A generalized approach of form error evaluation for sculptured surface within the framework of the new generation GPS standards system

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Abstract— **The form error is an important index used to evaluate the form precision of parts; the accuracy of its evaluation method has a significant influence on the quality and use performance of a mechanical product. With the development of modern measuring techniques, especially the coordinate measuring machines (CMM) and other precision measuring instruments have already been applied to practical production extensively, it has important practical meaning to study sculptured surface form error evaluation method based on the coordinate measuring data. In view of this current situation, based on the related principle of the form error evaluation within the framework of the new generation GPS standards system, a generalized mathematical model of form errors evaluation based on the least square method is proposed in this paper, and then the way of solving the evaluation model by using multiple nonlinear optimization function in MATLAB optimization toolbox is also studied. Finally, a numerical example based on the actual measurement data obtained by CMM is given to verify the evaluation model.**

Index Terms— **CMM, Evaluation, Form error, Least square method, MATLAB, New generation GPS standards system**

I. INTRODUCTION

The form error is the variation of actual shape of a geometric feature relative to its nominal shape of the part. The reason for causing the form errors is very complex, including the impacts of geometric errors of machining tools, cutting tools, fixtures, etc; and that of installation and adjustment errors of part; the elastic deformation of workpieces and machining equipments caused by cutting force and clamping force in the process of machining; and temperature change, vibration and cutting-tool wear during cutting etc [1]. The form errors have a great influence on the performance of mechanical parts. For example, the form error of the cylindrical surface will cause uneven gap distribution, accelerate the local wear and reduce the life of the part in clearance fit. The form error of the plane will reduce the actual supporting area of the contact parts, increase the pressure per unit area, and increase the deformation of the contact surface. Due to the massive impact of the form errors on the working accuracy and the life of products, it has great significance to correctly inspect and evaluate the form errors to ensure the quality of parts and mechanical products.

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Researchers at home and abroad have made great achievements in the errors evaluation of some geometric tolerance items such as straightness, flatness, roundness, sphericity, etc. And the corresponding evaluation techniques are becoming more and more mature [2-3]. The evaluation methods of straightness errors include two points connection method, the minimum area method and the least square method, etc. The diagonal plane method, three far point plane method, least square method and minimum area method are used to evaluate flatness error. The evaluation methods of roundness errors evaluation have minimum circumscribed circle and maximum inscribed circle method, minimum zone method and least square method, etc. Along with the increasingly widespread application of complex curved surface parts, the research of form error evaluation for these parts is becoming more and more important. In this paper, a generalized approach for evaluating the form error of complex sculptured surface is studied based on the least square method.

II. ATHEMATICAL MODEL OF FORM ERRORS EVALUATION

In this section, a generalized mathematical model of form error evaluation for sculptured surface based on the least square method is studied.

The equation of any ideal surface S in space can be expressed as:

$$f(x, y, z) = 0 \qquad (1)$$

As shown in Fig. 1, the feature of any point (x, y, z) on surface *S* can be described by three vectors, namely radius vector **r**, unit normal vector **n** and spherical tangent vector **t** [4]:

$$\begin{cases} \mathbf{r} = x \cdot \mathbf{i} + y \cdot \mathbf{j} + z \cdot \mathbf{k} \\ \mathbf{n} = \cos\alpha \cdot \mathbf{i} + \cos\beta \cdot \mathbf{j} + \cos\gamma \cdot \mathbf{k} \\ \mathbf{t} = \mathbf{r} \times \mathbf{n} = (y\cos\gamma - z\cos\beta)\mathbf{i} + (z\cos\alpha - x\cos\gamma)\mathbf{j} + (x\cos\beta - y\cos\alpha)\mathbf{k} \end{cases} \qquad (2)$$

Where $\cos\alpha = f_x \big/ \sqrt{f_x^2 + f_y^2 + f_z^2}$, $\cos\beta = f_y \big/ \sqrt{f_x^2 + f_y^2 + f_z^2}$, $\cos\gamma = f_z \big/ \sqrt{f_x^2 + f_y^2 + f_z^2}$, $f_x = \partial f / \partial x$, $f_y = \partial f / \partial y$, $f_z = \partial f / \partial z$. Vector **t** is located in the tangent plane of surface S and tangential to the intersecting line of the surface S and the sphere of radius $\|\mathbf{r}\|$ centered at the origin, $\|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$. Vectors **r**, **n**, **t** can also be regarded as the coordinate axes of surface *S* at this point.

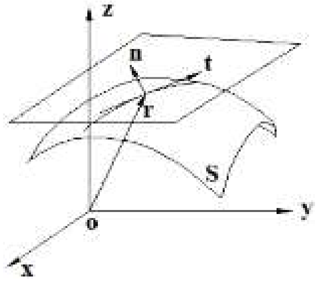

Fig. 1 Unit normal vector and spherical tangent vector of arbitrary space surface

Suppose the coordinate value of any measuring point on the surface is $\mathbf{r}_i = [x_i, y_i, z_i]^T$ (i = 1, 2, ⋯, p), p is the number of the measured points. Under the assumption of small deviation and small error, the actual measuring points are all located near the ideal surface *S*. The distance $d(r_i)$ from the actual measuring point $\mathbf{r}_i$ to the surface *S* is formulized as

$$d(r_i) = d(x_i, y_i, z_i) = f(x_i, y_i, z_i) \Big/ \sqrt{f_x^2 + f_y^2 + f_z^2} \qquad (3)$$

Due to the deviation between the measuring datum and evaluation datum, the coordinate values of the measured points should be adjusted slightly in the evaluation. In general, the tiny adjustment consists of three tiny translations Δ_x 、Δ_y 、Δ_z and three tiny rotations δ_x 、δ_y 、δ_z .Assume that the tiny adjustment variable is $\boldsymbol{\tau} = [\Delta_x, \Delta_y, \Delta_z, \delta_x, \delta_y, \delta_z]$, the coordinates $\mathbf{r}_i^* = [x_i^*, y_i^*, z_i^*]^T$ of the measured points after adjustment can be obtained by differential transformation formula:

$$\begin{bmatrix} x_i^* \\ y_i^* \\ z_i^* \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\delta_z & \delta_y & \Delta_x \\ \delta_z & 1 & -\delta_x & \Delta_y \\ -\delta_y & \delta_x & 1 & \Delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \qquad (4)$$

The distance from the actual measured point $\mathbf{r}_i^*$ to the ideal surface *S* is related with vector $\boldsymbol{\tau}$ and it is linear function of $\boldsymbol{\tau}$, which is represented as $d(r_i^*) = d(r_i^*; \boldsymbol{\tau})$, we have from Eq.(3):

$$d(r_i^*) = f(x_i^*, y_i^*, z_i^*) \Big/ \sqrt{f_x^2 + f_y^2 + f_z^2} \qquad (5)$$

Because surface $f(x,y,z)$ is fully smooth and $\boldsymbol{\tau} = [\Delta_x, \Delta_y, \Delta_z, \delta_x, \delta_y, \delta_z]$ is a tiny variable, we can get:

$$\begin{aligned} f(x_i^*, y_i^*, z_i^*) &= f(x_i, y_i, z_i) + f_x(\Delta_x - y_i\delta_z + z_i\delta_y) \\ &\quad + f_y(\Delta_y + x_i\delta_z - z_i\delta_x) + f_z(\Delta_z - x_i\delta_y + y_i\delta_x) \end{aligned} \qquad (6)$$

In order to evaluate the surface form error by applying the least square method, we define the mean square deviation function of the distance from the actual measured point to the evaluation datum:

$$v^2(\boldsymbol{\tau}) = \frac{1}{p}\sum_{i=1}^{p} d^2(r_i^*; \boldsymbol{\tau}) \qquad (7)$$

According to the definition of least square method, our objective is to minimize the mean square deviation function, that is:

$$\min\left[v^2(\boldsymbol{\tau})\right] = \min\left[\frac{1}{p}\sum_{i=1}^{p} d^2(r_i^*; \boldsymbol{\tau})\right] \qquad (8)$$

With minimizing Eq. (8), we can obtain:

$$\boldsymbol{\tau}^* = [\Delta_x^*, \Delta_y^*, \Delta_z^*, \delta_x^*, \delta_y^*, \delta_z^*] \qquad (9)$$

Let

$$\begin{aligned} v(\boldsymbol{\tau}^*)^+ &= \max\left\{d(r_i^*; \boldsymbol{\tau}^*) \mid i = 1, 2, \cdots, p\right\} \\ v(\boldsymbol{\tau}^*)^- &= \min\left\{d(r_i^*; \boldsymbol{\tau}^*) \mid i = 1, 2, \cdots, p\right\} \end{aligned} \qquad (10)$$

The result of the form error evaluation for sculptured surface will be:

$$e(\boldsymbol{\tau}^*) = v(\boldsymbol{\tau}^*)^+ - v(\boldsymbol{\tau}^*)^- \qquad (11)$$

III. Implementation Error Evaluation Based on MATLAB

According to above analysis, the optimization model of form error evaluation for sculptured surface is formulized as Eq. (8). We use the MATLAB optimization toolbox to solve the model. The MATLAB optimization toolbox provides a complete solution to various optimization problems, including linear programming, quadratic programming, nonlinear programming, least squares problems, nonlinear equation solving, multi-objective decision-making and other optimization problems. The main functions for solving unconstrained nonlinear programming problems include ***fminbnd***, ***fminunc*** and ***fminsearch***. The main functions for solving constrained nonlinear programming problems are ***fgoalattain*** and ***fminimax.*** This error evaluation problem belongs to the unconstrained nonlinear programming problem, the result can be obtained by calling the appropriate function, and we will call function ***fminunc*** to solve the model.

[***x***, ***fval***, ***exitflag***, ***output***] = **fminunc**(***fun***, ***x***$_0$, ***options***…)

where ***x*** is the optimal solution of returning target function; ***fval*** is the optimal value of returning target function; ***exitflag*** is ending flag of return algorithm; ***output*** is a data structure for optimizing algorithmic information; ***fun*** is the function name of calling the target function; x_0 is the initial point; ***options*** are used to set optimize options parameters.

The detailed evaluation steps are as follows:

Step 1: Assume that the equation of any ideal surface S in space is expressed as $f(x, y, z) = 0$, and import measuring coordinate data;

Step 2: The sum of the absolute values of the distances from the measured points to the ideal surface is taken as the objective function; and call the MATLAB unconstrained optimization function ***fminunc*** to determine the parameter values of the surface equation;

Step 3: Calculate $d(r_i)$ corresponding to each measurement point by using Eq. (3).

Step 4: Let $v^2(\boldsymbol{\tau}) = \frac{1}{p}\sum_{i=1}^{p} d^2(r_i^*;\boldsymbol{\tau})$ be the objective function, $\boldsymbol{\tau} = [\Delta_x, \Delta_y, \Delta_z, \delta_x, \delta_y, \delta_z]$ design variable, call unconstrained optimization function ***fminunc***, run optimization design program to obtain least squares solution $\boldsymbol{\tau}^* = [\Delta_x^*, \Delta_y^*, \Delta_z^*, \delta_x^*, \delta_y^*, \delta_z^*]$;

Step 5: Calculate $d(r_i^*)$ for each actual measuring point by using Eq. (5) and calculate $v(\boldsymbol{\tau}^*)^+ = \max\{d(r_i^*;\boldsymbol{\tau}^*) \mid i = 1, 2, \cdots, p\}$ and $v(\boldsymbol{\tau}^*)^- = \min\{d(r_i^*;\boldsymbol{\tau}^*) \mid i = 1, 2, \cdots, p\}$. Then the difference between $v(\boldsymbol{\tau}^*)^+$ and $v(\boldsymbol{\tau}^*)^-$ is the form error of the desired surface based on proposed method.

IV. CASE STUDY

In this section, the form error of an ellipsoid is evaluated based on the proposed method. Suppose the Cartesian coordinate equation of an ellipsoid is: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, the coordinate values of the ellipsoid surface measuring points are shown in Table 1

Table 1 Coordinates of measuring points

No	x	y	z	No	x	y	z
1	0.000	40.000	0.001	51	-50.000	0.002	0.000
2	15.461	38.044	0.000	52	-40.457	0.001	-17.634
3	12.502	38.045	5.449	53	-15.453	0.002	-28.533
4	4.774	38.044	8.816	54	15.455	0.000	-28.532
5	-4.776	38.042	8.817	55	40.458	0.004	-17.634
6	-12.502	38.043	5.447	56	50.003	0.002	0.000
7	-15.453	38.043	0.000	57	47.556	-12.361	0.000
8	-12.503	38.043	-5.446	58	38.476	-12.365	16.771
9	-4.776	38.042	-8.816	59	14.693	-12.363	27.136
10	4.774	38.043	-8.817	60	-14.698	-12.362	27.135
11	12.501	38.043	-5.447	61	-38.476	-12.364	16.770
12	15.452	38.042	0.000	62	-47.554	-12.366	0.001
13	29.387	32.362	0.000	63	-38.478	-12.361	-16.773
14	23.776	32.363	10.365	64	-14.696	-12.363	-27.134
15	9.083	32.362	16.770	65	14.695	-12.363	-27.135
16	-9.084	32.362	16.771	66	38.475	-12.367	-16.772
17	-23.777	32.361	10.365	67	47.554	-12.364	0.000
18	-29.384	32.360	0.001	68	40.455	-23.512	0.000
19	-23.775	32.362	-10.365	69	32.721	-23.511	14.265
20	-9.081	32.362	-16.770	70	12.500	-23.512	23.084
21	9.083	32.363	-16.770	71	-12.503	-23.511	23.082
22	23.774	32.366	-10.365	72	-32.726	-23.512	14.265
23	29.383	32.361	0.000	73	-40.453	-23.511	0.001
24	40.452	23.510	0.000	74	-32.723	-23.512	-14.265
25	32.723	23.512	14.265	75	-12.500	-23.513	-23.083
26	12.501	23.512	23.083	76	12.500	-23.516	-23.083
27	-12.503	23.512	23.083	77	32.724	-23.515	-14.265
28	-32.722	23.511	14.266	78	40.453	-23.512	0.001
29	-40.456	23.512	0.000	79	29.385	-32.362	0.000
30	-32.723	23.514	-14.266	80	23.772	-32.362	10.365
31	-12.505	23.511	-23.083	81	9.083	-32.361	16.775
32	12.506	23.513	-23.082	82	-9.081	-32.360	16.772
33	32.723	23.512	-14.265	83	-23.776	-32.362	10.366
34	40.458	23.512	0.001	84	-29.386	-32.365	0.000
35	47.557	12.362	0.000	85	-23.776	-32.362	-10.364
36	38.476	12.362	16.770	86	-9.081	-32.362	-16.772
37	14.697	12.362	27.135	87	9.083	-32.362	-16.770
38	-14.696	12.362	27.136	88	23.775	-32.363	-10.368
39	-38.477	12.360	16.770	89	29.389	-32.364	0.002
40	-47.554	12.362	0.001	90	15.457	-38.042	0.001
41	-38.471	12.361	-16.770	91	12.500	-38.043	5.447
42	-14.695	12.362	-27.136	92	4.775	-38.042	8.816
43	14.695	12.364	-27.135	93	-4.776	-38.042	8.817
44	38.475	12.366	-16.770	94	-12.501	-38.042	5.449
45	47.554	12.365	0.000	95	-15.452	-38.043	0.001
46	50.004	0.000	0.000	96	-12.502	-38.044	-5.446
47	40.453	0.000	17.634	97	-4.776	-38.042	-8.816
48	15.451	0.001	28.532	98	4.779	-38.043	-8.814
49	-15.452	0.000	28.532	99	12.502	-38.045	-5.449
50	-40.455	0.001	17.633	100	15.456	-38.043	0.000

Firstly, take $\sum_{i=1}^{p}\left(\left|\frac{x_i^2}{a^2}+\frac{y_i^2}{b^2}+\frac{z_i^2}{c^2}-1\right|\right)$ as objective function and (a, b, c) as design variable; import the coordinates of measuring points and call unconstrained optimization function ***fminunc*** to calculate the optimum design variable (a^*, b^*, c^*), according to the measured data above the calculated a^*, b^*, c^* are 50, 40, 30, respectively. Calculate $d(r_i)$ corresponding to each measuring point by using Eq. (3). Then, let $v^2(\boldsymbol{\tau})=\frac{1}{p}\sum_{i=1}^{p}d^2(r_i^*;\boldsymbol{\tau})$ be the objective function, $\boldsymbol{\tau}=[\Delta_x, \Delta_y, \Delta_z, \delta_x, \delta_y, \delta_z]$ design variable, call unconstrained optimization function ***fminunc***, run optimization design program to obtain $\boldsymbol{\tau}^*=[-0.0011, 0.0002, 0.0003, -0.0000, -0.0000, -0.0000]$. Finally, calculate $v(\boldsymbol{\tau}^*)^+=\max\{d(r_i^*;\boldsymbol{\tau}^*)\,|\,i=1,2,\cdots,p\}$ and $v(\boldsymbol{\tau}^*)^-=\min\{d(r_i^*;\boldsymbol{\tau}^*)\,|\,i=1,2,\cdots,p\}$ to obtain the difference between $v(\boldsymbol{\tau}^*)^+$ and $v(\boldsymbol{\tau}^*)^-$ is 0.0101, that is the final error evaluation result. Three-dimensional figure of fitted ellipsoid and measured data points is shown as Fig. 2.

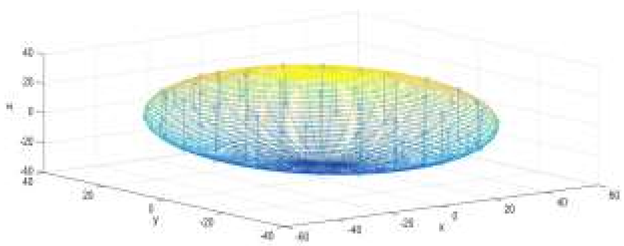

Fig. 2 Three-dimensional figure of fitted ellipsoid and measured data points

V. SUMMARY

Based on the least square method, a generalized mathematical model for evaluating the form error of complex surfaces is established. And the nonlinear optimization function of MATLAB optimization toolbox is used to solve the evaluation model. The method proposed in this paper can be applied to the shape measurement and data processing of three coordinate measuring machines, it is easy to execute by computer, and opens a new way for evaluating the form error of complex surface. It will provide a more practical way for manufacturing and inspection engineers to deal with the form error evaluation for sculptured surface in fields such as mechatronic products, robots and intelligent equipments, automobile manufacturing, etc.

ACKNOWLEDGMENTS

The author would like to acknowledge the financial support of the National Natural Science Foundation of the People's Republic of China (No. 51575235).

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