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Improvement and Simulation Analysis of Wavelet Threshold Algorithm

Jun Guo, Yao Wang, Wenjun Yi

***Abstract*— By studying the traditional threshold function , it is found that the hard threshold function is discontinuous , the pseudo - Gibbs phenomenon occurs , and the soft threshold function is too much eliminating the detail coefficient , which causes the loss of high - frequency signal . The simulation experiment of Wavelet threshold noise reduction for different sonar signals is carried out. The experimental results show that the improved threshold function is better than the conventional algorithm , and the noise - reducing signal has higher SNR and smaller RMSE.**

***Index Terms*— Wavelet transform, Threshold function, Sonar signal de-noising.**

I. INTRODUCTION

Sonar signal is easily polluted by underwater noise in the process of transmission and reception. How to extract sonar signal in underwater noise environment is an important way to improve sonar detection efficiency. Wavelet transform effectively solves the problem of Fourier transform in time-frequency joint analysis. Wavelet transform theory is widely used in the field of signal processing. There are three kinds of noise reduction methods based on Wavelet analysis, which are Modulus maximum method, Wavelet coefficient correlation method and Threshold de-noising algorithm.In 1995, Donoho and Johnstone[1] put forward Wavelet threshold de-noising method, which has obvious effect and less computation. However, the hard-threshold function is not continuous at the threshold point and there will be a Pseudo-Gibbs phenomenon. The soft threshold function can overcome the pseudo-Gibbs phenomenon and reduce the signal variance. But the soft threshold function lose some high-frequency signals and form an incomplete reconstruction sign. On the basis of this, Bruce [3]proposed a semi-soft threshold function which reduced the loss of signal. However, it still didn't solve the pseudo-Gibbs phenomenon. Jian li [4] has smoothed the traditional hard threshold function and made the signal more continuous. Li Jiaojun [5] theoretically studies the wavelet threshold de-noising algorithm of one-dimensional signal and summarizes the influencing factors of wavelet threshold de-noising algorithm.

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II. WAVELET THRESHOLD DE-NOISING ALGORITHM

A. *Wavelet Transform*

When the function $f(t)$ is expanded under the Wavelet basis, this expansion is called continuous wavelet transform. It is shown as follows：

$$CWT_f(a,\tau) =< f(t), \psi_{a,\tau}(t) >= \frac{1}{\sqrt{a}} \int_R f(t)\psi^*(\frac{t-\tau}{a})dt \qquad (1)$$

Where the real number a is the scale parameter, and the real number$\tau$is the translation parameter.As can be seen from the expression, wavelet transform is an integral transform.

Let $f(t)$ be a square integrable function whose discrete wavelet transform expression is:

$$DWT_f(a_0^j, k\tau_0) = \int f(t)\psi^*_{a_0^j,k\tau_0}(t)dt \quad j=0,1,2,\cdots,k\in Z \qquad (2)$$

In order to achieve fast wavelet transform, Mallat proposed a classical fast algorithm of wavelet transform.Mallat algorithm is the basic algorithm in the Application of Wavelet transform.The decomposition algorithm is shown as follows：

$$\begin{cases} \langle f,\phi_{j-1}\rangle = 2^{-\frac{1}{2}}\sum \overline{p_{k-2l}}\langle f,\phi_{jk}\rangle \\ \langle f,\psi_{j-1}\rangle 2^{-\frac{1}{2}}\sum(-1)^k p_{1-k+2l}\langle f,\phi_{jk}\rangle \end{cases} \qquad (3)$$

The restructuring algorithm is shown as follows：

$$\langle f,\phi_{jk}\rangle = 2^{-1/2}\sum p_{k-2l}\langle f,\phi_{j-1,l}\rangle + 2^{-1/2}\sum(-1)^k \overline{p_{1-k+2l}}\langle f,\psi_{j-1,l}\rangle \qquad (4)$$

B. *Threshold Noise Reduction Theory*

The signal is s (n). After adding noise, the signal becomes f(n).

$$f(n) = s(n) + \delta e(n) \qquad (5)$$

Where δ is the noise intensity, e (n) is Gaussian white noise.The process of signal noise reduction is to process the noise signal f (n), reduce the intensity of the noise component e (n), and restore the process of the original signal s (n).The basic idea of Wavelet threshold de-noising method is to use Wavelet analysis theory to decompose the noisy signal. We can study the difference between the Wavelet coefficients formed by the signal and the noise in the dyed signal, and use the difference to estimate the noise area and set the threshold to filter the noise signal.The frequency distribution of the signal is relatively concentrated, and the Wavelet coefficients are also relatively concentrated after Wavelet transform.The noise signal is distributed in the entire frequency domain, and the high-frequency signal occupies the majority, so the Wavelet coefficient value of the noise after Wavelet transform is smaller.By selecting the appropriate threshold and filtering the Wavelet coefficients with smaller amplitude, the purpose of noise reduction can be accomplished.

The main steps of Wavelet threshold de-noising algorithm are as follows:

1）Selection of Wavelet basis function and decomposition layer number according to the characteristics of signal. Then we use Mallat algorithm to decompose it.
2）We select the threshold variable to determine the threshold function,and then carry out quantitative processing on the decomposed Wavelet coefficients by using the threshold function .
3)The Mallat reconstruction algorithm is used to complete the signal reconstruction by combining the Wavelet detail coefficients and the scale coefficients after processing.

III. WAVELET THRESHOLD DE-NOISING IMPROVED ALGORITHM

There are three main factors which affect the Wavelet threshold noise reduction[6]: The selection of Wavelet basis function, the selection of decomposition layer number (scale), the determination of threshold and threshold function.In this paper, a new type of threshold function is proposed by studying soft threshold, hard threshold and other threshold functions.This function has continuity and is closer to the hard threshold function, keeping more details of the signal.

The hard threshold function is shown as follows:

$$\tilde{t}_{j,k} = \begin{cases} t_{j,k} & |t_{j,k}| \ge \lambda \\ 0 & |t_{j,k}| < \lambda \end{cases} \tag{6}$$

The soft threshold function is shown as follows:

$$\tilde{t}_{j,k} = \begin{cases} t_{j,k} - \lambda \operatorname{sgn}(t_{j,k}) & |t_{j,k}| \ge \lambda \\ 0 & |t_{j,k}| < \lambda \end{cases} \tag{7}$$

where λ denotes threshold, $\operatorname{sgn}()$ denotes sign function.

The threshold function in document [7] is:

$$\tilde{t}_{jk} = \begin{cases} t_{jk} - 0.5 \cdot \lambda \cdot \operatorname{sgn}(t_{jk}) & |t_{jk}| \ge \lambda \\ 0.5 \cdot k \cdot \lambda \cdot \tan(\frac{\pi t_{jk}}{4\lambda}) & |t_{jk}| < \lambda \end{cases} \tag{8}$$

The improved threshold function in this paper is designed as:

$$\begin{cases} \tilde{t}_{jk} = \dfrac{t_{jk}\left|\dfrac{t_{jk}}{\lambda}\right|^{m}}{\left(\left|\dfrac{t_{jk}}{\lambda}\right| + \varepsilon\right) m!} & |t_{jk}| \ge \lambda \\ \tilde{t}_{jk} = \dfrac{t_{jk}}{|t_{jk}|}\sqrt{t_{jk}^{2} - \lambda^{2}} + \dfrac{t_{jk} \cdot \lambda}{|t_{jk}|(1+\varepsilon) \cdot m!} & |t_{jk}| < \lambda \end{cases} \tag{9}$$

where $\varepsilon \downarrow 0$, $m \in N^{+}$

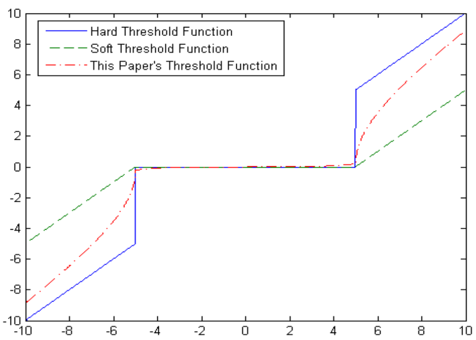

Fig.1 Comparison of the Soft, Hard and this paper's threshold functions

In this paper, the threshold function is used in the threshold region to suppress the upward trend of the exponential function. This allows the threshold function to be close to 0 in the threshold region, and a part of the high frequency signal can be retained. In this way, the signal integrity can be increased and the signal-to-noise ratio of the signal can be improved effectively. The function is closer to the hard threshold function outside the $[-\lambda, \lambda]$ region so that more high frequency Wavelet coefficients can be retained.At the same time , the function of this paper eliminates the discontinuity of the hard threshold function and eliminates the pseudo - Gibbs phenomenon .The variable coefficient m in this paper makes the use of the threshold function more flexible, so that it can choose whether to retain more Wavelet high frequency coefficients.

IV. ILLUSTRATIVE EXAMPLES

In the process of simulation, the normal distribution white noise sequence is added to the common sonar signal, and the signal is de-noised to verify the effectiveness of the proposed Wavelet de-noising algorithm.In figure 2 and figure 3, the noise reduction effect diagram is output, which directly reflects the result of noise reduction.Through the numerical calculation of signal signal-to-noise ratio (SNR) and root mean square error (RMSE), the validity of this algorithm is determined by more accurate measurement. The definitions of SNR and RMSE [8]are as follows:

$$SNR = 10 \cdot \log_{10}\left(\frac{\sum_{i=1}^{N} s_i^2}{\sum_{i=1}^{N}(f_i - s_i)^2}\right) \tag{10}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(f_i - s_i)^2} \tag{11}$$

where f is the signal after de-noising, and s is the original signal.From the definition of the formula, the SNR is the ratio of the original signal to the noise reduction signal on the power.The larger SNR and the smaller RMSE mean that the difference between the noise-reduced signal and the original signal is smaller, indicating that the better the noise reduction effect is.

In this paper, two typical sonar signals are selected as simulation signals.CW signal is the most common signal in sonar.Doppler signal is the simulation of sonar signal with Doppler translation.In the noise reduction simulation, the db4 Wavelet basis function is used in this paper.In the noise reduction simulation, we use db4 Wavelet basis function, and set the decomposition level to 4 layers. The same general threshold is used to compare the effect of each threshold de-noising algorithm.

The general threshold expression is as follows:

$$\lambda = \sigma\sqrt{2\ln(M)} \tag{12}$$

where M is the signal length , σ is the estimation of standard deviation of noise .

the standard deviation estimate is shown as follows :

$$\sigma = \frac{med(|d_1(k)|)}{0.6745} \tag{13}$$

where $d_1(k)$ is the detail factor after Wavelet decomposition, $med(\delta)$ means the median of δ .

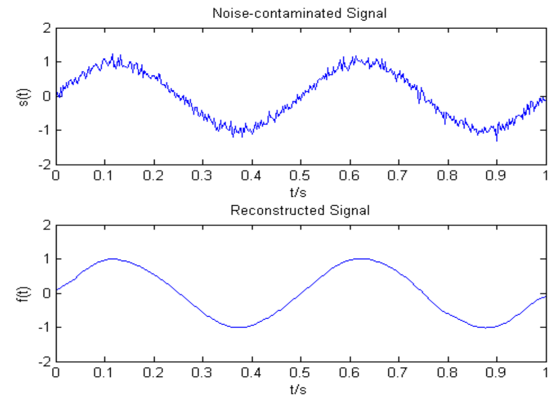

Fig. 2 Noise reduction effect of CW signal

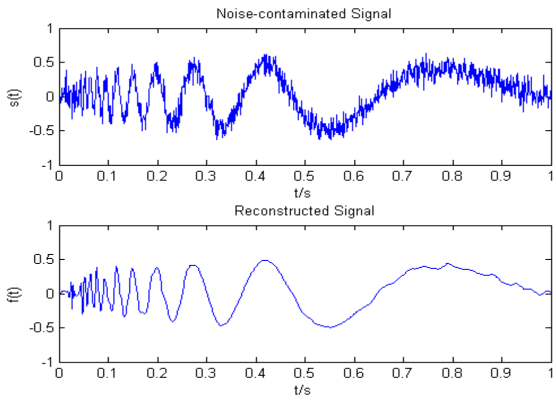

Fig. 3 Noise reduction effect of Doppler signal

The noise reduction results of the CW signal are compared as shown in the following table:

Threshold function	SNR	RMSE
Soft threshold function	24.7977	0.1219
Hard threshold function	22.7929	0.1536
The threshold function in Literature[7]	21.4797	0.1787
The threshold function when m =3	24.1057	0.1321
The threshold function when m=4	26.7491	0.0974

The results of the Doppler signal de-noising are compared to the following table:

Threshold function	SNR	RMSE
Soft threshold function	15.3627	1.5984
Hard threshold function	16.9152	1.3368
The threshold function in Literature[7]	16.8556	1.346
The threshold function when m =3	16.9116	1.3374
The threshold function when m =4	17.1204	1.3013

It can be seen from figs. 2 and 3 that the threshold function in this paper has better noise reduction effect in the process of sonar signal de-noising, and the reconstructed signal is more smooth. And the reconstructed signal has a high degree of similarity with the original signal.From the numerical comparison table, it can be seen that this paper's threshold function of the m=4 is better than the soft threshold, hard threshold and the threshold function in the literature [7].

V. Conclusion

The experimental results show that the improved threshold function proposed in this paper overcomes the pseudo-Gibbs phenomenon of hard threshold function and reserves more high frequency coefficients by designing the threshold function.Together with the advantages of a soft threshold function and a hard threshold function.It is proved that the threshold function of this paper has good flexibility and noise reduction in the noise reduction process of common sonar signals.

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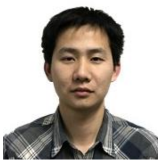

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