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Availability Analysis of a Markovian System with Preventive Maintenance

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***Abstract*— This paper deals with a Markovian queueing system, in which the system can fail either partially or completely. The partially failed system can fail completely, or still be operative during maximum operting time. After maximum operintig time, the partially failed system is shutdown for preventive maintenance. When completely failed, the system is repaired. The system works as new after preventive maintenance or repair. The steady-state availability and the expected profit is analized analytically.**

***Index Terms*— Availability, Markovian system, preventive maintenance, maximum operating time.**

I. INTRODUCTION

Several researchers have considered systems with server subject to breakdowns and repairs, making assumption that the normal system directly breaks down with a failure rate [1-7].

Kadyan [1] considered the profit analysis of a single-unit system, in which the system fails completely either directly from normal state or via partial failure. As numerical results, they derived the expressions of various performance measures with Markovian assuptions. The model considered in [1] is described as follows: There is a single-unit system, in which the system can fail either partially or completely. The partially failed system can fail completely, or still be operative during maximum operating time. After maximum operating time, the partially failed system is shutdown for preventive maintenance. When completely failed, the system is repaired. The system works as new after preventive maintenance or repair. For numerical results, Kadyan [1] assumed that the failure time from normal state to complete failure, the failure time from normal state to partial failure, the failure time from partial failure to complete failure, the repair time of the failed system, the maximum operating time after partial failure, and the preventive maintenance time of the system are all exponentially distributed with rate λ, λ_1, λ_2, θ, β, and α, respectively. They showed that the steady-state availability is given by

$$A_0 = \frac{\theta\beta(\alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1)}{[\theta\beta(\alpha\lambda + \lambda\lambda_2 + \alpha\lambda_1 + \lambda_1^2 + \lambda_1\lambda_2 + \lambda\lambda_1) + (\beta\lambda_1^2\lambda_2 + \alpha\theta\lambda_1^2 + \beta\lambda\lambda_1\lambda_2 + \alpha\theta\lambda\lambda_1)]} \quad (1)$$

However, there must be some errors in the above results: When $\lambda_1 = 0$, the system becomes a simple on-off system with failure rate λ and repair rate θ, where the steady state availability A_0 should be

$$A_0 = \frac{\theta}{\theta + \lambda}. \quad (2)$$

However, A_0 given by the above equation is 0.

In Section 2, the modified analysis are presented under Markovian assumption, and in Section 3, some numerical results are given.

II. ANALYSIS

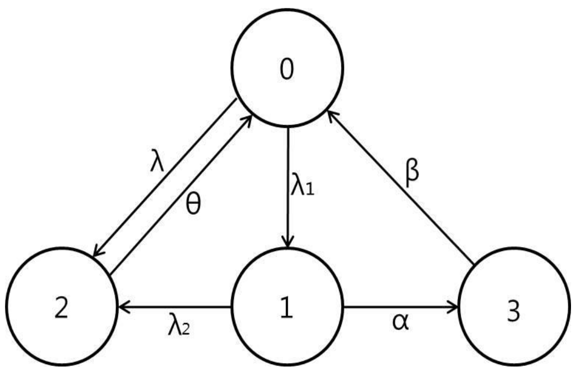

Figure 1. State transition diagram

Now consider probabilistic state transition diagram of the system described in previous section as shown in Figure 1, where state 0 is the normal state, state 1 the partially failed state, state 2 the completely failed state, and state 3 the preventive maintenance state. Solving the probability equations generated from this model, we get the following expressions for steady-state probabilities:

$$\pi_0 = \frac{1}{1 + \frac{\lambda}{\theta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \quad (3)$$

$$\pi_1 = \frac{\frac{\lambda_1}{\lambda_2 + \alpha}}{1 + \frac{\lambda}{\theta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \quad (4)$$

$$\pi_2 = \frac{\frac{\lambda}{\theta} + \frac{\lambda_2}{\theta}\frac{\lambda_1}{\lambda_2 + \alpha}}{1 + \frac{\lambda}{\theta} + \left(1 + \frac{\lambda_2}{\theta} + \frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2 + \alpha}}, \quad (5)$$

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α	$\lambda = .13$, $\lambda_1 = .17, \lambda_2 = .21$, $\theta = 2.1, \beta = 2.7$		$\lambda = .16$, $\lambda_1 = .17, \lambda_2 = .21$, $\theta = 2.1, \beta = 2.7$		$\lambda = .13$, $\lambda_1 = .20, \lambda_2 = .21$, $\theta = 2.1, \beta = 2.7$		$\lambda = .13$, $\lambda_1 = .17, \lambda_2 = .21$, $\theta = 2.6, \beta = 2.7$		$\lambda = .13$, $\lambda_1 = .17, \lambda_2 = .21$, $\theta = 2.1, \beta = 3.7$	
	our result	Ref. [1]	our result	Ref. [1]	our result	Ref. [1]	our result	Ref. [1]	our result	Ref. [1]
5	0.891564	0.546645	0.880701	0.499655	0.883569	0.578855	0.901317	0.546958	0.904315	0.554911
10	0.890324	0.540117	0.879323	0.492417	0.881995	0.571840	0.899959	0.540277	0.903512	0.548583
15	0.889891	0.537835	0.878842	0.489887	0.881444	0.569380	0.899485	0.537943	0.903231	0.546369
20	0.889671	0.536673	0.878597	0.488598	0.881163	0.568126	0.899244	0.536754	0.903088	0.545242
25	0.889537	0.535969	0.878449	0.487818	0.880992	0.567366	0.899098	0.536034	0.903001	0.544559
30	0.889448	0.535497	0.878349	0.487294	0.880878	0.566855	0.899000	0.535551	0.902943	0.544101
35	0.889383	0.535158	0.878278	0.486919	0.880796	0.566489	0.898929	0.535205	0.902902	0.543772
40	0.889335	0.534904	0.878224	0.486636	0.880735	0.566214	0.898876	0.534944	0.902870	0.543524
45	0.889297	0.534705	0.878183	0.486416	0.880687	0.565999	0.898835	0.534741	0.902846	0.543332
50	0.889267	0.534546	0.878149	0.486239	0.880648	0.565827	0.898802	0.534578	0.902826	0.543177

$$\pi_3 = \frac{\frac{\alpha}{\beta}\frac{\lambda_1}{\lambda_2+\alpha}}{1+\frac{\lambda}{\theta}+\left(1+\frac{\lambda_2}{\theta}+\frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2+\alpha}}, \quad (6)$$

The steady-state availability A is

$$A = \pi_0 + \pi_1 = \frac{1+\frac{\lambda_1}{\lambda_2+\alpha}}{1+\frac{\lambda}{\theta}+\left(1+\frac{\lambda_2}{\theta}+\frac{\alpha}{\beta}\right)\frac{\lambda_1}{\lambda_2+\alpha}}. \quad (7)$$

III. Numerical Examples

For numerical analysis, we present the steady-state availability of the system. The steady-state availability obtained in this paper is coincide with the availability in Table 3 of [1]. However, the numerical values in the table of [1] do not coincide with those derived from their corresponding equations in [1]. Table 1 presents the numerical results obtained in this paper as well as those derived from the corresponding equations in [1], which show that our results are largely different from those derived from the equations in [1].

IV. Conclusion

This paper dealt with a Markovian queueing system, in which the system can fail either partially or completely. The partially failed system can fail completely, or still be operative during maximum operating time. After maximum operating time, the partially failed system is shutdown for preventive maintenance. When completely failed, the system is repaired. The system works as new after preventive maintenance or repair. The steady-state availability were analyzed analytically and compared with that of [1].

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