A Strong Convex Atmost 2-Distance Dominating Sets In Graphs

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***Abstract*— Let G be a connected graph of order p ≥ 2. We study about the convex and dominating sets of G. We define strong convex sets and strong convex atmost 2-distance dominating sets and we prove a theorem to develop new convex sets with domination number. Finally we present in this paper, various bounds for it and characterize the graphs, with bounds attained.**

***Index Terms*— convex sets, domination number, distance, corona, strong convex set.**

I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite, undirected, connected graph without loop or multiple edges. For a graph G, Let $V(G)$ and $E(G)$ denote is vertex and edge sets respectively. Let p and q be the number of vertices and edges respectively. For $S \subset V(G)$), the set $I[S]$ is the union of all sets $I[u,v]$. For $u, v \in S$ we say that a non-empty subset S is convex if $I[S] = S$ [5]. In this paper, we study about strong convex sets [2] and in this section, some basic definitions and important results on convex sets and domination number [4], [5] are presented.

A set S of vertices is called geodesically convex, g- convex, or simply convex, if $I[S] = S$, that is every pair $u, v \in S$ the interval $I[u, v] \subset S$. In any graph the empty set, the whole vertex set, every singleton, and every two-path are convex.

A set $S \subset V(G)$ is a *dominating set* for G if every vertex of G either belongs to S or is adjacent to a vertex of S.

For a set S of vertices, let the closed interval $I[S]$ of S be the union of the closed intervals $I[u, v]$ over all the pairs of vertices u and v in S. A set of vertices S is called *geodetic set* if $I[S] = V(G)$ and the minimum cardinality of the geodetic set is the *geodetic number* and is denoted by $g(G)$. A geodetic set of cardinality $g(G)$ is called a *minimum geodetic set* (or) g- *set* of G.

For a graph given in Figure 1, the minimum geodetic sets are $\{a, f, h\}$. The cardinality of the minimum geodetic set of G is 3.

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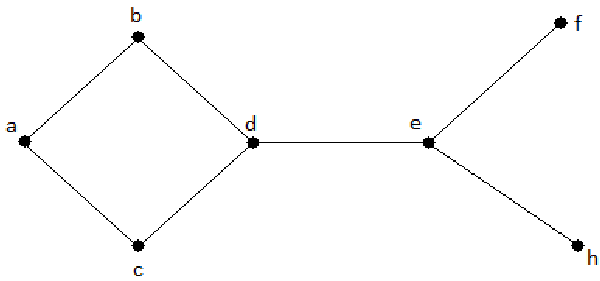

Figure 1: A graph *G* for geodetic set

II. STRONG CONVEX SETS

In this section we define strong convex sets and study their properties. Next we define the concepts of domination in strong convex sets.

Definition 2.1

A set $D \subseteq V$ is a strong convex set if for any two vertices u, v in D, $d_{<S>}(u, v) = d_{<G>}(u, v)$.

Definition 2.2

A strong convex set $S \subset V(G)$ is a 2-*distance dominating set* for G if every vertex of G either belongs to S or is adjacent to *atmost* 2-*distance* to a vertex of S.

Definition 2.3

A strong convex set $D \subseteq V$ is a strong convex atmost 2–distance dominating set of G if every vertex in V-D is strongly dominated by atmost 2–distance in D. The minimum cardinality of D is called the strong convex atmost 2- distance domina-
tion number of G and it is denoted by $\gamma_{scd\leq 2}(G)$.

Example 2.4

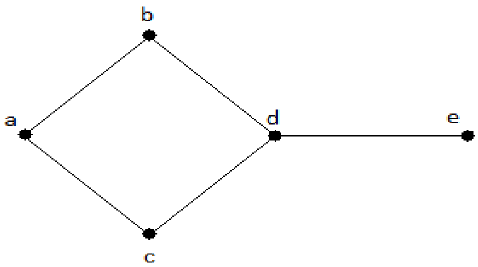

Figure 2 A graph with strong convex atmost 2-distance dominating set

Here {d, e}is the strong convex set and $d(d, e)$=1 , d(e, b) = d(e, c) = 2, $d(d, a)$ = 2. So the vertices are dominated by at most 2-distance. Hence {d, e} is the strong convex atmost 2-distance dominating set.

Theorem 2.5

For $n \geq 3$, a path P_n, then the strong convex atmost 2-distance dominating set

$$\gamma_{scd\leq 2}(P_n) = \begin{cases} 2 & \text{if } n \leq 6 \\ n-4 & \text{if } n > 6 \end{cases}$$

Proof:

Case 1: For $n \leq 6$ we have deal two cases

Sub case 1.1: $n \leq 5$

If n = 3, {v_1, v_2, v_3 } be the vertices and {e_1, e_2} be the edges. Here {v_1, v_2}, {v_2, v_3 } and {v_1, v_2, v_3 } these are the strong convex sets of P_3. But {v_i, v_{i+1}}, i =1, 2 are the minimum strong convex sets. Notice that $d(v_i, v_{i+1})$ = 1, i = 1, 2. Hence {v_i, v_{i+1}} are dominate other vertices by 1-distance. Therefore the minimum cardinality of the strong convex atmost 2-distance dominating set is 2. It is denoted by $\gamma_{scd\leq 2}(G)$ = 2.

If n = 4, {v_1, v_2, v_3, v_4 } be the vertices and {e_1, e_2, e_3} be the edges. Here {v_1, v_2} and {v_3, v_4} are the minimum strong convex sets and by definition {v_i, v_{i+1}} i= 1, 2, 3 these set of vertices are dominated other vertices in P_n, where n = 4 by atmost 2-distance. Hence {v_i, v_{i+1}} is the minimum strong convex atmost 2-distance dominating set. Therefore $\gamma_{scd\leq 2}(G)$ = 2.

Sub case 1.2:

If n = 5, we know that {v_i, v_{i+1}} i = 1, 2 ,3, 4 is the minimum strong convex sets. If we take i=1, {v_1, v_2} is the strong convex set and it dominates all other vertices in P_5 atmost 3-distance so we cannot choose i = 1 and obviously i = 4 cannot dominates all other vertices atmost 2-distance . Notice that if k = 2 and 3 then {v_k, v_{k+1} } dominates all the vertices in this path atmost 2-distance. Hence $\gamma_{scd\leq 2}(G)$ = 2.

If n = m+1, where m = 5 we know that {v_i, v_{i+1}} i = 1, 2, 3, 4, 5 is the minimum strong convex sets. For i =1, 2, 4 and 5 it dominates all other vertices in P_6 atmost 3-distance so we cannot choose i = 1, 2, 4 and 5. Clearly i = 3 dominates all other vertices in P_6 atmost 2-distance. Hence $\gamma_{scd\leq 2}(G)$ = 2.

Case 2: For $n > 6$

For any path {v_i, v_{i+1}} is the minimum strong convex sets. If n = 7, {v_i, v_{i+1}} it does not dominates other vertices by atmost 2-distance. So we choose next minimum strong convex sets, therefore { v_i, v_{i+1}, v_{i+2} } is the minimum strong convex sets for path P_7. Clearly i = 3 dominates all other vertices in P_7 atmost 2-distance. Hence cardinality of {v_i, v_{i+1}, v_{i+2}}, i = 3 is 3. If n = 8, {$v_i, v_{i+1}, v_{i+2}, v_{i+3}$} is the minimum strong convex sets dominates remaining vertices by at most 2-distance when i = 3 and its cardinality equal to 4. Proceeding like this we get the minimum cardinality of strong convex atmost 2- distance dominating sets is l+k, where l=4 and k=1,2,3,…. In general$\gamma_{scd\leq 2}(G)$ = $n-4$. □

The following theorem can be proved as above for cycles. We observe that for $n \geq 3$, a cycle C_n, then the $\gamma_{scd\leq 2}(C_n)$ = $\begin{cases} 2 & \text{if n} \leq 6 \\ \text{n}-4 & \text{if n} > 6 \end{cases}$

Theorem 2.7

For a star graph, complete graph, complete bipartite graph G, then the strong convex atmost 2-distance dominating set $\gamma_{scd\leq 2}(G)$ = 2

Proof:

Case: (i) For a star graph

Let v_i be the root vertex and $v_1, v_2, \ldots\ldots, v_{i-1}, v_{i+1}, \ldots, v_n$ be the pendent vertices of $K_{1,n}$. Take v_i and any one of the pendent vertices v_k where k = 1, …,i-1, i+1,…., n. we know that both are adjacent to each other, so $d(v_i, v_k)$ = 1. The set of vertices v_i and v_k are taken us D, D={ v_i, v_k} by definition D is the strong convex set. Notice that $d(v_k, v_{k+1})$ = 2. Since $K_{1,n}$ is star graph, v_i dominates all the vertices by 1-distance and v_k dominates every vertices by 2-distance, therefore by definition D={v_i, v_k}is the minimum strong convex atmost 2-distance dominating set and its cardinality of D = 2. Hence the $\gamma_{scd\leq 2}(G)$ = 2.

Case: (ii) For a complete graph

Let $v_1, v_2, \ldots\ldots, v_{i-1}, v_i, v_{i+1}, \ldots, v_n$ be the vertices of K_n. since K_n is complete, the distance between every vertex in K_n is 1, therefore {v_i, v_{i+1}} is the strong convex set and it is dominating all the vertices by 1-distance and 2-distance. Hence { v_i, v_{i+1} } is the minimum strong convex atmost 2-distance dominating set and its cardinality is 2. Therefore $\gamma_{scd\leq 2}$(G) = 2.

Case: (iii) For a complete bipartite graph

Let $K_{m,n}$ be the complete bipartite graph. It can be partitioned into two sets W_1 and W_2, W_1={$u_1, \ldots, u_m$}and W_2 = { $v_1, \ldots, v_n$ }. Choose any one vertex from W_1 as u_i and also from W_2 as v_j. Since $K_{m,n}$ be the complete bipartite graph, u_i and v_j both of them are adjacent to each other. Therefore we can choose u_i and v_j is the set of strong convex set by definition. We know that d(u_i, v_j)=1, d(u_i, u_{i+1}) = 2 and d(v_i, v_{i+1}) = 2. Hence the set {u_i, v_j} is the minimum strong convex atmost 2-distance dominating set and its cardinality is 2. Hence $\gamma_{scd\leq 2}(G)$ = 2. □

Theorem 2.8

Let G be any connected graph then $\gamma_{scd\,\leq\,k}(G) < diam(G)$.

Proof:

Suppose that $\gamma_{scd\leq k}(G) > diam(G)$. Let S be a strong convex set. We know that $d_{<S>}(u, v) = d_{<G>}(u, v)$ for every $u, v \in G$ and |S| ≤ $C(G)$ +1. Hence S dominates all other vertices at most 2-distance. It contradicts the fact that $\gamma_{scd\,\leq\,k}(G) > diam(G)$. It completes the proof. □

III. CORONA GRAPH

In this section we discuss the concept on the corona graph.

Definition 3.1

The corona $G_1 \circ G_2$ of two graphs was defined as the graph G obtained by taking one copy of G_1 and P_1 copies of G_2, and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Theorem 3.1

If $G = K_n \circ K_1$ then $\gamma_{scd\leq 2}(G) = 2$

Proof

First we construct this corona $K_n \circ K_1$ as follows. Let K_n be a complete graph, each vertex in K_n dominates atmost 1 distance to all other vertices. Take one copy of K_n and p_1 copies of K_1 then join each point of K_n to K_1. Now choose any two vertices from K_n as v_i, and v_j. Since K_n is complete the set of vertices $\{v_i, v_j\}$ of this corona $K_n \circ K_1$ is the strong convex set dominates all other vertices by atmost 2-distance. We choose any one of the vertices from K_n as u_l and any one of K_1 as u_m and this set $\{u_l, u_m\}$ is also the strong convex set and it dominate all other vertices by atmost 2-distance. Hence we can take either $\{v_i, v_j\}$ or $\{u_l, u_m\}$ as a minimum strong convex set. Therefore $\gamma_{scd\leq 2}(K_n \circ K_1) = 2$. □

The following theorem can proved as above.

Theorem 3.2

If $G = C_n \circ K_1$ then $\gamma_{scd\leq 2}(G) = n - 2$.

Proof :

It is similar to the above proof. □

IV. Conclusion

In this paper we have studied the strong convex set of a finite, undirected, connected graph without loops or multiple edges, whose dominating sets are known. We have investigated strong convex sets, strong convex atmost 2 - distance dominating sets and various bounds. Some results are useful to develop new convex dominating sets. Then we have presented various theorems to find strong convex atmost 2 - distance dominating sets and based on diameter, as well as degree.

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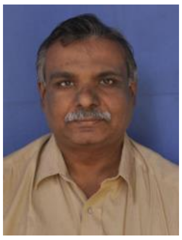

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