Response of Thin Walled Double Spine Mono-Symmetric Box Girder Structures to Torsional-Distortional Loads

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Abstract— **in this work, the response of thin-walled double spine mono-symmetric box girder frame to torsional and distortional loads was studied using Vlasov's theory of thin walled structures. The potential energy of the system at equilibrium was used to obtain the governing differential equations of equilibrium for torsional-distortional analysis of the box structure, by minimizing same using the principle of variational calculus and Euler-lagrange equation. The torsional-distortional strain modes interactions were considered and used to obtain the coefficients of the governing differential equations of equilibrium which were solved using method of Fourier sine series to obtain the torsional and distortional displacements of the box girder section. The maximum torsional and distortional displacements of the 20m simple supported box girder were found to be 2.97mm and 8.77mm respectively**

Index Terms—**box girder, distortion, double spine, mono-symmetric**

I. INTRODUCTION

A thin-walled structure is one which is made from thin plates joined along their edges. The plate thickness however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. Thin walled structures have gained special importance and notably increased application in recent years. The wide use of these thin walled structures is due to their great carrying capability and reliability and to the economic advantage they have over solid (column and beam) structures. Initially the design of box girder bridges is related to the design of plate girder bridges. However, such design knowledge does not contain important primary conditions of cross sectional deformations such as warping and distortion.

The thin walled beam theory by Vlasov's [1] marked the birth of all research efforts published to date on the analysis and design of straight and curved box-girder bridges. Many technical papers, reports and books have been published in the literature concerning various applications of, and even modifications to, the two theories.

There are several methods available for the analysis of box girder bridges.

Fortunately structural designer are careful enough not to ignore the effects of torsion on a structural member. Unfortunately however, the effects of warping and distortion on a structural component are poorly evaluated or ignored in the analysis, simply because of the rigorous mathematics involved in their evaluation. There is therefore the need to develop a simple analytical model to enable designers put into consideration the primary condition of cross sectional deformations in the analysis and design of box girder structures. Thus a better and more elaborate assessment of all the effects of loads on a thin walled box girder bridge structure can be achieved by a consideration of phenomena of warping torsion and distortion.

Every box girder bridge structure needs to be designed both longitudinally and transversely. Transverse bending moment, shear and warping torsion (distortion) are vital components

of the analysis and design. Computation of the above is not easily done. Therefore no amount of work will be too much on the transverse design of box girder structures.

II. REVIEW OF PAST WORK

Before the advent of Vlasov's 'theory of thin-walled beams the conventional method of predicting warping and distortional stresses is by beam on elastic foundation (BEF) analogy. This analogy ignores the effect of shear deformations and takes no account of the cross sectional deformations which are likely to occur in a thin walled box girder structure.

A modification of BEF analogy was developed by Hsu et al [2] as a practical approach to the distortional analysis of steel box girders. The equivalent beam on elastic foundation (BEF) method as it is called is an enhancement of the BEF method. It is adoptable to the analysis of closed (or quasi-closed) box girders and provides a simplified procedure to account for deformation of the cross section, the effect of rigid or flexible interior diaphragms and continuity over the supports.

Osadebe and Mbajiogu [3] employed the variational principles of cross sectional deformation on the assumption of Vlasov's theory and developed a fourth order differential equation of distortional equilibrium for thin walled box girder structures. Their formulation took into considerations shear deformations which were reflected in the equation of equilibrium by second derivative term. Numerical analysis of a single cell box girder subjected to distortional loading enabled them to evaluate values of distortional displacement, distortional warping stresses and distortional shear which they compared with BEF analogy results and concluded that the effect of shear deformations can be substantial and should not be disregarded under distortional loading.

Several investigators; Chidolue et al [4] Osadebe and Chidolue [5] Chidolue and Osadebe [6] and Mbachu and Osadebe [7] considered torsional, distortional and flexural stresses in thin-walled mono symmetric box girder structures involving single cell and multicell sections on the other hand, Xian and Xu [8] Sarode and Vesmawala [9] Ozgur [10] and Rubeena [11] considered horizontally curved reinforced concrete box girder bridges for torsional-distortional and flexural stresses while [12] Zhang Yuan-hai and Li Qiao Arici et al [13] considered horizontally curved steel box girder structures. Eze [14] studied reinforced concrete box column based on the numerous literatures consulted in the literature survey the following observations and comments can be made.

1. Research work done on thin-walled box girder structures covers essentially single cell box girder structure and multi-cell box girder structure either straight or curved.
2. Literature on multiple spine box girders appears to be scarce. Thus, there appears to be a dearth of information on the torsional-distortional behavior of thin-walled double spine box girder bridge structure.

III. VLASOV'S STRESS - STRAIN RELATIONS

The longitudinal warping and transverse (distortional) displacements given by Vlasov (1958) are

$$u(x,s) = U(x)\,\varphi(s) \qquad (1a)$$

$$v(x,s) = V(x)\,\Psi(s) \qquad (1b)$$

Where U(x) and V(x) are unknown functions governing the displacements in the longitudinal and transverse directions respectively, and φ and Ψ are generalized warping and distortional strain modes respectively. These strain modes are known functions of the profile coordinates, and are chosen in advance for any type of cross section. The displacements may be represented in series form as;

$$u(x,s) = \sum_{i=1}^{m} U_i(x)\varphi_i(s) \qquad (2a)$$

$$v(x,s) = \sum_{K=1}^{n} V_k(x)\,\Psi_k(s) \qquad (2b)$$

Where, $U_i(x)$ and $V_k(x)$ are unknown functions which express the laws governing the variation of the displacements along the length of the space frame.

φ_i *(s)* and Ψ_k (s) are elementary displacements of the strip frame, respectively out of the plane (m displacements) and in the plane (n displacements).

These displacements are chosen among all displacements possible, and are called the generalized strain coordinates of a strip frame.

From the theory of elasticity the strain in the longitudinal and transverse directions are given by;

$$\frac{\partial u}{\partial x}(x,s) = \sum_{i=1}^{m} U_i'(x)\varphi_i(s) \text{ and} \qquad a$$

$$\frac{\partial v}{\partial x}(x,s) = \sum_{K=1}^{n} V_k'(x)\Psi_k(s) \qquad b \qquad (3)$$

The expression for shear strain is $\gamma(x,s) = \frac{\partial u}{\partial s} + \frac{\partial v}{\partial x}$

$$\text{Or } \boldsymbol{\gamma}(x,s) = \sum_{i=1}^{m} \varphi'_i(s)U_i(x) + \sum_{K=1}^{n} \Psi_k(s)V_k'(x) \qquad (4)$$

Using the above displacement fields and basic stress-stain relationships of the theory of elasticity the expression

$$\sigma(x,s) = \frac{\partial u}{\partial x}(x,s) = E\sum_{i=1}^{m} \varphi_i(s)U_i'(x) \qquad (5)$$

$$\tau(x,s) = G\gamma(x,s) = \left(G\sum_{i=1}^{m}\varphi_i'(s)U_i(x) + \sum_{k=1}^{n}\Psi_k(s)V_k'(x) \right) \qquad (6)$$

The (m+n) functions sought for, u_i (x) and v_k (x), are determined from (m+n) equations for the strip frame, obtained by equating to zero the work done by external and internal forces in (m+n) independent virtual displacements

Every virtual displacement is as a result of an infinitesimal variation experienced by one of the generalized strain coordinates which determine the position of all joints and bars of the frame. This application of the principle of virtual displacements is called the method of variations.

Transverse bending moment generated in the boxes structure due to distortion is given by;

$$M(x,s) = \sum_{k=1}^{n} M_k(s)V_k(x) \qquad (7)$$

Where $M_k(s)$ = bending moment generated in the cross sectional frame of unit width due to a unit distortion, V(x) = 1

IV. POTENTIAL ENERGY FUNCTIONAL

The potential energy of a box structure under the action of a distortion load of intensity q is given by:

$$\Pi = U + W_E \qquad (8)$$

Where,

Π = the total potential energy of the box structure,

U = Strain energy

W_E = External potential or work done by the external loads.

From strength of material, the strain energy of a structure is given by

$$U = \frac{1}{2}\int_L\int_S \left[\left(\sigma^2(x,s)/E + \tau^2(x,s)/G \right) t(s) + M^2(x,s)\,EI_{(s)} \right] dxds \qquad (9)$$

And work done by external load is given by;

W_E = qv(x,s)dxds

$$= \int_s\int_x q\Sigma V_h(x)\varphi_h(s)dsdx = \int_x \Sigma q_h V_h dx \qquad (10)$$

Substituting expressions (9) and (10) into Eqn. (8) we obtain that,

$$\Pi = \frac{1}{2}\left\{ \int_L\int_s \left[\alpha^2(x,s)/E + \tau^2(x,s)/G\; t(s) + M^2(x,s)/EI_{(s)} - qv(x,s) \right] dxds \right\} \qquad (11)$$

Where,

$\sigma(x,s)$ = Normal stress

$\tau(x,s)$ = Shear stress

M (x,s) = Transverse distortion bending moment

q = Line load per unit area applied in the plane of the plate

$I_{(s)} = \frac{t^3(s)}{12(1-v^2)}$ = moment of inertia

E = Modulus of elasticity

G = Shear modulus

v = poisson ratio

t = thickness of plate

Substituting the expression for $\sigma(x,s)$ (eqn (5), $\tau(x,s)$ eqn. (6), M (x,s) eqn. (7) and v(x,s) eqn. (1) into eqn (11) we obtain that:

$$\Pi = E\Sigma\varphi_i(s)U'_i(x) * \Sigma\varphi_i(s)U_j''(x) * t(s)dsdx +$$

$$+G[\Sigma\varphi_i'(s)U_i(x) + \Sigma\Psi_k(s)V_k'(x)] * [\Sigma\varphi_j'(s)U_j(x) + \Sigma\Psi_h(s)V_h'(x)] * t(s)dsdx \; +$$

$$+1/EI\,[\sum_{k=1} M_k(s)V_k * \sum_{h=1} M_h(s)V_h(x)]dsdx \quad -\int_x \Sigma q_h V_h dx \qquad (12)$$

Simplifying further nothing that t(s)ds=dA we obtain;

$$\Pi = \frac{1}{2}E\sum_{1=1}^{m} U_i'(x)U_i'(x) * \sum_{j=i}^{m} \varphi_i'(s)\varphi_j'(s) * dAdx$$

$$+\frac{1}{2}G\sum_{i=1}^{m} U_i(x)U_j(x)\sum_{j=i}^{m} \varphi_i'(s)\varphi_j'(s)dAdx$$

$$+1/2G\sum_{J=i}^{n} U_j(x)V_k'(x) * \sum_{K=1}^{n} \varphi_j'(s)\Psi_k(s) * dAdx$$

$$+1/2G\sum_{h=i}^{n} U_j(x)U_h'(x) * \sum_{i=1}^{m} \varphi_i'(s)\Psi_h(s) * dAdx$$

$$+\frac{1}{2}G\sum_{h=1}^{n} V_k'(x)V_h'(x) * \sum_{k=1}^{n} \Psi_k(s)\Psi_h(s) * dAdx$$

$$+\frac{1}{2}\sum_{k=1}^{n} \frac{M_k(x)M_h(x)}{EI_{(s)}} * \sum_{k=1}^{n} V_k(x)V_h(x) * dxds$$

$$-\Sigma q_h V_h dx \qquad (13)$$

Let,

$a_{ij} = a_{ji} = \int \varphi_i(s)\varphi_j(s)dA$ (a)

$b_{ij} = b_{ji} = \int \varphi_i'(s)\varphi_j'(s)dA$ (b)

$c_{kj} = c_{jk} = \int \varphi_k'(s)\Psi_j(s)dA$ (c)

$c_{ih} = c_{hi} = \int \varphi_i'(s)\Psi_k(s)dA$ (d)

$r_{kh} = r_{hk} = \int \Psi_k(s)\Psi_h(s)dA;$ (e)

$s_{kh} = s_{hk} = \dfrac{1}{E}\displaystyle\int \frac{M_k(s)M_h(s)}{EI_{(s)}}\,ds$ (f)

$q_h = \int q\Psi_h ds$ (g) (14)

Substituting eqns. (14) into eqn. (13) gives the potential energy functional:

$$\Pi = \frac{1}{2}E\Sigma a_{ij}\, U_i'(x)U_i'(x)dx$$

$$+\frac{1}{2}G\left[\Sigma b_{ij}U_i(x)U_j(x) + \Sigma c_{kj}U_k(x)V_j'(x)\right]$$

$$+\frac{1}{2}G\left[\Sigma c_{ih}U_i(x)V_h'(x) + \Sigma r_{kh}V_k'(x)V_h'(x)\right]dx$$

$$+\frac{1}{2}E\Sigma s_{hk}\, V_k(x)V_h(x)dx \quad -\Sigma q_h V_h dx \qquad (15)$$

The governing equations of distortional equilibrium are obtained by minimizing the above functional eqn. (15), with respect to its functional variables u(x) and v(x) using Euler Largange technique, eqns. (15) and (16).

$$\frac{\partial\Pi}{\partial u_j} - \frac{d}{dx}\left[\frac{\partial\Pi}{\partial u_j}\right] = 0 \qquad (a) \qquad (16)$$

$$\frac{\partial\Pi}{\partial V_h} - \frac{d}{dx}\left[\frac{\partial\Pi}{\partial V_h}\right] = 0 \qquad (b)$$

Carrying out the partial differentiation of eqn. (15) with respect to U_j and U_j' gives

$$\frac{\partial\Pi}{\partial U_j} = G[\Sigma b_{ij}U_i(x) + \Sigma c_{kj}V_k'(x)],$$

$$\frac{\partial\pi}{\partial U_j} = E\Sigma a_{ij}U_i'(x); \quad \frac{d}{dx}\frac{\partial\Pi}{\partial U_j} = Ea_{ij}U_i''(x)$$

Therefore $\dfrac{\partial\Pi}{\partial U_i} - \dfrac{d}{dx}\left[\dfrac{\partial\Pi}{\partial U_j'}\right] = 0$

$$\Rightarrow G[\Sigma b_{ij}U_i(x) + \Sigma c_{kj}V_k'(x)] - E\Sigma a_{ij}\, U_i''(x) = 0$$

Or $E\Sigma a_{ij}U_i''(x) - G\Sigma b_{ij}U_i(x) - G\Sigma c_{kj}V_k'(x) = 0$

Diving through by G, and re-arranging we obtain;

$$k\sum_{i=1}^{m} a_{ij}U_i''(x) - \sum_{i=1}^{m} b_{ij}U_i(x) - \sum_{k=1}^{n} c_{kj}V_k'(x) = 0 \qquad (17)$$

Where $k = \dfrac{E}{G} = 2(1+\nu)$

Performing similar operations with respect to V_h and V_h' we obtain the second equation as follows.

$$\frac{\partial\Pi}{\partial V_h} = E\Sigma s_{hk}\, V_k(x) - \Sigma q_h$$

$$\frac{\partial\Pi}{\partial V_h'} = G[\Sigma c_{ih}\, U_i(x) + \Sigma r_{kh}\, V_k'(x)]$$

$$\frac{d}{dx}\left[\frac{\partial\Pi}{\partial V_h'}\right] = G[\Sigma c_{ih}\, U_i'(x) + \Sigma r_{kh}\, V_k'(x)]$$

$$\therefore \frac{\partial\Pi}{\partial V_h} - \frac{d}{dx}\left[\frac{\partial\Pi}{\partial V_h'}\right] = -G[\Sigma c_{ih}\, U_i'(x) + \Sigma r_{kh}V_k'(x)] + E\Sigma s_{hk}V_k(x) - \Sigma q_h = 0$$

$$\Sigma c_{ih}\, U_i'(x) + \Sigma r_{kh}\, V_k''(x) - k\Sigma s_{hk}V_k(x) + \frac{1}{G}\Sigma q_h = 0 \qquad (18)$$

Equations (17) and (18) are vlasov's differential equations of distortional equilibrium for a box girder. The matrix form of eqns.(17 and 18) are:

$$k\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{Bmatrix} U_1'' \\ U_2'' \\ U_3'' \end{Bmatrix} - \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}\begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} -$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \end{pmatrix}\begin{Bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{Bmatrix} = 0 \qquad (19a)$$

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}\begin{Bmatrix} U_1' \\ U_2' \\ U_3' \end{Bmatrix} - k\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix}\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{Bmatrix} +$$

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix} \begin{Bmatrix} V_1'' \\ V_2'' \\ V_3'' \\ V_4'' \end{Bmatrix} = \frac{1}{G} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = 0 \qquad (19b)$$

V. STRAIN MODES

From the energy formulation of the equilibrium it was noted that φ and Ψ represent generalized warping and distortional strain modes respectively and from eqns. (2a and 2b) $\varphi_i(s)$ and $\Psi_k(s)$ are elementary displacements) respectively. It was also noted that these displacements are chosen among all displacements possible and are called the generalized strain coordinates of a strip frame. Thus, Vlasov's coefficients of differential equations of equilibrium, eqn.(14), which involve a combination of these elementary displacements and their derivatives may be obtained by consideration of the box girder bridge cross section as a strip frame and then applying unit displacement one after the other at the nodal points of the frame in longitudinal direction, to determine the corresponding out of plane displacements at the joints in **n** possible transverse directions, the corresponding transverse (in-plane) displacements can also be obtained. The first order derivatives of these displacement functions may be obtained by numerical differentiation and used for computation of the coefficients with the aid of Morh's integral for displacement computations.

Consideration of the double spine mono-Symmetric strip frame in fig 1.shows that it has eight degrees of freedom in the longitudinal direction and seven in the transverse direction. From equation (2a and 2b), where in this case m = 8 and n = 7, it follows that we have fifty-six displacement quantities to compute and hence, fifty-six differential equations of distortional equilibrium will be required.The application of Vlasov's generalized strain modes as modified by Varbanov (1970) reduces the number of displacement quantities and hence the differential equations of equilibrium required to solve for them to seven, irrespective of the number of degrees of freedom possessed by the structure.

In the generalized strain modes, there are three strain fields in the longitudinal direction φ_1,φ_2, and φ_3. Thus, from eqn.

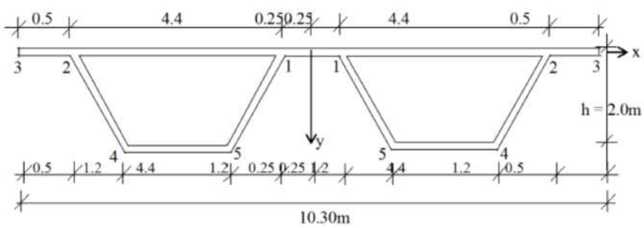


Fig.1 Double Spine Mono-Symmetric Box Girder Section

(2a) we have $\varphi(x,s) = \varphi_1(x)\, \varphi_1(s) + \varphi_2(x)\, \varphi_2(s) + \varphi_3(x)\, \varphi_3(s)$ Or

$$\varphi(x,s) = \sum_{i=1}^{3} \varphi_i(x)\, \varphi_i(s) \qquad (20a)$$

In the transverse direction four strain modes are also recognized Ψ_1,Ψ_2 and Ψ_3. Thus, we have $\Psi(x,s) = \Psi_1(x)\, \Psi_1(s) + \Psi_2(x)\, \Psi_2(s) + \Psi_3(x)\, \Psi_3(s) + \Psi_4(x)\, \Psi_4(s)$ Or

$$\Psi(x,s) = \sum_{k=1}^{4} \Psi_k(x)\, \Psi_k(s) \qquad (20b)$$

Where φ_1 = out of plane displacement parameter when the load is acting (vertically) normal to the top flange of the girder, i.e. bending is about horizontal axis.

φ_2 = out of plane displacement parameter when the load is acting tangential to the plane of the flanges i.e. bending is about vertical axis.

φ_3 = out of plane displacement parameter due to distortion of the cross section i.e; the warping function.

Ψ_1 = In-plane displacement parameter due to the load giving rise to φ_1

Ψ_2 = In-plane displacement parameters due to the load giving rise to φ_2

Ψ_3 = In-plane displacement parameter due to the distortion of the cross section i.e non uniform torsion.

Ψ_4 In-plane displacement functions due to pure rotation or Saint Venant torsion of the cross section.

VI. STRAIN MODE DIAGRAMS

Consider a simply supported girder loaded as shown in Fig.2a. if we assume the normal bean theory, i.e.; neutral axis remaining neutral before and after bending then the distortion of the cross section will be as shown in Fig.2 where, θ is the distortion angle (rotation of the vertical axis). The displacement φ_1 at any distance R, from the centroid is given by $\varphi_1 = R\theta$. If we assure a unit rotation of the vertical (z) axis then $\varphi_1 = R$, at any point on the cross section. Note that φ_1 can be positive or negative depending on the value of R, in the tension or compression zone of the girder. Thus, φ_1 is a property of the cross section obtained by plotting the displacement of the members of the cross section when the vertical (z-z) axis is rotated through a unit radian.

Similarly, if the load is acting in a horizontal (y-y) direction, normal to the x-z plane in Fig.2, then the bending is in x-z plane and y axis is rotated through angle θ_2 giving rise to φ_2 , displacement out of plane. The values of φ_2 , are obtained for the members of the cross section by plotting the displacement of the cross section when y-axis is rotated through a unit radian.

The warping function $\varphi_{3,}$ of the beam cross section is obtained as detailed in Fig.3a it has been explained that the warping function is the out of plane displacement of the cross section when the beam is twisted about its axis through the pole, one radian per unit length without bending in either x or y direction and without longitudinal extension. Ψ_1 and Ψ_2 are in-plane displacement of the cross section in x-z and x-y planes respectively while Ψ_3 is the distortion of the cross section. They can be obtained by numerical differentiation of Ψ_1 Ψ_2 and Ψ_3 diagrams respectively.

Ψ_4 is the displacement diagram of the beam cross section when the section is rotated one radian in say, a clockwise direction, about its centroidal axis. Thus, Ψ_4 is directly proportional to the perpendicular distance (radius of rotation) from the centroidal axis to the members of the cross section. It is assumed to be positive if the member moves in the positive directions of the coordinate axis and negative otherwise.

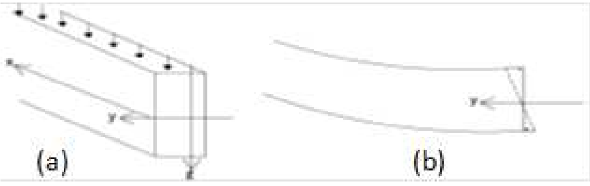
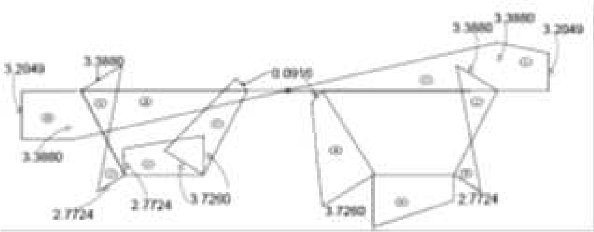
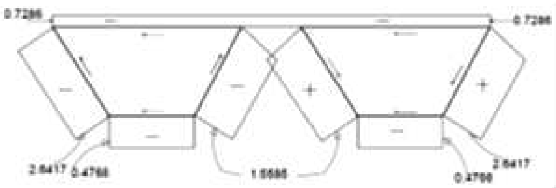


Fig.2 Simply Supported Girder Section and Cross Section Distortion

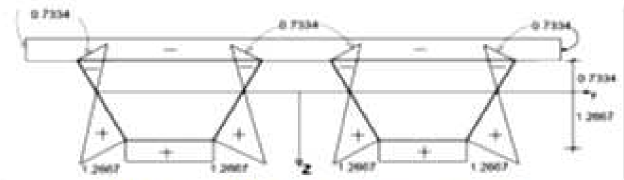


(a) Warping function ω_m (φ_3)

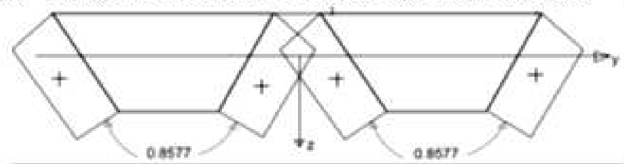
Fig.3 Generalized Strain Mode Diagram



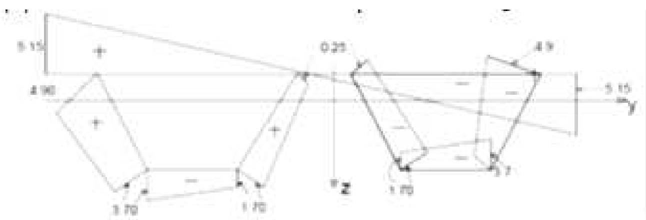
(b) Distortion diagram $\varphi^1_3 = \Psi_3$



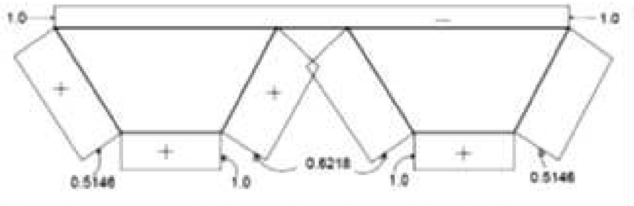
(c) Longitudinal strain mode diagram y-y axis bending φ_1



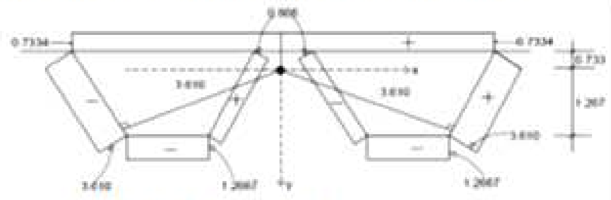
(d) Transverse strain mode in y direction Ψ_1



(e) Longitudinal strain mode z-z axis bending φ_2



(f) Transverse strain mode in z direction Ψ_2



(g) Rotation strain mode , Ψ_4

For monosymmetric section, the relevant Vlasov's coefficients for Torsional-distortional equilibrium are a_{33}, $b_{33} = r_{33}$, $r_{34} = r_{43}$ r_{44}

$a_{33} = \int \varphi_3(s)\, \varphi_3(s)\, dA \quad = 24.682$

$b_{33} = \int \varphi^1_3 \quad \varphi^1_3\, (s)\, dA \quad = 9.918$
$r_{34} = \int \varphi_3\, \varphi_3\, dA \quad = 7.107$
$r_{44} = \int \varphi_4\, \varphi_4\, dA \quad = 15.33$
Note,
$b_{33} = C_{33} = r_{33} = 9.918$
$r_{34} = r_{43} = 7.107$

The coefficient $S_{hk} = S_{kh} = \frac{I}{E}\int \frac{M_3\,(S)M_3\,(S)}{EIs}$ (21)

Where M_3 (s) is the distortional bending moment

VII. DETERMINATION OF DISTORTIONAL BENDING MOMENT FOR THE BOX GIRDER

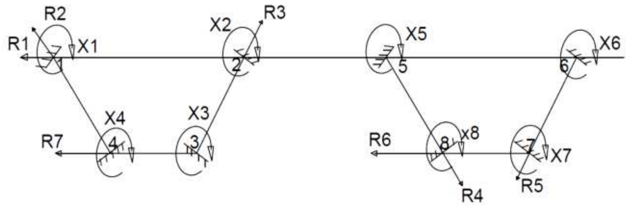


Fig. 4 Base System for Evaluation of Distortion Bending Moment

Fig.3a shows the base system for the evaluation of distortional bending moment for the double spine mono-symmetric box girder. The evaluation of the distortional bending moment involves the application of unit rotation $X_1$to X_8 at joint 1 to 8 respectively and applying unit transverse displacement of joints based on distortion diagram

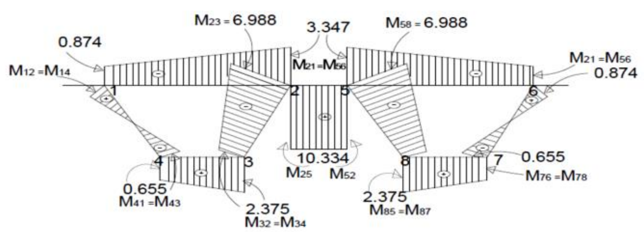


Fig.5 Bending Moment Diagram Due to Distortion of the Cross Section

Shk = shk = $\frac{1}{E}\int\frac{M_k(s)M_h(s)}{EIs}$ ds

Shk = Skh = S_{33} = 2.891x$10^{-3}I_s$ only S_{33} has value

VIII. FORMULATION OF DIFFERENTIAL EQUATIONS OF EQUILIBRIUM

The relevant coefficients for torsional-distortional equilibrium are a_{33}, b_{33}, c_{33}, c_{34}, r_{33}, r_{34}, r_{43}, r_{44}, and s_{33}. Substituting these into the matrix notation equation (8) and (9) we obtain:

$$k\begin{bmatrix}0&0&0\\0&0&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}U_1''\\U_2''\\U_3''\end{bmatrix}-\begin{bmatrix}0&0&0\\0&0&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}U_1\\U_2\\U_3\end{bmatrix}-\begin{bmatrix}0&0&0\\0&0&0\\0&0&b_{33}\end{bmatrix}\begin{bmatrix}V_1'\\V_2'\\V_3'\\V_4'\end{bmatrix}=0$$

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&C_{33}\\0&0&C_{43}\end{bmatrix}\begin{bmatrix}U_1''\\U_2''\\U_3''\end{bmatrix}-K\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&S_{33}&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\\V_4\end{bmatrix}+$$

$$\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&r_{33}&r_{34}\\0&0&r_{43}&r_{44}\end{bmatrix}=-\frac{1}{G}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\end{bmatrix}$$

Multiplying out we obtain

$Ka_{33}\,U_3'' - b_{33}U_3 - C_{33}V_3' - C_{34}V_4' = 0$

$C_{33}\,U_3' - KS_{33}V_3 + r_{33}V_3' + r_{34}V_4' = -\frac{q_3}{G}$

$C_{43}\,U_3' - r4_3V_3' + r_{44}V_4' = -\frac{q_4}{G}$

Simplifying further we obtain

$\beta_1 V_4'' - \gamma_1 V_3 = K_1$ (22a)

$\propto_1 V_3^{'v} + \propto_2 V_4^{'v} - \beta_2 V_4'' = K_2$ (22b)

Where $\propto_1 = ka_{33}c_{43}$; $\propto_2 = ka_{33}\,r_{44}$;

$\beta_1 = r_{34}c_{43} - c_{33}r_{44}$

$\beta_2 = b_{33}\,r_{44} - c_{34}\,c_{43}$;

$\gamma_1 = c_{43}ks_{33}$ (23)

$K_1 = -C_{33}\frac{q_4}{G} - C_{43}\frac{q_3}{G}$; $K_2 = \left(\frac{b_{33}q_4}{G}\right)$ (24)

Torsional – Distortional Analysis of Mono-Symmetric Box Girder Structure

In this section the solutions of the differential equations of equilibrium are obtained for the double spine mono-symmetric box girder section shown in fig.1. Live loads are considered according to AASHTO-LRFD following the HL-93 loading. [15] Uniform lane load of 9.3N/mm distributed over a 3m width plus tandem load of two 110KN axles. The loads are positioned at the outermost possible location to generate the maximum torsional effects. A two span simply supported bridge deck structure, 20m per span, was considered.

The obtained torsional loads are as follows

q_3 = 1410.318KN, q_4 = 3732.202 KN

Parameters for the governing equations (22a and 22b) are:

$\propto_1 = K\,a_{33}\,C_{43}$; $\propto_2 = Ka_{33}\,r_{44}$

$\beta_1 = r_{34}C_{43} - C_{33}\,r_{44}$; $\beta_2 = b_{33}\,r_{44} - C_{34}\,C_{43}$

$\gamma_1 = C_{43}\,K\,S_{33}$; $K_1 = C_{33}\frac{E4}{G} = C_{43}\frac{E3}{G}$

$K_2 = b_{33}\frac{q_4}{G}$; $S_{33} = 2.891 \times 10^{-2} I_s$

k = 2 (1 + V); k = 2 (1 + 0.25) = 2.5; ν = 0.25 for concrete

E = 24 x 10^9 N/m^2; G = 9.6 x 10^9 N/m^2

∴ $\propto_1$ = 2.5 x 24.682 x 7.107 = 438.537

$\propto_2$ = 2.5 x 24.682 x 15.153 = 935.016

β_1 = 7.107 x 7.107 – 9.918 x 15.153 = - 99.778

β_2 = 9.918 x 15.153 – 7.107 x 7.107 = 99.778

γ_1 = 7.107 x 2.5 x 2.891 x 10^{-2} = 0.5137

$K_1 = 9.918 \times \frac{3732.202 \times 10^3}{9.6 \times 10^9} - 7.107 \times \frac{1410.318 \times 10^3}{9.6 \times 10^9}$ =0.0028109

$K_2 = \frac{(9.918 \times 3732.202 \times 10)}{9.6 \times 10^9} = 3.856\times10^{-3}$

Substituting the coefficients $\propto_1$, $\propto_2$, β_1, β_2, γ_1, K_1 and K_2

We obtain equations (25) and (26) below

438.537 V_3^{IV} + 935.016 V_4^{IV} – 99.778 V_4^{II} = 3.856 X 10^{-3} (25a)

-99.778V_4^{II} – 0.5137 V_3 = 2.811 x 10^{-3} (25b)

Integrating by method of Trigonometric Series with accelerated convergence we obtain

$V_3(x) = 8.773 \times 10^{-3} \sin\frac{\pi x}{20}$ (26)

$V_4(x) = 2.972 \times 10^{-3} \sin\frac{\pi x}{20}$

Table 1: Variation of torsional and distortional displacements along the length of the girder (20m simply supported)

Distance x from left support (m)	Sin $\frac{\pi x}{20}$	Distortional Displacement V_3 (x) x 10^{-3} m	Torsional Displacement V_4 (x) x10^{-3} m
0	0.000	0.000	0.000
2	0.309	2.711	0.918
4.	0.588	5.158	1.747
6.	0.809	7.097	2.404
8.	0.951	8.343	2.826
10	1.000	8.773	2.972
12	0.951	8.343	2.826
14.	0.809	7.097	2.404
16	0.588	5.158	1.747
18	0.309	2.711	0.918
20	0.000	0.000	0.000

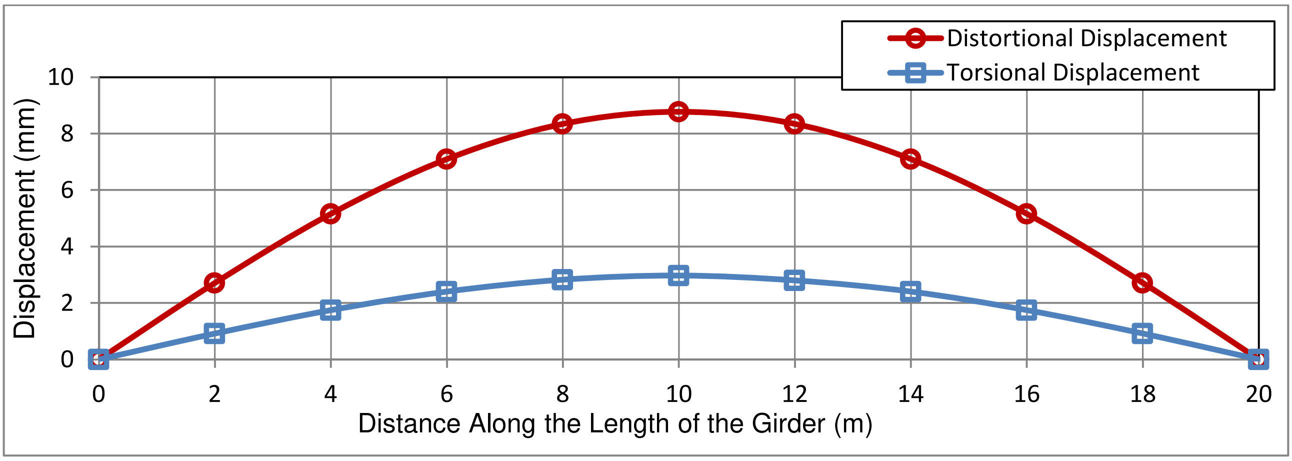


Fig. 6 Variation of torsional and distortional displacement along the length of the girder

IX. DISCUSSION OF RESULTS:

The governing differential equations of torsional-distortional equilibrium for the double spine mono-symmetric box girder structures are given by eqn. (25a and 25b).

The solution of the torsional-distortional equations of equilibrium for the double spine monosymmetric box girder studied is given by: $V_3 = 8.773x10^{-3} \sin \frac{\pi x}{L}$

$$V_4 = 2.972x10^{-3} \sin \frac{\pi x}{L} \qquad (26)$$

Where L represents the span of the girder

The torsional and distortional deformations obtained by integration of eqn. (25) are given by eqn. (26). The results of the analysis are presented in table 1 with graphical presentation in fig.3. The maximum (mid-span) torsional displacement was 2.97mm while the mid-span distortional displacement was 8.77mm. Thus the maximum distortional deformation is about 3 times that of torsional deformation. This explains why torsional stresses may be neglected but not distortional stresses.

The obtained governing differential equations of torsional-distortional equilibrium are fourth order coupled linear differential equations. The coupling of the equations of torsional –distortional equilibrium reveal a strong interaction between torsional strain mode and distortional strain mode such that torsional analysis of a mono symmetric double spine box girder structure cannot be carried out independent of distortional analysis without introducing errors in the analysis.

X. CONCLUSION:

The distortional deformations were found to be about three times that of torsional deformation.

The response of double spine mono-symmetric box girder structure to torsional and distortional loads is similar to that of single and multi cellular box girders obtained from earlier studies by other researchers; Chidolue and Osadebe 2012.

The generalized forth order differential equations for torsional-distortional analysis of double spine mono-symmetric box girder structure and indeed, all mono-symmetric box girder structures are given by eqns.(22a) and (22b)

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