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# Rates of Heat and Mass Transfers for a Non Darcy Porous Medium Subject to Double Dispersion and Saturated by a Nanofluid

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***Abstract*— This work aims to quantify the rates of heat and mass transfers occurred between a vertical and nonisothermal plate immersed into a non-Darcy porous medium and saturated with a weak nanofluid. Double dispersion is assumed and natural convection is the exchange mode. The similarity transformations are involved and the governing system of nonlinear partial differential equations is converted into a set of nonlinear ordinary differential equations via similarities. Results are displayed graphically to illustrate the influence of δ and ξ on the velocity, the temperature and concentration of the species profiles. For a weak nanofluid, the rate of mass transfer is affected strongly by the double dispersion while the rate of heat transfer coefficient is less sensitive to it.**

***Index Terms*— Double dispersion, weak nanofluid, double diffusion, non-Darcy porous medium, nonisothermal plate.**

## I. INTRODUCTION

It is known that the hydrodynamic dispersion plays an important role in the porous medium. The dispersion can be caused by flow and geometrical obstruction as well [1]. It is generally attributed to the velocity distribution in each variable pore. As a result, the non-uniformity of the temperature or the concentration of the solute distributions are identified by experimental data and recognized as thermal and solutal dispersion effects.

In the porous medium, a nanofluid can be used in many attractive applications, and in the recent years, nanofluids with weak concentration of nanoparticles are experienced because they have a greater thermal conductivity than the classical fluids. Theoretical and experimental studies are conducted to collect the knowledge about these new fluids. An enhancement in the range of 10 – 50 % relatively to the base fluid is observed for the thermal conductivity with very low concentration of particles. Among the models when single phase motion is considered, mechanisms have retained as the Brownian diffusion and the thermophoresis as the two most important, Buongiorno [2].

The flows of these nanofluids are then naturally extended to the porous medium, in which inertial effects can be described by the Darcy-Forchheimer model. At a second step, the dispersion substantially important, but generally ignored, are here not omitted in the case of natural convective heat and mass transfer.

The extremely large impact of the molecular collisions on a fine particle with size less than 0.1 μm generates a force that leads to the dispersion, Cheng [3] presented a model for the longitudinal and transversal dispersion coefficients added to the stagnant thermal conductivity and linked to the velocity. Hong and Tien [4] suggested that the dispersion diffusivity is also expressed linearly with the velocity and the pore diameter of the porous medium. Up to unity the dispersion coefficient is used in an analytic approach developed by Wang et al. [5]. The double dispersion effects on a simultaneous heat and mass transfer is studied with a classical fluid by Amin [6].

In this area of a nanofluid in a porous medium, Nield and Kusnetsov [7-10] examined the diffusive, the double diffusive boundary layer and the related revised model in a porous medium.

The objective of this study is to known the rates of heat and mass transfers in the case of double diffusion and double dispersion effects in a non-Darcy porous medium saturated by a weak nanofluid. The more realistic convective boundary condition with an immersed plate is considered.

## II. STATEMENT OF THE PROBLEM

A laminar free convection flow is considered along a vertical and nonisothermal plate and immersed in a porous medium with porosity $\varepsilon$. Highly porous medium is assumed so that the Forshheimer extension of the Darcy's law is used. Buongiorno [2] are taking into account in modelling the behaviour of the nanofluid the volume fraction of the nanoparticles, and an equation is added to the mathematical developed model.

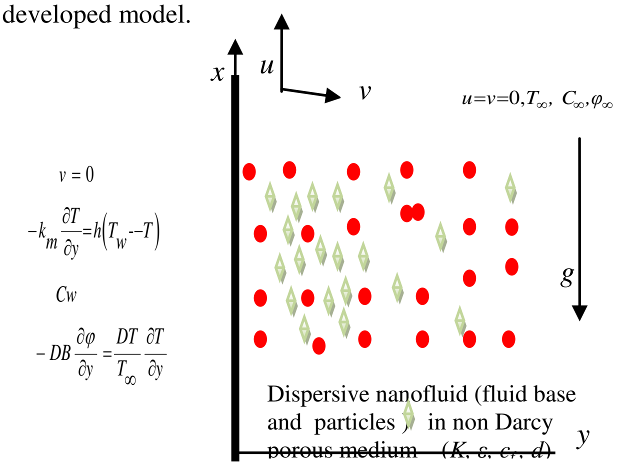


Figure 1: Sketch of the physical model and the coordinate system.

The convective boundary condition (non-isothermal plate) is studied. The setting of the prescribed wall nanoparticle volume fraction is not reasonable and the revised model or passive boundary condition presented by Nield and kuznetsov

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[6, 9-10] is adopted here, while the corresponding value at ambient is denoted $\phi_\infty$.

A sketch of the physical domain and coordinate systems for the present problem is shown in Fig. 1

## III. MATHEMATICAL MODEL

### A. Dimension model

Following the model developed in the work of Nield and Kuznetsov [8], and using the Oberbeck-Boussinesq's approximation, the naturally governing conservation equations for the four field variables, the vector of the velocity $\mathbf{v} = (u, v)$, the temperature $T$, the concentration of the species $C$ and the volume fraction of the nanoparticles $\phi$, can be written as

$$\nabla.\mathbf{v} = 0 \tag{1}$$

$$0 = -\nabla p - \frac{\mu}{K}\mathbf{v} - \rho_{f\infty}\frac{c_f}{\sqrt{K}}\mathrm{v}^2 + \left[\phi\rho_p + (1-\phi)\rho_f\left(1-\beta_T(T-T_\infty) - \beta_C(C-C_\infty)\right)\right]\boldsymbol{g} \tag{2a}$$

$$(\rho c)_f\,\mathbf{v}.\nabla T = k_m\nabla^2 T + \varepsilon(\rho c)_p\left[D_B\nabla\phi.\nabla T + \left(\frac{D_T}{T_\infty}\right)\nabla T.\nabla T\right] \tag{3}$$

$$\frac{1}{\varepsilon}\,\mathbf{v}.\nabla C = D_S\nabla^2 C \tag{4}$$

$$\frac{1}{\varepsilon}\,\mathbf{v}.\nabla\phi = D_B\nabla^2\phi + \left(\frac{D_T}{T_\infty}\right)\nabla^2 T \tag{5}$$

These equations are subject to the following boundary conditions

$$at\ y = 0, v = 0, -k_m\frac{\partial T}{\partial y} = h(T_w - T)$$

$$C = C_w,\ D_B\frac{\partial\phi}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial T}{\partial y} \tag{6a}$$

$$as\ y\to\infty,\ u = v = 0,\ T\to T_\infty,\ C\to C_\infty,\ \phi\to\phi_\infty \tag{6b}$$

The porous medium have a porosity $\varepsilon$, permeability $K$ and the empirical constant associated with inertia effect term $c_f$. The gravitational acceleration vector is indicated by $\mathbf{g}$. $\beta_T$ and $\beta_C$ are the volumetric thermal expansion and the equivalent solutal coefficient of the base fluid. The letters $\rho$ and $p$ are the density and pressure of the nanofluid. Subscript $p$ represents the particle and $f$ the base fluid. The coefficients $D$ and $\mu$ are the diffusion coefficient and the viscosity of the nanofluid. Subscripts $B$, $T$, and $S$ referred to the Brownian motion, the thermophoretic effect and the solutal diffusivity of the porous medium, respectively. The effective thermal conductivity is $k_m$ for the porous medium and heat capacity is $(\rho c)$. $h$ is the heat transfer coefficient between the plate and the nanofluid.

Considering that the nanoparticle concentration is diluted, so that the equation (2a) can be linearized and reformulated as

$$0 = -\nabla p - \frac{\mu}{K}\mathrm{v} - \rho_{f\infty}\frac{c_f}{\sqrt{K}}\mathrm{v}^2 + \left[(\rho_p - \rho_{f\infty})(\phi-\phi_\infty) + (1-\phi_\infty)\rho_{f\infty}\left(\beta_T(T-T_\infty) - \beta_C(C-C_\infty)\right)\right]\boldsymbol{g} \tag{2b}$$

According to the above presentation of the thermal and solutal dispersion, the diffusivities $\alpha = k_m/(\rho c)_f$ and $D_S$ are expressed in two parts, one representing the molecular diffusivities and the second part the dispersion linked linearly to the velocity [4]. For a rigorous statement, the dispersion part should contain the parameters of the nanofluid, $\phi$ and the nanoparticle size.

It is assumed that the constants $\delta$ and $\sigma$ incorporate them, because not any model is available now and corroborate from experimental data. With the known concept of the boundary-layer and the related classical approximations, the conservation equations (1, 2b-5) and after cross-differentiation of the momentum equations, become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial}{\partial y}\left[u + \frac{c_f\sqrt{K}}{\nu}u^2\right] = \frac{(1-\phi_\infty)\rho_{f\infty}gK}{\mu}\left[\beta_T\frac{\partial T}{\partial y} + \beta_C\frac{\partial C}{\partial y}\right] - (\rho_p - \rho_{f\infty})\frac{gK}{\mu}\frac{\partial\phi}{\partial y} \tag{8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left((\alpha_m + \delta du)\frac{\partial T}{\partial y}\right) + \frac{\varepsilon(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial\phi}{\partial y}.\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \varepsilon\frac{\partial}{\partial y}\left((D_{Sm} + \sigma du)\frac{\partial C}{\partial y}\right) \tag{10}$$

$$u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = \varepsilon\left(D_B\frac{\partial^2\phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}\right) \tag{11}$$

Where $\delta, \sigma, d, \alpha_m$ and $D_{Sm}$ are respectively the constants of the dispersion, the pore diameter, the molecular thermal diffusivity and molecular solutal diffusivity of the saturated porous medium.

### A. Dimensionless model

Following the works [8, 10] for porous medium, dimensionless quantities are introduced to obtain the similarity solutions

$$\eta = \frac{y}{x}Ra_x^{1/2}, f(\eta) = \frac{\psi}{\alpha_m Ra_x^{1/2}}\left(u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}\right),$$

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}, \gamma = \frac{C-C_\infty}{C_w-C_\infty}, s = \frac{\phi-\phi_\infty}{\phi_w-\phi_\infty} \tag{12}$$

The local Rayleigh number is then introduced and pertinently defined by

$$Ra_x = \frac{(1-\phi_\infty)\rho_{f\infty}\beta_T(T_w-T_\infty)gKx}{\mu\alpha_m} \tag{13}$$

Where $\eta$ is the independent similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless

temperature, $\gamma(\eta)$ is the dimensionless concentration species and $s(\eta)$ is the dimensionless volume fraction.
Using the non-dimensional quantities (12-13), the equations (7-11) with the boundary conditions (6a-6b) can be further reduced to a set of ordinary differential equations for which numerical solutions will be more easily obtained

$$f''[1+2F_0 Ra_d f'] = \theta' + N_c\gamma' - N_r s' \quad (14)$$
$$\theta''[1+\delta Ra_d f'] = -\frac{1}{2}f\theta' - N_b s'\theta' - N_t(\theta')^2 - \delta Ra_d \theta' f'' \quad (15)$$
$$\gamma''[1+\xi Ra_d f'] = -\frac{1}{2}L_e f\gamma' - \xi L_e Ra_d \gamma' f'' \quad (16)$$
$$s'' = -\frac{1}{2}L_n f s' - \frac{N_t}{N_b}\theta'' \quad (17)$$

The boundary conditions become

$$at\ \eta = 0,\ f = 0,\ \theta' + Bi(1-\theta) = 0,\ \gamma = 1,\ N_b s' + N_t\theta' = 0 \quad (18a)$$

$$as\ \eta \to \infty,\ f' = 0,\ \theta \to 0,\ \gamma \to 0,\ s \to 0 \quad (18b)$$

Where, the parameters involved in these equations are defined by

$$F_0 = \frac{c_f\sqrt{K}}{\nu d}\alpha_m \quad Ra_d = \frac{(1-\phi_\infty)\rho_{f\infty}\beta_T(T_w-T_\infty)gKd}{\mu\alpha_m} \quad N_c = \frac{\beta_C(C_w-C_\infty)}{\beta_T(T_w-T_\infty)} \quad N_r = \frac{(\rho_p-\rho_{f\infty})(\phi_w-\phi_\infty)}{(1-\phi_\infty)\rho_{f\infty}\beta_T(T_w-T_\infty)}$$
$$N_b = \frac{\varepsilon(\rho c)_p D_B(\phi_w-\phi_\infty)}{(\rho c)_f\alpha_m} \quad N_t = \frac{\varepsilon(\rho c)_p D_T(T_w-T_\infty)}{(\rho c)_f T_\infty\alpha_m} \quad L_e = \frac{\alpha_m}{\varepsilon D_{Sm}} \quad L_n = \frac{\alpha_m}{\varepsilon D_B} \quad Bi = \frac{Bi_x}{Ra_x^{1/2}} = \frac{hx/k_m}{Ra_x^{1/2}} \quad (19)$$

$F_0$ is a parameter that represents the structure of the porous medium in a term of the inertia effect conjugated to the diffusivity of the saturated porous medium. $Ra_d$, is the Rayleigh number expressed with $d$, rather than $x$. $N_c$, $N_r$, $N_b$, and $N_t$ are the regular double-diffusive buoyancy ratio, the nanofluid buoyancy ratio, the Brownian motion parameter and the thermophoresis parameter respectively. $L_e$ denotes the classical Lewis number, while $L_n$ is its corresponding for nanofluid Lewis number. The new constant of solutal dispersion $\xi$ is set equal to $\varepsilon\sigma$. The interaction between the plate and the porous medium in the boundary condition case is related by the Biot number $Bi$.

### A. Quantities of interest

For practical applications, the physical quantities of most interest are the wall heat flux $q_w$ and the wall mass flux $q'_w$ and the corresponding local dimensionless physical quantities may be quantified by

$$Nu_x Ra_x^{-1/2} = -[1+\delta Ra_d f'(0)]\,\theta'(0) \quad (20)$$
$$Sh_x Ra_x^{-1/2} = -[1+\xi Ra_d f'(0)]\,\gamma'(0) \quad (21)$$

## IV. RESULTS

One recognizes that the set of the equations derived is highly nonlinear and cannot be solved analytically. An iterative finite difference method that implements for example the 3-stage Lobatto collocation formula is used. Especially, when the relative difference between the previous iteration and the current reached $10^{-5}$, the solution is considered to be converged and constitutes a criterion.

From the table 1, the key values of $-\theta'(0)$ are compared with those reported by [8] and are found in the good agreement.

Next, numerical computations have been carried out for the velocity, the temperature, the concentration species and the volume fraction of the nanoparticles profiles.

Fig. 2 displays the velocity profiles for different values of $Bi$. Low exchange thermal flux between the plate and the nanofluid causes a decrease in the velocity.

**Table 1.** Comparison of the values of $-\theta'(0)$ for monodiffusive regular fluid and nanofluid cases. $L_e$=$L_n$=10.0

| | $-\theta'(0)$ | |
|---|---|---|
| | Nield and Kusnetsov [8] | Present results |
| Monodiffusive regular fluid ($N_b$=0., $N_t$=0., $N_c$=0., $N_r$=0.) | 0.4439 | 0.44377568 |
| Monodiffusive nanofluid ($N_b$=0.2, $N_t$=0.2, $N_c$=0., $N_r$=0.2) | 0.3343 | 0.33417924 |

As comprehensive and physical sense, it is evident that the energy of the nanofluid is supplied totally from the wall and a low exchange is synonymous of a reduced velocity.

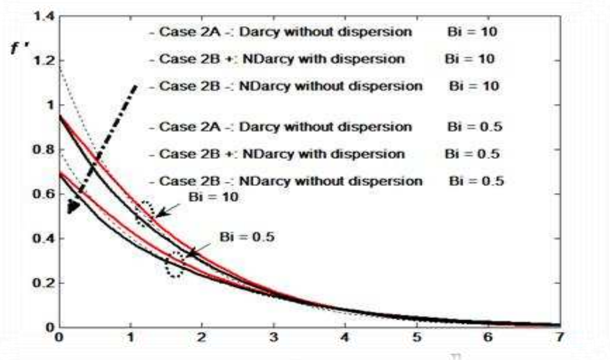


Figure 2: Velocity profile,
$N_b$=0.2, $N_t$=0.1, $N_c$=0.2, $N_r$=0.2, $L_e$=10.0, $L_n$=10.0

From fig. 3, the same explanation can be argued for the temperature profiles. Here again, the double dispersion tend to increase the temperature in the boundary layer, due to the intense Brownian motion and thermophoresis effect causing a supplementary thermal diffusion in the nanofluid. The level of the temperature is in dependency with the value of $Bi$.

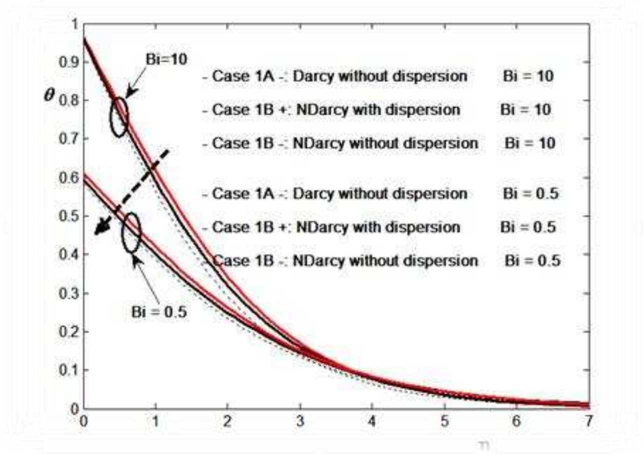


Figure 3: Temperature profile,
$N_b$=0.2, $N_t$ =0.1, $N_c$=0.2, $N_r$=0.2, $L_e$=10.0, $L_n$=10.0

It can be seen that the behavior of the concentration of the species profile is identical to the temperature profile, regarding the double dispersion effect, fig.4. However, the level of the concentration is non-Biot dependent. It seems that the double dispersion effect tend to homogenize the solute, without the effect of the thermal flux at the boundary.

In fig.5, the slight effect reported above is observed again with the different $Bi$. The volume fraction of the nanoparticles at the wall is more negative as $Bi$ increase. The maximum mean value is around 0.02, it is logical that a deeper gradient of the volume fraction is needed to respond to the greater gradient of the temperature at the wall.

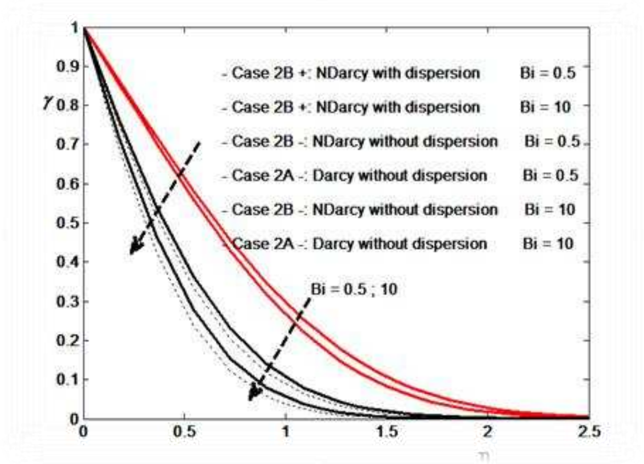


Figure 4: Concentration of species profile,
$N_b$=0.2, $N_t$ =0.1, $N_c$=0.2, $N_r$=0.2, $L_e$=10.0, $L_n$=10.0

The local Nusselt and Sherwood numbers for different values of $Bi$, $Ra_d$ and $F_0$ are illustrated in the table 2. For the purpose of comparison, the values obtained are listed in a form that highlighted the dispersion effect.

It can be seen from table 2 that the Nusselt number is enhanced when a double dispersion occurs. From quantitative point of view, this enhancement is great for a strong natural convection relative to a weak convection and lesser for porous medium with inertia relative to only the Darcy flow.

On the other hand, the mass transfer via $Sh_xRa_x^{-1/2}$ is more sensitive to the double dispersion effect, but in the inverse sense for the $Nu$ number.

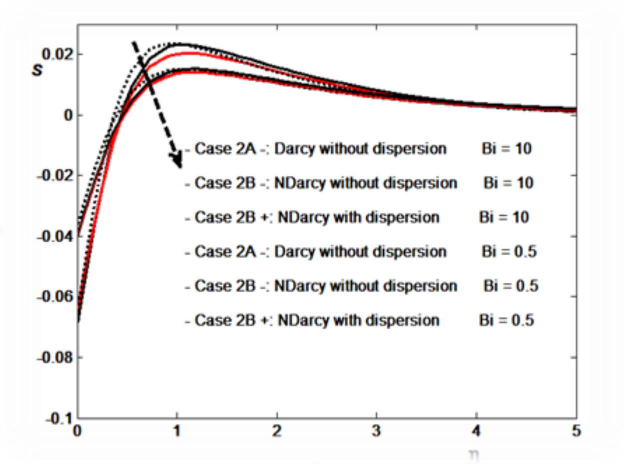


Figure 5: Volume fraction profile,
$N_b$=0.2, $N_t$ =0.1, $N_c$=0.2, $N_r$=0.2, $L_e$=10.0, $L_n$=10.0

**Table 2.** Values of $Nu_xRa_x^{-1/2}$ *and* $Sh_xRa_x^{-1/2}$ for selected values of $Bi$, $Ra_d$ and $Fo$ with ($N_b$=0.2, $N_t$ =0.1, $N_c$=0.2, $N_r$=0.2, $L_e$=$L_n$=10.0).

| | | | $Nu_xRa_x^{-1/2}$ | | $Sh_xRa_x^{-1/2}$ | |
|---|---|---|---|---|---|---|
| $Bi$ | $Ra_d$ | $Fo$ | $\delta=\xi=0$ | $\delta=\xi=0.5$ | $\delta=\xi=0$ | $\delta=\xi=0.5$ |
| 103 | 0.5 | 0 | 0.44935 | 0.50541 | 1.8074 | 1.03732 |
| | | 0.5 | 0.41579 | 0.45851 | 1.6391 | 0.99656 |
| | 2 | 0.5 | 0.36628 | 0.46619 | 1.41203 | 0.73435 |
| 10 | 0.5 | 0 | 0.42257 | 0.47781 | 1.77389 | 1.027 |
| | | 0.5 | 0.39357 | 0.43579 | 1.61495 | 0.98831 |
| | 2 | 0.5 | 0.34957 | 0.4489 | 1.39635 | 0.7289 |
| 0.5 | 0.5 | 0 | 0.20801 | 0.23842 | 1.44036 | 0.91232 |
| | | 0.5 | 0.20254 | 0.22777 | 1.35429 | 0.89069 |
| | 2 | 0.5 | 0.1921 | 0.26144 | 1.2126 | 0.6586 |

## V. CONCLUSION

We can conclude the following from our investigation:

- The double dispersion has a strong effect on the heat and mass convective transfers. This effect is more pronounced for a nanofluid than a clear fluid and for mass transfer than the heat transfer.

- Working with a nanofluid inside a non-Darcy porous medium leads to modifying the velocity, the temperature and the concentration of the species mass profiles, which in turn affect the rates of heat and mass, when the double dispersion acts.

- It is found that the rate of heat transfer increases and the mass transfer decreases strongly with the double dispersion.

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