# A Washout Filter Aided Design for the Stabilizing Control of Electric Power Systems

## Der-Cherng Liaw, Yun-Hua Huang

*Abstract*— A feedback stabilizing control law is proposed in this paper for the electric power systems to delay and/or eliminate the appearance of the so-called "voltage collapse." The phenomenon of voltage collapse is known to be possibly attributed to the occurrence of the saddle-node bifurcation or Andronov-Hopf bifurcation. Based on a previous study (Liaw et al, 2005), in this study a washout filter aided linear stabilizing control law is designed for the power systems to delay and/or eliminate the appearance of the bifurcation phenomena. Numerical simulations demonstrate the success of preventing the occurrence of voltage collapse by the proposed schemes.

Index Terms—power systems, voltage collapse, control

#### I. INTRODUCTION

In the recent years, the study of voltage collapse phenomena in electric power systems has attracted lots of attention [1]-[8]. The main concern is that the power systems are facing growing load demands but with little addition of the power generation and transmission facilities, which will then push the power systems to be operated near the stability limits. As the load demands become too heavy to offer, the magnitude of load voltage falls sharply to a very low level. Such a phenomena is referred as the so-called "voltage collapse." Among those existing studies, the occurrence of voltage collapse had been believed to be attributed to the existence of saddle-node bifurcation in electric power systems [1]-[4]. Based on the simple dynamical model proposed by Dobson and Chiang, et al [2]-[3], it had been shown that voltage collapse may arise from the existence of Andronov-Hopf bifurcation, which is prior to the appearance of saddle-node bifurcation [5]-[6]. The effect of tap changer ratio on the nonlinear behavior of an electric power system has been studied for the large scale electric power networks (e.g., [1], [7]). Based on the model of [3], an extra tap changer was proposed in [9] to be added in parallel to the nonlinear load of the power model for the bifurcation analysis of nonlinear dynamics for electric power system with respect to the variation of the tap changer ratio. Both of saddle-node bifurcation and Andronov-Hopf bifurcations were observed in [9] by treating the real power, reactive power and tap changer as system parameters, which make the appearance of static and dynamic voltage collapses, respectively. Those phenomena are found to generate a progressive decrease or a sharp change in load voltage magnitude of electric power system.

The Static Var Compensator (SVC) has been recently

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considered as a control actuator for improving system stability (e.g., [8], [10]). For instance, a washout filter-aided feedback control law was proposed in [8] to delay the occurrence of the system instability and/or voltage collapse while a sliding-mode based design was proposed to regulate the load voltage [10]. Instead of controlling the system behavior via the tuning of the SVC, in this paper we consider a different approach by using the ratio of the tap changer as the solely control to prevent and/or delay the occurrence of voltage collapse.

The organization of the paper is as follows. First, the model of the electric power systems given in [3] and [9] are recalled. It is followed by the design of the proposed control laws for an example system. Numerical simulations are then presented in Section IV to demonstrate the success of the proposed scheme. Finally, conclusion is given in Section V to highlight the major contributions and possible applications.

#### II. ELECTRIC POWER SYSTEM DYNAMICS

In this paper, we will focus on the control design for electric power systems by using the mathematical model proposed in [3] and tap changer as a solely control input. First, we recall the electric power system model from [3] in this section. It will then be used in Section III to develop the control law. As recalled from [3], we have the electric power system model as given by

$$\delta_m^{\mathbf{x}} = \omega_m \tag{1}$$

$$M \mathscr{A}_{m} = P_{m} - d_{m} \omega_{m}$$
<sup>(2)</sup>

$$+E_m^{-2}Y_m\sin\theta_m + E_m^{-2}Y_m^{-1}V\sin(\delta - \delta_m - \theta_m)$$

$$k_{qw} \delta = -k_{qv^2} V^2 - k_{qv} V + Q(\delta_m, \delta, V) - Q_0 - Q_1$$
(3)  
$$T k_{aw} k_{mv} \delta = k_{nw} k_{-2} V^2 + (k_{nw} k_{av} - k_{aw} k_{mv}) V$$

$$+k_{q\omega}(P(\delta_m, \delta, V) - P_0 - P_1)$$

$$-k_{p\omega}(Q(\delta_m, \delta, V) - Q_0 - Q_1)$$

$$(4)$$

where  $\delta_m$ ,  $\omega_m$ ,  $\delta$  and V denote the generator phase angle, the generator phase angle velocity, the phase angle of the load voltage and the load voltage, respectively, and the nonlinear PQ load are given as

$$P(\delta_{m}, \delta, V) = (Y_{0} '\sin \theta_{0} '+ Y_{m} \sin \theta_{m})V^{2}$$

$$-E_{m}Y_{m}V \sin(\delta - \delta_{m} + \theta_{m}) - E_{0} 'Y_{0} 'V \sin(\delta + \theta_{0} ')$$

$$Q(\delta_{m}, \delta, V) = -(Y_{0} '\cos \theta_{0} '+ Y_{m} \cos \theta_{m})V^{2}$$

$$+E_{m}Y_{m}V \cos(\delta - \delta_{m} + \theta_{m}) + E_{0} 'Y_{0} 'V \cos(\delta + \theta_{0} ')$$
(6)

with

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$$E_0' = E_0 \left[ 1 + C^2 Y_0^{-2} - 2C Y_0^{-1} \cos \theta_0 \right]^{-\frac{1}{2}}$$
(7)

$$Y_0' = Y_0 [1 + C^2 Y_0^{-2} - 2C Y_0^{-1} \cos \theta_0]^{\overline{2}}$$
(8)

$$\theta_0' = \theta_0 + \tan^{-1} \{ \frac{CY_0^{-1} \sin \theta_0}{1 - CY_0^{-1} \cos \theta_0} \}$$
(9)

Definitions of each system parameter and derivations of the model equations above can be referred to [2]-[3]. Based on the parameter values given in Table I, a bifurcation diagram was obtained via code AUTO [12] as shown in Fig. 1 for the electric power system (1)-(4) with  $P_1=0$ . Note that, the symbols of HB and SNB in Fig. 1 denote the Andronov-Hopf bifurcation and saddle-node bifurcation, respectively. In addition, the solid-line denotes the stable equilibrium point, while the dashed-line is unstable one. The corresponding values of the two bifurcation points are given in Table II. Nonlinear dynamical behaviors with different values of PQ-load around the bifurcation point HB were also obtained as shown in Figs. 2 and 3. It is observed from Fig. 2-3 that the system might exhibit chaos-like behavior for  $P_1 = 0$  and  $Q_1 = 2.98984$ .

Table I System parameter values

$K_{pw} = 0.4$ p.u.	$K_{pv} = 0.3$ p.u.	$K_{qw} = -0.03$ p.u.
$K_{qv} = -2.8$ p.u.	$K_{qv2} = 2.1$ p.u.	T = 8.5 p.u.
$P_0 = 0.6$ p.u.	$Q_0 = 1.3$ p.u.	M = 0.01464
$Y_0 = 3.33$ p.u.	$\theta_0 = 0$ deg.	$E_0 = 1.0$ p.u.
C = 3.5 p.u.	$Y_m = 5.0$ p.u.	$\theta_m = 0$ deg.
$E_m = 1.05$ p.u.	$P_m = 1.0$ p.u.	$d_m = 0.05$ p.u.



Fig. 1 Bifurcation diagram of open-loop system

Table II Location of bifurcation points

	$Q_{1}$ (p.u.)	$\delta_m$ (rad)	$\delta$ (rad)	V (p.u.)
HB	2.98021	0.267426	0.0475028	0.873124
SNB	3.02578	0.30292	0.0610163	0.795136



Fig. 2 Phase-diagram of open-loop system: (a)  $Q_1 = 2.98840$ , (b)  $Q_1 = 2.98920$ , (c)  $Q_1 = 2.98954$ , (d)  $Q_1 = 2.98984$ .



Fig. 3 Chaos alike timing response of open-loop system at  $Q_1 = 2.98984$ : (a)  $\delta_m$ , (b)  $\omega_m$ , (c)  $\delta$ , and (d) V.

## III. DESIGN OF CONTROL LAWS

As presented in [9], we have obtained a nonlinear analysis of bifurcation phenomena for a power system with tap changer as depicted in Fig. 4 below. In that study, numerical simulations demonstrate that the occurrence of subcritical Andronov-Hopf bifurcation or saddle-node bifurcation might lead to the voltage collapse of power system. In addition, it was also found that the Andronov-Hopf bifurcation occurs first as the PQ-load increases. Thus, the control of Hopf bifurcation becomes a major issue in the design of prevention of voltage collapse. In this study, we will focus on the design of control laws to eliminate and/or delay the occurrence of Hopf bifurcation as well as voltage collapse phenomena. Here, for practical implementation we only consider the tap changer ratio as the solely control input for the electric power systems.



Fig. 4 Schematic diagram of electric power system with tap changer

Let the control input u be defined as the difference of tap changer ratio, i.e.,  $u = 1/n - 1/n_0$ , where  $n_0$  denotes the nominal value of the tap changer. We can then adopt the modified model of electric power systems from [9] as given

## International Journal of Engineering and Applied Sciences (IJEAS) ISSN: 2394-3661, Volume-5, Issue-5, May 2018

below:

$$\delta_m^{\mathbf{X}} = \omega_m \tag{10}$$

$$H \partial_{m} = I_{m} - u_{m} \partial_{m} + E_{m} I_{m} \sin \theta_{m} + E_{m} Y_{m} V(n_{0} + u) \sin(\delta - \delta_{m} - \theta_{m})$$
(11)

$$k_{qw}\delta = -k_{qv^2}V^2 - k_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1$$
(12)

$$Tk_{qw}k_{pv}V^{\mathbf{X}} = k_{p\omega}k_{qv^{2}}V^{2} + (k_{p\omega}k_{qv} - k_{q\omega}k_{pv})V + k_{q\omega}[P(\delta_{m}, \delta, V) - P_{0} - P_{1}] - k_{p\omega}[Q(\delta_{m}, \delta, V) - Q_{0} - Q_{1}]$$
(13)

$$P(\delta_m, \delta, V) = [Y_0 '\sin \theta_0 ' + Y_m (n_0 ' + u)^2 \sin \theta_m] V^2$$
  
-  $E_m Y_m V(n_0 ' + u) \sin(\delta - \delta_m + \theta_m)$   
-  $E_0 'Y_0 ' V \sin(\delta + \theta_0 ')$  (14)

$$Q(\delta_m, \delta, V) = -[Y_0' \cos \theta_0' + Y_m (n_0' + u)^2 \cos \theta_m] V^2 + E_m Y_m V(n_0' + u) \cos(\delta - \delta_m + \theta_m) + E_0' Y_0' V \cos(\delta + \theta_0')$$
(15)

with  $n_0' = 1/n_0$ .

It is known that the existence and location of system equilibria depend on the given system dynamics. Thus, one may expect the system equilibria will be affected if a static type state feedback control law is applied. As discussed in [11], the advantages of using washout filter-aided control law include the preservation of system equilibria and automatic equilibrium following. Motivated by [11] and the design presented in [8], in the following we will design a washout filter-aided feedback control law to delay or eliminate the occurrence of Andronov-Hopf bifurcation. The major difference between the design proposed in this paper below and those in [8] is that the tap changer ratio is used as solely control instead of the SVC given in [8].

In the following discussion, we assume that the values of extra real power demand  $P_1$  is fixed but the values of the extra reactive power demand  $Q_1$  and tap changer ratio n can be varied. The parameter values listed in Table I is used in Eqs. (10)-(13) as an example system for control design. Let  $x_1 = \delta_m$ ,  $x_2 = \omega_m$ ,  $x_3 = \delta$ ,  $x_4 = V$  and  $x_5$  denote the washout state.

Follow the design in (e.g., [8], [11]), we take the output of the washout filter *y* as  $y = k_1x_1 - dx_5 + k_2x_2 + k_3x_3 + k_4x_4$ . From Eqs. (10)-(13) with washout filter dynamics, we then have the following five state equations:  $x_2 = x_2$ 

$$\begin{aligned} & & & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\$$

It is clear from the first four state equations in (16) above that the system equilibrium of the original power system will be the same for u = 0. But, there is an extra equilibrium value  $x_{50}$  for the washout filter state variable  $x_5$ , which is solvable by letting  $x_5 = 0$  and will be a linear function of the original equilibrium state variables.

Now, consider the tap changer control system (16) with the control input u = y. Let  $X_e = [x_{10}, x_{20}, x_{30}, x_{40}, x_{50}]^T$  be the equilibrium point of system (16), where  $[x_{10}, x_{20}, x_{30}, x_{40}]^T$  is the Andronov-Hopf bifurcation point HB as given in Table II. Note that, as discussed above the value of  $x_{50}$  can be solved from x = 0 and will be a linear function of  $x_{i0}$  for i = 1, ..., 4.

Let  $\hat{x} = X - X_e$  with  $X = [x_1, x_2, x_3, x_4, x_5]^T$ . Taking the linearization of (16) at  $X_e$ , we then have the following linearized model :

 $\hat{x} = A\hat{x} + Bu \tag{17}$  where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ a_{31} & 0 & a_{33} & a_{34} & 0 \\ a_{41} & 0 & a_{43} & a_{44} & 0 \\ k_1 & k_2 & k_3 & k_4 & -d \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ b_2 \\ b_3 \\ b_4 \\ 0 \end{bmatrix}.$$

with

$$\begin{split} a_{21} &= -358.607n_0 'x_{40} \cos(-x_{10} + x_{30}) \\ a_{22} &= -3.4153 \\ a_{23} &= 358.607n_0 'x_{40} \cos(-x_{10} + x_{30}) \\ a_{24} &= 358.607n_0 'x_{40} \sin(-x_{10} + x_{30}) \\ a_{31} &= -175.0n_0 'x_{40} \sin(-x_{10} + x_{30}) \\ a_{33} &= -35(3.17143x_{40} \sin(-x_{30}) + 5n_0 'x_{40} \sin(x_{10} - x_{30})) \\ a_{34} &= -35(2.66667 - 2(2.16190 + 4.7619n_0 '^2)x_{40} \\ &+ 3.17143\cos(x_{30}) + 5n_0 '\cos(x_{10} - x_{30}) \\ &+ 2n_0 'x_{40} \sin(x_{10} - x_{30})) \\ a_{43} &= -13.7225(-0.15n_0 'x_{40} \cos(x_{10} - x_{30})) \\ &+ 1.26885x_{40} \sin(x_{30}) - 2n_0 'x_{40} \sin(x_{10} - x_{30})) \\ &+ 1.26885x_{40} \sin(x_{30}) - 2n_0 'x_{40} \sin(x_{10} - x_{30})) \\ a_{44} &= -13.7225(-1.05833 + 1.72990x_{40} + 3.81036n_0 '^2 x_{40} \\ &- 1.26885\cos(x_{30}) + 2n_0 '\cos(x_{10} - x_{30}) \\ &- 0.09516\sin(-x_{30}) - 0.15n_0 '\sin(x_{10} - x_{30})) \\ b_2 &= 358.607x_{40} \sin(-x_{10} + x_{30}) \\ b_3 &= -35(-9.52381n_0 'x_{40}^2 + 5x_{40}\cos(x_{10} - x_{30})) \\ b_4 &= -13.7225(3.81035n_0 'x_{40}^2 - 2x_{40}\cos(x_{10} - x_{30})) \\ -0.15x_{40} \sin(x_{10} - x_{30})). \end{split}$$

The characteristic equation of the closed-loop system given in (17) is then calculated as

 $\lambda^{5} + (39.4335 + d + 68.3067k_{2} - 104.995k_{3} + 16.0776k_{4})\lambda^{4} + (560.216 + 39.4335d + 68.3067k_{1} - 30880.0k_{2} - 349.187k_{3} + 347.412k_{4})\lambda^{3} + (998.074 + 560.216d - 30880.0k_{1} - 11018.8k_{2} - 29773.4k_{3} + 5674.36k_{4})\lambda^{2} + (13542.8 + 998.074d - 11018.8k_{1} - 5355.16k_{3} + 33945.5k_{4})\lambda + (13542.8d) = 0.$ By applying the Partic Lurging the life origina to (19)

By applying the Routh-Hurwitz stability criterion to (18), we then have the following results.

**Lemma 1.** The equilibrium point HB of system (10)-(13) is asymptotically stabilizable by the washout filter aided control with  $u = -dx_5 + k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4$  if the following conditions hold :

- (i)  $39.4335 + d + 68.3067k_2 104.995k_3 + 16.0776k_4 > 0$
- (ii)  $(560.216 + 39.4335d + 68.3067k_1 30880.0k_2 349.187k_3 + 347.412k_4)(39.4335 + d + 68.3067k_2 104.995k_3 + 16.0776k_4) (998.074 + 560.216d 30880.0k_1 11018.8k_2 29773.4k_3 + 5674.36k_4) > 0,$
- $\begin{array}{ll} ({\rm iii}) & (998.074+560.216d-30880.0k_1-11018.8k_2\\ -29773.4k_3+5674.36k_4)((560.216+39.4335d\\ +68.3067k_1-30880.0k_2-349.187k_3+347.412k_4)\\ & (39.4335+d+68.3067k_2-104.995k_3+16.0776k_4)\\ -(998.074+560.216d-30880.0k_1-11018.8k_2\\ -29773.4k_3+5674.36k_4))-(39.4335+d+68.3067k_2\\ -104.995k_3+16.0776k_4)((13542.8+998.074d\\ -11018.8k_1-5355.16k_3+33945.5k_4)(39.4335+d\\ +68.3067k_2-104.995k_3+16.0776k_4)-13542.8d)>0, \end{array}$
- (iv)  $(13542.8d (998.074d 11018.8k_1 5355.16k_3 + 33945.5k_4 + 13542.8)(1.0d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335))(560.216d 30880.0k_1 11018.8k_2 29773.4k_3 + 5674.36k_4 + 998.074)(560.216d 30880.0k_1 11018.8k_2 29773.4k_3 + 5674.36k_4 (1.0d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335)(39.4335d + 68.3067k_1 30880.0k_2 349.187k_3 + 347.412k_4 + 560.216) + 998.074) (13542.8d (998.074d 11018.8k_1 5355.16k_3 + 33945.5k_4 + 13542.8) (d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335))^2(d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335))^2(d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335) 13542.8d (560.216d 30880.0k_1 11018.8k_2 29773.4k_3 + 5674.36k_4 (d + 68.3067k_2 104.995k_3 + 16.0776k_4 + 39.4335) (39.4335d + 68.3067k_1 30880.0k_2 349.187k_3 + 347.412k_4 + 560.216) + 998.074)^2 > 0, and$

(v) 13542.8d > 0.

It is clear from the condition (v) of Lemma 1 above that we need to have d > 0 for the design of stabilizing control laws. For simplicity and without loss of generality, let d = 1. Next result follows readily from Lemma 1 to relax the design of control laws.

**Corollary 1.** The equilibrium point HB of system (10)-(13) is guaranteed to be asymptotically stable by the control law of  $u = -x_5 + k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4$  if one of the following four conditions hold:

(i)  $-0.6252 < k_1 < 0$  and  $k_2 = k_3 = k_4 = 0$ ,

- (ii)  $-0.0398 < k_2 < 0$  and  $k_1 = k_3 = k_4 = 0$ ,
- (iii)  $k_3 < 0$  and  $k_1 = k_2 = k_4 = 0$ ,
- (iv)  $k_4 > 0$  and  $k_1 = k_2 = k_3 = 0$ .

## IV. NUMERICAL SIMULATIONS

In this paper, we choose  $n_0 = 1$  for numerical study. As presented in Lemma 1 and Corollary 1 above, the stability of system equilibria of (10)-(13) can only be guaranteed at which near the equilibrium point HB. In order to study the

non-local stability range of system equilibria with respect to

the variation of the PQ-load and the advantages of using washout filter aided control design, in the following we will study the effect of the control gain  $k_i$  for each i = 1,..,4 on the nonlocal system stability.

First, consider the condition (i) of Corollary 1. It is clear from the bifurcation diagram shown in Fig. 5 that the whole branch of system equilibria will be stabilized by choosing  $k_1 = -0.2$ . Time responses of the four system states for two typical examples at  $Q_1 = 2.98$  and  $Q_1 = 3$  are given in Figs. 6 and 7, respectively. It is observed from those figures that system states will be pushed to approach the desired system equilibrium by the applied control input. In addition, it is found that the system equilibrium branch shown in Fig. 5 is the same as those in Fig. 1 for the open-loop system. That demonstrates the system equilibria will not be changed by the applied washout filter aided control.

Next, we consider the condition (ii) of Corollary 1. As stated in the condition (ii) of Corollary 1, the feasible range of  $k_2$  for system stabilization is very small. As depicted in Fig. 8, the system equilibria can only be stabilized up to some extent of  $Q_1$  for  $k_2 = -0.02$  and  $k_2 = -0.03$ . That means the Andronov-Hopf bifurcation can only be delayed but not be eliminated by the condition (ii) of Corollary 1. Time responses of the four system states for two typical examples at  $Q_1 = 2.98$  and  $Q_1 = 2.985$  are given in Figs. 9 and 10, respectively. It is observed from those figures that system states will be finally approaching the desired system equilibrium by the applied control input.

Now, we consider another two conditions of Corollary 1. It is clear from the bifurcation diagram shown in Fig. 11 that the whole branch of system equilibria will be stabilized by choosing  $k_3 = -0.2$  or  $k_4 = 3$ . Time responses of the four system states for two typical examples at  $Q_1 = 2.98$  and  $Q_1 = 3$  with different control strategies are given in Figs. 12-15, respectively. It is observed from those figures that system states will be pushed to approach the desired system equilibrium by any of the four control schemes listed in Corollary 1 at  $Q_1 = 2.98$ . However, the system equilibrium will not be stabilized at  $Q_1 = 3$  by the condition (ii) of Corollary 1. Moreover, it is also observed from those figures that the control law from the condition (iv) of Corollary 1 will provide better timing performance with the expense of large magnitude of the control gain.



Fig. 5: Bifurcation diagram with respect to  $Q_1$  for  $k_1 = -0.2$ .

International Journal of Engineering and Applied Sciences (IJEAS) ISSN: 2394-3661, Volume-5, Issue-5, May 2018



Fig. 6: Time responses at  $Q_1 = 2.98$  for  $k_1 = -0.2$ .



Fig. 7: Time responses at  $Q_1 = 3$  for  $k_1 = -0.2$ .







Fig. 9: Time responses at  $Q_1 = 2.98$  for  $k_2 = -0.02$ .



Fig. 10: Time responses at  $Q_1 = 2.985$  for  $k_2 = -0.02$ .



Fig. 11: Effect of  $k_i$  for i = 3 and 4 on the bifurcation diagram: (a)  $k_3 = -0.2$  and (b)  $k_4 = 3$ .



Fig. 12: Comparison of time responses at  $Q_1 = 2.98$  for  $k_1 = -0.2$  and  $k_2 = -0.02$ .



Fig. 13: Comparison of timing responses at  $Q_1 = 2.98$  for  $k_3 = -0.2$  and  $k_4 = 3$ .



Fig. 14: Comparison of timing responses at  $Q_1 = 3$  for  $k_1 = -0.2$  and  $k_2 = -0.02$ .



Fig. 15: Comparison of timing responses at  $Q_1 = 3$  for  $k_3 = -0.2$  and  $k_4 = 3$ .

## V. CONCLUSION

In this paper, we focus on the washout filter aided stabilizing control design of bifurcation phenomena for a power system with tap changer. The simulations demonstrate the success of the proposed schemes. It was found that the Andronov-Hopf bifurcation can only be delayed up to some extent if the control input is a function of the washout state and the state  $\omega_m$ . In contrast, the Andronov-Hopf bifurcation can be eliminate by the control input with another three system states of the electric power system as feedback signals. That means the dynamic type of voltage collapse caused by the Andronov-Hopf bifurcation can be totally prevented via suitable choice of the control laws. Those finding might give a guide in the practical applications.

#### ACKNOWLEDGMENT

The authors would like to thank Mr. Hung-Tse Lee for helping the redo and check of numerical simulations as well as the editing of the manuscript.

## REFERENCES

- S. Abe, Y. Fukunaga, A. Isono and B. Kondo, "Power system voltage stability," IEEE Transactions on Power Apparatus and Systems, Vol. PAS-101, No. 10, pp. 3830–3840, 1982.
- [2] I. Dobson, H.-D. Chiang, J. S. Throp and L. Fekih-Ahmed, "A model of voltage collapse in electric power systems," Proc. 27th IEEE Conference on Decision and Control, Austin, Texas, pp. 2104-2109, Dec. 1988.
- [3] H.-D. Chiang, I. Dobson, R. J. Thomas, J. S. Throp and L. Fekih- Ahmed, "On voltage collapse in electric power systems," IEEE transactions on Power Systems, vol. 5, no. 2, pp. 601-611, 1990.
- [4] V. Ajjarapu and B. Lee, "Bifurcation theory and it's application to nonlinear dynamical phenomena in an electrical power system," IEEE Transactions on Power Systems, Vol. 7, No. 1, pp. 424–431, 1992.
- [5] H. O. Wang, E. H. Abed, and A. M. A. Hamdan, "Bifurcations, chaos, and crises in voltage collapse of a model power system," IEEE Transactions on Circuits and Systems–I: Fundamental Theory and Applications, Vol. 41, No. 3, pp. 294–302, 1994.
- [6] M. Yue and R. Schlueter, "Bifurcation subsystem and its application in power system analysis," IEEE Transactions on Power Systems, Vol. 19, No. 4, pp. 1885–1893, 2004.
- [7] C.-C. Liu and K. T. Vu, "Analysis of tap-changer dynamics and construction of voltage stability regions," IEEE Transactions on Circuits and Systems, Vol. 36, No. 4, pp. 575–589, 1989.
- [8] M. S. Saad, M. A. Hassouneh, E. H. Abed, and A. Edris, "Delaying instability and voltage collapse in power systems using svc's with washout filter-aided feedback," Proc. 2005 American Control Conf., pp. 4357-4362, Portland, June 8-10, 2005.
- [9] D.-C. Liaw, K.-H. Fang, and C.-C. Song, "Bifurcation Analysis of Power Systems with Tap Changer," Proc. 2005 IEEE International Conference on Networking, Sensing and Control (ICNSC'05), Tucson, Arizona, U.S.A., March 19-22, 2005.
- [10] D.-C. Liaw, S.-T. Chang, and Y.-H. Huang, "Voltage tracking design for electric power systems via SMC approach," Proc. 48th IEEE Conference on Decision and Control, 28th Chinese Control Conference, Shanghai, China, pp. 7854–7859, Dec. 2009.
- [11] H.-C. Lee, Robust Control of Bifurcating Nonlinear Systems with Applications, Ph.D. Dissertation, Dept. of Elec. Engr., Univ. of Maryland, College Park, MD, USA, 1991.
- [12] E. Doedel, AUTO 86 User Manual, Computer Science Dept., Soncordia Univ., Jan. 1986.

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