{H}(t)],$$

$$C = C_k[1 + \varepsilon \mathcal{H}(t)], Z \rightarrow \infty: q = 0; \theta = 1, C = 1 \quad [12]$$

and $q = u + iv, i = \sqrt{-1}$, E is the rotational parameter, \mathcal{M}^2 is the magnetic parameter

G_c and G_r are the free convection parameters for concentration and temperature respectively.

\mathcal{R}_T is the total radiation parameter. The mathematical statement of the problem embodies the solution of equation (9), (10) and (11) subject equation (12).

IV. METHOD OF SOLUTION

The problem posed in equation (9), (10) and (11) very non-linear and generally will involve a step by step numerical integration by (say) the explicit finite difference scheme. if however ε is small analytical solution could be possible by adopting regular perturbation. A great physical

insight could be shed on the whole of the CR transport by such a perturbation scheme and this is the problem we shall pursue in this investigation. We write:

$$\begin{aligned}\theta &= \theta^{(0)}(\mathcal{Z}) + \varepsilon \theta^{(1)}(\mathcal{Z}, t) + \dots \\ C &= C^{(0)}(\mathcal{Z}) + \varepsilon C^{(1)}(\mathcal{Z}, t) + \dots \\ q &= q^{(0)}(\mathcal{Z}) + \varepsilon q^{(1)}(\mathcal{Z}, t) + \dots\end{aligned}\quad [13]$$

Substituting equation (13) into equation (9)–(11) we have the sequence of approximations equations.

$$\begin{aligned}2iEq &= \frac{d^2 q^{(0)}}{d\mathcal{Z}^2} - \mathcal{M}^2 q^{(0)} + G_r(\theta^{(0)} - 1) + G_c(C^{(0)} - 1) \\ \frac{d^2 \theta^{(0)}}{d\mathcal{Z}^2} - 4\mathcal{R}_T(\theta^{(0)} - 1) &= 0 \\ \frac{d^2 C^{(0)}}{d\mathcal{Z}^2} - 4k_r(C^{(0)} - 1) &= 0\end{aligned}\quad [14]$$

$$\frac{d^2 \theta^{(0)}}{d\mathcal{Z}^2} - 4\mathcal{R}_T(\theta^{(0)} - 1) = 0 \quad [15]$$

$$\frac{d^2 C^{(0)}}{d\mathcal{Z}^2} - 4k_r(C^{(0)} - 1) = 0 \quad [16]$$

$$\begin{aligned}Z = 0; \theta^{(0)} &= \theta_k, C^{(0)} = C_k \\ q^{(0)} &= 1, Z \rightarrow \infty; \theta^{(0)} = 1; q^{(0)} = 0, C^{(0)} = 1\end{aligned}\quad [17]$$

$$\begin{aligned}\text{and} \\ \frac{\partial q^{(1)}}{\partial t} + 2iEq^{(1)} &= \frac{\partial^2 q^{(1)}}{\partial \mathcal{Z}^2} - \mathcal{M}^2 q^{(1)} + G_r\theta^{(1)} + G_cC^{(1)}\end{aligned}\quad [18]$$

$$\frac{\partial \theta^{(1)}}{\partial t} = \frac{\partial^2 \theta^{(1)}}{\partial \mathcal{Z}^2} - 4\mathcal{R}_T\theta^{(0)}\theta^{(1)} \quad [19]$$

$$\frac{\partial C^{(1)}}{\partial t} = \frac{\partial^2 C^{(1)}}{\partial \mathcal{Z}^2} - \mathcal{R}_TC^{(0)}C^{(1)} \quad [20]$$

$$\begin{aligned}Z = 0; \theta^{(1)} &= \theta_k; C^{(1)} = C_k; q^{(1)} = 0; Z \rightarrow \infty; \theta^{(1)} = 0; C^{(1)} = 0 \\ q^{(1)} &= 0; t = 0; \theta^{(1)} = 0 = C = q^{(1)}; Z > 0\end{aligned}\quad [21]$$

In equation (21) we assume

$t >$

0. Thus by asymptotic approximation.,

the problem now splits into a steady flow on which is superimposed a first order transient component, the solution for $\theta^{(0)}$ and $C^{(0)}$ are given.

$$Z = \sqrt{\left(\frac{5}{2\mathcal{R}_T}\right)} \int_{\theta^{(0)} C^{(0)}}^{\theta_k, C_k} \frac{ds}{(s^5 - 5s + 4)^{\frac{1}{2}}} \quad [22]a$$

$$Z = \sqrt{\left(\frac{1}{2\mathcal{R}_T}\right)} \int_{C^{(0)}}^{C_k} \frac{ds}{(s^5 - 5s + 1)^{\frac{1}{2}}} \quad [22]b$$

and

$$\begin{aligned}q^{(0)} &= \exp\left[-(\mathcal{M}^2 + 2iE)^{\frac{1}{2}}Z\right] - \sqrt{\left(\frac{5}{2\mathcal{R}_T}\right)} \int_{\theta^{(0)} C^{(0)}}^{\theta_k, C_k} dz \\ \frac{\sinh\left\{\left[\mathcal{M}^2 + 2iE\right]^{\frac{1}{2}}\left[Z(\theta^{(0)}, C^{(0)}) - Z(S)\right]\right\}(S-1)ds}{(S^5 - 5S + 4)^{\frac{1}{2}}}\end{aligned}\quad [23]$$

To solve equation (18), (19) and (20) and also consider the boundary conditions in equation (21) we take the Laplace transform with respect to time. Denoting the transformed variable by ξ and placing a tilde over the transformed function, the equation satisfied by $q^{(1)}, \theta^{(1)}$ and $C^{(1)}$ are;

$$\frac{d^2 \tilde{q}^{(1)}}{d\mathcal{Z}^2} - (\mathcal{M}^2 + 2iE)\tilde{q}^{(1)} = -[G_r\tilde{\theta}^{(1)} + G_c\tilde{C}^{(1)}] \quad [24]$$

$$\frac{d^2 \tilde{\theta}^{(1)}}{d\mathcal{Z}^2} - (4\mathcal{R}_T\tilde{\theta}^{(0)^2} + \xi)\tilde{\theta}^{(1)} = 0 \quad [25]$$

$$\frac{d^2 \tilde{C}^{(1)}}{d\mathcal{Z}^2} - (4\mathcal{R}_TC^{(0)^2} + \xi)\tilde{C}^{(1)} = 0 \quad [26]$$

with boundary conditions

$$\begin{aligned}Z = 0, \tilde{\theta}^{(1)} &= \tilde{\theta}_k, \tilde{C}^{(1)} = \tilde{C}_k, \text{ and} \\ q^{(1)} &= -[G_r\tilde{\theta}^{(1)} + G_c\tilde{C}^{(1)}] \\ Z \rightarrow \infty, \tilde{\theta}^{(1)} &= \tilde{C}^{(1)} = 0 = q^{(1)}\end{aligned}\quad [27]$$

The solution for $\tilde{\theta}^{(1)}$ and $\tilde{C}^{(1)}$ are

$$\tilde{\theta}^{(1)} = \frac{\exp[-(4\mathcal{R}_T + \xi)^{\frac{1}{2}}Z]}{\xi} \quad [28]$$

and

$$\tilde{C}^{(1)} = \frac{\exp[-(4\mathcal{R}_T + \xi)^{\frac{1}{2}}Z]}{\xi} \quad [29]$$

Hence, taking the inverse Laplace transform

$$\begin{aligned}\tilde{\theta}^{(1)} &= \frac{1}{2}\left\{e^{-2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} - (4\mathcal{R}_T t)^{\frac{1}{2}}\right] + e^{2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} + (4\mathcal{R}_T t)^{\frac{1}{2}}\right]\right\} \\ \tilde{C}^{(1)} &= \frac{1}{2}\left\{e^{-2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} - (4\mathcal{R}_T t)^{\frac{1}{2}}\right] + e^{2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} + (4\mathcal{R}_T t)^{\frac{1}{2}}\right]\right\}\end{aligned}\quad [30]$$

$$\begin{aligned}\tilde{\theta}^{(1)} &= \frac{1}{2}\left\{e^{-2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} - (4\mathcal{R}_T t)^{\frac{1}{2}}\right] + e^{2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} + (4\mathcal{R}_T t)^{\frac{1}{2}}\right]\right\} \\ \tilde{C}^{(1)} &= \frac{1}{2}\left\{e^{-2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} - (4\mathcal{R}_T t)^{\frac{1}{2}}\right] + e^{2\mathcal{R}_T^{\frac{1}{2}}Z} \operatorname{erfc}\left[\frac{Z}{2t^{\frac{1}{2}}} + (4\mathcal{R}_T t)^{\frac{1}{2}}\right]\right\}\end{aligned}\quad [31]$$

Next we consider when θ_k, C_k are arbitrary and approximate $\theta^{(0)}, C^{(0)}$ by

$$\theta^{(0)^{\frac{1}{2}}} = (\theta_k^3 - 1)e^{-2\beta Z} + 1 \quad [32]$$

$$C^{(0)^{\frac{1}{2}}} = (C_k^3 - 1)e^{-2\gamma Z} + 1 \quad [33]$$

By virtue of equation (32) and (33) the solution for $\tilde{\theta}^{(1)}$ and $\tilde{C}^{(1)}$ now reduces to

$$\frac{\tilde{\theta}^{(1)}}{\theta_k} = \frac{J}{\xi J_{(4\mathcal{R}_T + \xi)(i\eta)}} \frac{1}{(i\eta e^{-\beta Z})^{\frac{1}{2}}} = \frac{I}{\xi I_{(4\mathcal{R}_T + \xi)^{\frac{1}{2}}}} \frac{1}{(\eta e^{-\beta Z})^{\frac{1}{2}}} \quad [34]$$

$$\frac{C^1}{C_k} = \frac{I_{\frac{1}{2}(i\eta e^{-\gamma Z})}}{(4\mathcal{R}_T + \xi)^{\frac{1}{2}}} = \frac{I_{\frac{1}{2}\eta e^{-\gamma Z}}}{\xi I_{\frac{1}{2}(\eta)}} \quad [35]$$

$$\eta = \mathcal{R}_T(\theta_k^3 - 1)$$

and

$J_n(x)$ and $I_n(x)$ are the *b*Bessel and Modified Bessel

function of the first kind respectively. Equation (34) and (35)

have simple pole at $\xi = 0$ and another branch at

$\xi = 4\mathcal{R}_T$ and \mathcal{R}_T respectively. These Equations could then be inverted in the (39) and Bromwich contour with a suitable branch cut and we obtain

$$\frac{\theta^1}{\theta_K} = \frac{I_{4\mathcal{R}_T\frac{1}{2}\eta e^{-\beta Z}}}{I_{4\mathcal{R}_T\frac{1}{2}\eta}} + \frac{e^{-4R_T t}}{2\pi i} \left\{ \int_0^\infty \frac{e^{-xI_{ix\frac{1}{2}\eta e^{-\beta Z}}}}{x+4R_T I_{ix\frac{1}{2}(\eta)}} - \int_0^\infty \frac{e^{-xtI_{ix\frac{1}{2}\eta e^{-\beta Z}}}}{(x+4R_T)I_{-ix(\eta)}} \right\} dZ \quad [36]$$

$$\frac{C^{(1)}}{C_K} = \frac{I_{4R_T\frac{1}{2}\eta e^{-\gamma Z}}}{I(4R_T\frac{1}{2}(\eta))} + \frac{e^{-R_T t}}{2\pi i} \left\{ \int_0^\infty \left[\frac{e^{-xI_{ix\frac{1}{2}\eta e^{-\gamma Z}}}}{(x+4R_T)I_{ix\frac{1}{2}(\eta)}} - \int_0^\infty \frac{e^{-xtI_{ix\frac{1}{2}\eta e^{-\gamma Z}}}}{(x+4R_T)I_{-ix(\eta)}} \right] dZ \right\} \quad [37]$$

Because of the complexity of the above equations (36) and (37) we find it expedient to consider limiting values with

$$J_{n(x)} \approx \frac{1}{(2\pi n)^{\frac{1}{2}}} \left(\frac{ex}{2n} \right)^n, \quad n \rightarrow \infty \quad [38]$$

We can show that

$$\frac{\theta^{(1)}}{\theta_k} \approx \exp \left[-\frac{\beta Z (4R_T + \xi)^{\frac{1}{2}}}{\xi} \right]$$

$$\frac{C^{(1)}}{C_K} \approx \exp \left[-\frac{\gamma Z (R_T + \xi)^{\frac{1}{2}}}{\xi} \right] \quad [40]$$

We the condition that $\xi \rightarrow \infty$ is the form of equations (28) and (28) we can now write.

$$\frac{\theta^{(1)}}{\theta_k} \approx \frac{1}{2} \left\{ e^{-2\beta R_T^{\frac{1}{2}} Z} \operatorname{erfc} \left[\frac{\beta Z}{(2t)^{\frac{1}{2}}} - (4R_T t)^{\frac{1}{2}} \right] + e^{-2\beta R_T^{\frac{1}{2}} Z} \operatorname{erfc} \left[\frac{\beta Z}{(2t)^{\frac{1}{2}}} + (4R_T t)^{\frac{1}{2}} \right] \right\} \quad [41]$$

and

where

$$\frac{C^{(1)}}{C_k} \approx \frac{1}{2} \left\{ e^{-2\gamma R_T^{\frac{1}{2}} Z} \operatorname{erfc} \left[\frac{\gamma Z}{(2t)^{\frac{1}{2}}} - (R_T t)^{\frac{1}{2}} \right] + e^{2\gamma R_T^{\frac{1}{2}} Z} \operatorname{erfc} \left[\frac{\gamma Z}{(2t)^{\frac{1}{2}}} + (R_T t)^{\frac{1}{2}} \right] \right\} \quad [42]$$

When R_T is large and of order 0(1) then

$$\frac{\theta^{(1)}}{\theta_k} \sim \operatorname{erfc} \left(\frac{\beta Z}{2t^{\frac{1}{2}}} \right) \text{ And}$$

$$\frac{C^{(1)}}{C_k} \sim \operatorname{erfc} \left(\frac{\gamma Z}{2t^{\frac{1}{2}}} \right) \text{ as } t \rightarrow 0 \quad [43]$$

Also for $n \rightarrow 0$; following Watson (1981)

$$I_n(x) = I_0(x) - n\mathcal{K}_0(x)$$

where $\mathcal{K}_0(x)$ is now the modified Bessel function of the second of order zero. Therefore

$$\frac{\theta^{(1)}}{\theta_k} \approx \frac{1}{I_0(\eta)} \left\{ I_0(\eta e^{-\beta Z}) \cdot \frac{1}{\xi} + \left[\frac{I_0(\eta e^{-\beta Z})}{I_0(\eta)} \right] \mathcal{K}_0(\eta) - \mathcal{K}_0(\eta e^{-\beta Z}) \right\} (\xi + 4R_T)^{\frac{1}{2}} \quad [44]$$

This on inversion gives.

$$\frac{\theta^{(1)}}{\theta_K} \approx \frac{1}{I_0(\eta)} \left\{ I_0(\eta e^{-\beta Z}) + [\mathcal{K}_0(\eta)I_0(\eta e^{-\beta Z}) - \mathcal{K}_0(\eta e^{-\beta Z})] \cdot e^{-4R_T t} \left[\frac{1}{\pi} + (4R)^{\frac{1}{2}} e^{-4R_T t} \right] \right\} \quad [45]$$

$$\operatorname{erfc} (4R_T t)^{\frac{1}{2}} \} \text{ as } t \rightarrow \infty$$

$$\text{and } \frac{C^{(1)}}{C_K} \approx \frac{1}{I_0(\eta)} \left\{ I_0(\eta e^{-\gamma Z}) + [\mathcal{K}_0(\eta)I_0(\eta e^{-\gamma Z}) - \mathcal{K}_0(\eta e^{-\gamma Z})] \cdot e^{-R_T t} \left[\frac{1}{\pi t} + (R_T)^{\frac{1}{2}} e^{-R_T t} \right] \right\} \quad [46]$$

$$\operatorname{erfc} (4R_T t)^{\frac{1}{2}} \} \text{ as } t \rightarrow 0$$

The solution for $q^{(1)}$ in equation (24) can now be solved putting

$$S^2 = \xi + \mathcal{M}^2 + 2iE,$$

We get

$$q^{(1)} = \frac{1}{2} G \int_0^Z e^{-\frac{s(z-z)}{s}} \theta(z) dz \quad [47]$$

where

$$LT^{-1}\left[e^{\frac{-S(z-z)}{S}}\right] = e^{-(M^2+2iE)t} LT^{-1}\left[e^{\frac{-\xi^2(z-z)}{\xi}}\right]$$

$$= e^{-(M^2+2iE)t} \frac{1}{(\pi t)^{\frac{1}{2}}} e^{-(z-z)^2/4t} \quad [48]$$

and by convolution theorem we can deduce that.

$$q^{(1)} = \frac{1}{2\pi^{\frac{1}{2}}} \int_0^z dz \int_0^t \frac{1}{\tau^{\frac{1}{2}}} e^{-(M^2+2iE)\tau} e^{-(Z-z)^2/4\tau} \times$$

$$[\theta^{(1)}(t-\tau) d\tau + C^{(1)}(t-\tau) d\tau] \quad [49]$$

The solution is now complete.

Fig 1: plot of variation of concentration and Temperature with Z for R is 1.0

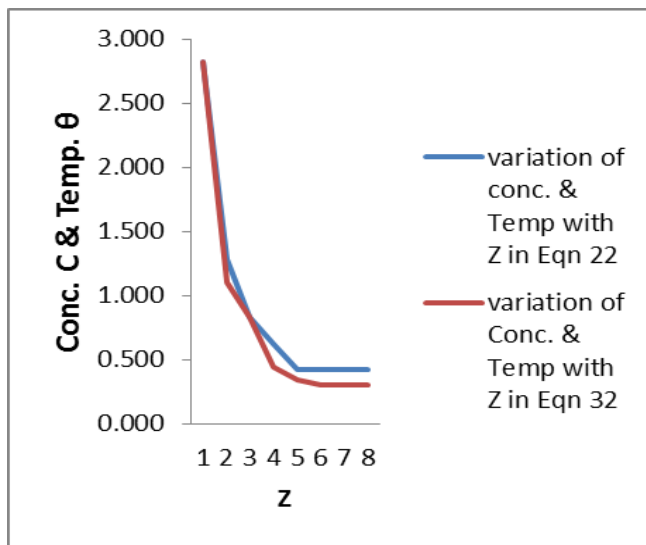


Fig 2: plot of variation of concentration and Temperature with Z for R is 0.5

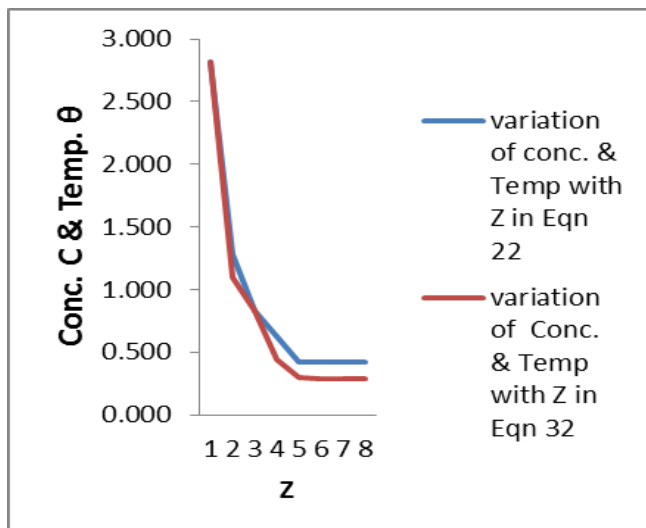


Fig 3: plot of variation of concentration and Temperature with Z for R is 0.25

V. RESULTS AND DISCUSSIONS.

The problem of the effect of radiative heat transfer on cosmic rays (CR) in rotating cloudy interstellar medium has been solved making fairly realistic assumptions, for a small

time-dependent perturbation of the temperature and adiabatic concentration, the non-linear problem is tackled by asymptotic approximation, giving solutions for steady state flow on which a first order transient component is superimposed.

Equations (22) and (23) give solutions for the steady-state components of the temperature $\theta^{(0)}$ adiabatic concentration $C^{(0)}$ and velocity $q^{(0)}$ fields. Equations (22) have been evaluated by numerical integration using three values of R_T (0.5, 1.0 and 1.5) to show the dependence of $\theta^{(0)}, C^{(0)}$ on Z. These are shown in figure 1, 2, and 3. It can be seen from these curves that their cube values say $\theta^{(0)3}$ and $C^{(0)3}$ approach unity asymptotically. Furthermore, when the parameters are $\gamma = 10$ and $\beta = 12$, the variation of the adiabatic concentration $C^{(0)3}$ and temperature $\theta^{(0)3}$ with Z for steady state flow are given by equations (32) and (33). These are almost identical to that deduced from equation (22a) and (22b). They are illustrated by the red line dashed curves in figures 1, 2 and 3.

Also for the case where $\theta^{(0)} = 1$ the transient component of the temperature exhibit a standing wave structure. The magnitude of the standing wave increases intensely when the solute concentration $C^{(0)} = 1$. This means that for $\theta^{(0)} = 1 = C^{(0)}$, the intense radiative heat is large, that is when R_T is large. There is contrary to a relatively low radiation when R_T is of order 0(1) in which the standing wave structure is no longer exhibited by both the temperature and concentration fields. These are shown in equations (41) - (43). However, equation (49) give a complete solution of the transient component of the velocity field.

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