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HOLOGRAPHY FOR ROTATING BLACK HOLES

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Abstract

Holographic principle and its realization in string theory through AdS/CFT correspondence has given us a new way to study the theory of quantum gravity. Shortly, holography in this context is understood as a duality between a gravitational theory in $D+1$ dimensions with a field theory with no gravity living on the boundary with D dimensions. More precisely, the statement of AdS/CFT correspondence is the duality between a gravitational theory in $D+1$ dimensional anti-de Sitter (AdS) space and a conformal field theory (CFT) on the boundary of AdS with D dimensions. In the same spirit of holography, Kerr/CFT correspondence states that the physics of rotating black holes is dual to a two dimensional conformal field theory on a hypersurface near the black hole's horizon. In this paper, I review the Kerr/CFT correspondence. In this regard, some basic materials about black holes and conformal field theory that are needed to understand Kerr/CFT correspondence are presented.

Keywords: konformal symmetry, black holes

Abstrak

Prinsip holografi dan realisasinya dalam teori string melalui korespondensi AdS/CFT telah memberikan cara pandang baru dalam mempelajari teori kuantum gravitasi. Secara ringkas, holografi dalam konteks ini diartikan sebagai dualitas antara teori gravitasi di ruangwaktu berdimensi $D+1$ dengan teori medan tanpa gravitasi di daerah batas dari ruangwaktu ini dengan dimensi D. Secara lebih tepat, pernyataan korespondensi AdS/CFT yaitu dualitas antara teori gravitasi di ruang anti-de Sitter (AdS) berdimensi $D+1$ dengan teori medan conformal (CFT) di ruang batas AdS ini yang berdimensi D. Seiring dengan semangat holografi, korespondensi Kerr/CFT menyatakan bahwa fisika lubang hitam berotasi bersifat dual kepada teori medan konformal dua dimensi yang berada pada sebuah irisan ruangwaktu dekat dengan horison lubang hitam terkait. Dalam makalah ini, saya mengulas kembali korespondensi Kerr/CFT. Terkait dengan hal ini, disajikan beberapa materi dasar tentang lubang hitam dan teori medan konformal yang diperlukan dalam memahami korespondensi Kerr/CFT.

Kata Kunci: holografi, simetri konformal, lubang hitam.

1. Extremal Kerr/CFT

1.1. Introduction

Quite recently several theorists [1, 2, 3, 4] proposed a new correspondence, still in the same spirit with AdS/CFT duality [5, 6, 7], namely Kerr/CFT correspondence. This Kerr/CFT correspondence is considered to be more *down to the earth*, since it is related to the general astrophysical rotating black holes that exist in the sky, even the near extremal ones. Unlike AdS/CFT, the Kerr/CFT does not need any extra dimension(s) since it was constructed purely from Einsteinian gravity whose duals are 2 dimensional CFTs. This type of correspondence had also been extended to some more general cases of black holes, for example the extremal Kerr-AdS metrics in 4, 5, 6, and 7 dimensions [8], and also for Kerr-Sen black holes [9]. In this section we will give a review of this correspondence, restricted to the case of extremal Kerr black holes.

1.2. The NHEK geometry

NHEK in the title is an abbreviation for "Near Horizon Extreme Kerr", hence our discussion in this section is restricted to the case of extremal Kerr black holes only. The Kerr solution in Boyer-Lindquist coordinates is

$$ds^2 = -\frac{\Delta}{\rho}\left(d\hat{t} - a\sin^2\theta d\hat{\phi}\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(\left(\hat{r}^2 + a^2\right)d\hat{\phi} - ad\hat{t}\right)^2 + \frac{\rho^2}{\Delta}d\hat{r}^2 + \rho^2 d\hat{\theta}^2. \qquad (1)$$

where $\Delta \equiv \hat{r}^2 - 2M\hat{r} + a^2$ and $\rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta$. The mass and rotational parameter of black holes are denoted by M and a. The angular momentum black hole is $J = Ma$. We are using the system where Newton's gravitational constant is unity, i.e. $G = 1$.

We would like to study the very near region to horizon of an extreme Kerr black hole (1). This region has been studied by Bardeen and Horowitz [10]. The starting point is defining new (dimensionless) coordinates

$$t = \frac{\lambda\hat{t}}{2M}\,, \qquad y = \frac{\lambda M}{\hat{r} - M}\,, \qquad \phi = \hat{\phi} - \frac{\hat{t}}{2M} \qquad (2)$$

and take $\lambda \to 0$ that keep (t, y, ϕ, θ) fixed. The outcome is the near-horizon extreme Kerr (NHEK) geometry

$$ds^2 = 2J\Omega^2\left(\frac{-dt^2 + dy^2}{y^2} + d\theta^2 + \Lambda^2(d\phi + \frac{dt}{y})^2\right) \qquad (3)$$

where

$$\Omega^2 \equiv \frac{1 + \cos^2\theta}{2}\,, \qquad \Lambda \equiv \frac{2\sin\theta}{1 + \cos^2\theta}, \qquad (4)$$

$\phi \sim \phi + 2\pi$ and $0 \leq \theta \leq \pi$. This NHEK geometry is not asymptotically flat, which means taking a very large y doesn't make the metric reduces to a Minkowski form. In fact, the metric (3) covers only part of the NHEK geometry which can be seen for example from the definition of t when we take the limit $\lambda \to 0$. We can have the global coordinates (r, τ, φ) by the following transformations

$$
\begin{aligned}
y &= \left(\cos\tau\sqrt{1+r^2}+r\right)^{-1}, & (5)\\
t &= y\sin\tau\sqrt{1+r^2}, & (6)\\
\phi &= \varphi+\ln\left(\frac{\cos\tau+r\sin\tau}{1+\sin\tau\sqrt{1+r^2}}\right). & (7)
\end{aligned}
$$

The transformations above yields (3) transforms to

$$
d\bar{s}^2 = 2J\Omega^2\left(-(1+r^2)d\tau^2+\frac{dr^2}{1+r^2}+d\theta^2+\Lambda^2(d\varphi+rd\tau)^2\right). \tag{8}
$$

The Killing vectors of isometry group for (8) is $SL(2,\mathbb{R})\times U(1)$. The $U(1)$ is generated by

$$
\zeta_0 = -\partial_\varphi. \tag{9}
$$

and the $SL(2,\mathbb{R})$ isometry group generated by the Killing vectors

$$
\begin{aligned}
\tilde{J}_0 &= 2\partial_\tau, & (10)\\
\tilde{J}_1 &= 2\sin\tau\frac{r}{\sqrt{1+r^2}}\partial_\tau-2\cos\tau\sqrt{1+r^2}\partial_r+\frac{2\sin\tau}{\sqrt{1+r^2}}\partial_\varphi, & (11)\\
\tilde{J}_2 &= -2\cos\tau\frac{r}{\sqrt{1+r^2}}\partial_\tau-2\sin\tau\sqrt{1+r^2}\partial_r-\frac{2\cos\tau}{\sqrt{1+r^2}}\partial_\varphi. & (12)
\end{aligned}
$$

The $SL(2,\mathbb{R})$ algebra for the last three generators above is

$$
\left[\bar{J}_0,\bar{J}_1\right]=-2\bar{J}_2 \quad , \quad \left[\bar{J}_0,\bar{J}_2\right]=2\bar{J}_1 \quad , \quad \left[\bar{J}_1,\bar{J}_2\right]=2\bar{J}_0. \tag{13}
$$

1.3. The Asymptotic Symmetry Group

As one can observe the metrics (3) or (8) have the AdS-like structure, hence it is natural to guess that the asymptotic symmetry group proposed by Brown et al [11] could work for NHEK spacetime. The fact that NHEK is not asymptotically flat allows us to impose an appropriate and consitent boundary condition when $r \to \infty$. It is clear that there are an infinite numbers of boundary condition that can be choosen, but different condition connects to different physics. For each boundary condition we pick up, there is an associated asymptotic symmetry group (ASG) which is defined as the set of allowed diffeomorphism modulo the set of trivial diffeomorphism,

$$
\text{ASG} = \frac{\text{Allowed Symmetry Transformations}}{\text{Trivial Symmetry Transformations}}. \tag{14}
$$

Here ‘allowed’ means the transformation that is consistent with the specified boundary conditions, and ‘trivial’ means the generator of transformations which vanishes after we have imposed the constraints and reduced it to a boundary integral.

To determine the allowed difeomorphisms, we need to specify a boundary condition by assigning the appropriate $p \in \mathbb{Z}$ in each component of NHEK deviation metric $h_{\mu\nu} = \mathcal{O}(r^p)$. We choose the boundary conditions

$$\begin{pmatrix} h_{\tau\tau} = \mathcal{O}(r^2) & h_{\tau\varphi} = \mathcal{O}(1) & h_{\tau\theta} = \mathcal{O}(\frac{1}{r}) & h_{\tau r} = \mathcal{O}(\frac{1}{r^2}) \\ h_{\varphi\tau} = h_{\tau\varphi} & h_{\varphi\varphi} = O(1) & h_{\varphi\theta} = \mathcal{O}(\frac{1}{r}) & h_{\varphi r} = \mathcal{O}(\frac{1}{r}) \\ h_{\theta\tau} = h_{\tau\theta} & h_{\theta\varphi} = h_{\varphi\theta} & h_{\theta\theta} = \mathcal{O}(\frac{1}{r}) & h_{\theta r} = \mathcal{O}(\frac{1}{r^2}) \\ h_{r\tau} = h_{\tau r} & h_{r\varphi} = h_{\varphi r} & h_{r\theta} = h_{\theta r} & h_{rr} = \mathcal{O}(\frac{1}{r^3}) \end{pmatrix} , \tag{15}$$

Correspondingly, the most general diffeomorphisms which preserve the boundary conditions (15) are of the form

$$\xi = [-r\epsilon'(\varphi) + O(1)]\partial_r + [C + O(\frac{1}{r^3})]\partial_\tau + [\epsilon(\varphi) + O(\frac{1}{r^2})]\partial_\varphi + O(\frac{1}{r})\partial_\theta \tag{16}$$

where $\epsilon(\varphi)$ is an arbitrary smooth function, and C is an arbitrary constant. The subleading terms correspond to trivial diffeomorphisms and the leading term

$$\zeta_\epsilon = \epsilon(\varphi)\partial_\varphi - r\epsilon'(\varphi)\partial_r \ . \tag{17}$$

is the ASG of NHEK. By the periodicity $\varphi \sim \varphi + 2\pi$, it is convenient to define $\epsilon_n(\varphi) = -e^{-in\varphi}$ and $\zeta_n = \zeta(\epsilon_n)$. Under Lie brackets[1], these symmetry generators obey the Virasoro algebra

$$i[\zeta_m, \zeta_n]_{L.B.} = (m-n)\zeta_{m+n} \ . \tag{18}$$

Note that ζ_0 generates the $U(1)$ rotational isometry.

1.4. Generators

For every diffeomorphism ξ there is an associated conserved charge Q_ξ which under Dirac bracket with a field Φ yields

$$\{Q_\xi, \Phi\}_{DB} = \mathcal{L}_\xi \Phi. \tag{19}$$

The Dirac bracket is an extension of Poisson bracket[2] that accomodates the local symmetries as well as the constraints related. If we have a number of constraints C_i, then a Dirac bracket between two functions is defined as

$$\{A, B\}_{DB} = \{A, B\}_{PB} - \{A, C_i\}_{PB} \{C_i, C_j\}^{-1}_{PB} \{C_j, B\}_{PB} . \tag{20}$$

From (19), the Dirac bracket between 2 conserved charges could be written as

$$\{Q_\zeta, Q_\xi\}_{DB} = Q_{[\zeta,\xi]} + c_{\zeta\xi} \tag{21}$$

where $c_{\zeta\xi}$ would be a central charge, and $\{Q_\zeta, c_{\zeta\xi}\}_{DB} = 0$.

[1] $[X, Y] = \mathcal{L}_X Y - \mathcal{L}_Y X$.

[2] $\{Q, P\}_{PB} = \sum_{i=1}^{N} \left[\frac{\partial Q}{\partial q_i} \frac{\partial P}{\partial p_i} - \frac{\partial Q}{\partial p_i} \frac{\partial P}{\partial q_i} \right]$ where q_i and p_i are the canonical coordinates in phase space.

The infinitesimal charge differences between neighboring geometries $g_{\mu\nu}$ and $g_{\mu\nu} + h_{\mu\nu}$ are given by

$$\delta Q_\zeta[g] = \frac{1}{8\pi G} \int_{\partial\Sigma} k_\zeta[h, g] \tag{22}$$

where the integral is over the boundary of a spatial slice and

$$\begin{aligned} k_\zeta[h, g] \;\; = \;\; & -\frac{1}{4}\epsilon_{\alpha\beta\mu\nu}\big[\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2} h D^\nu \zeta^\mu \\ & -h^{\nu\sigma} D_\sigma \zeta^\mu + \frac{1}{2} h^{\sigma\nu}(D^\mu \zeta_\sigma + D_\sigma \zeta^\mu)\big] dx^\alpha \wedge dx^\beta \,. \end{aligned} \tag{23}$$

Covariant derivatives D_μ and raising indices are performed by using global NHEK metric tensor (8) $\bar{g}_{\mu\nu}$.

1.5. Central Charge

If we assume $h = \mathcal{L}_\xi \bar{g}$, then from (23) we can have an explicit form for the central charge

$$c_{\zeta\xi} = \frac{1}{8\pi} \int_{\partial\Sigma} k_\zeta \left[\mathcal{L}_\xi \bar{g}, \bar{g}\right]. \tag{24}$$

Hence, the Dirac bracket algebra of the asymptotic symmetry group is obtained as

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{D.B.} = Q_{[\zeta_m, \zeta_n]} + \frac{1}{8\pi G} \int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n} \bar{g}, \bar{g}]. \tag{25}$$

The Lie derivatives for NHEK geometry are

$$\mathcal{L}_{\zeta_n} \bar{g}_{\tau\tau} = 4GJ\Omega^2(1 - \Lambda^2) r^2 i n e^{-in\varphi}, \tag{26}$$

$$\mathcal{L}_{\zeta_n} \bar{g}_{r\varphi} = -\frac{2GJ\Omega^2 r}{1 + r^2} n^2 e^{-in\varphi}, \tag{27}$$

$$\mathcal{L}_{\zeta_n} \bar{g}_{\varphi\varphi} = 4GJ\Lambda^2\Omega^2 i n e^{-in\varphi}, \tag{28}$$

$$\mathcal{L}_{\zeta_n} \bar{g}_{rr} = -\frac{4GJ\Omega^2}{(1 + r^2)^2} i n e^{-in\varphi} \tag{29}$$

It follows that

$$\frac{1}{8\pi G} \int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n} \bar{g}, \bar{g}] = -i(m^3 + 2m)\delta_{m+n} J. \tag{30}$$

Now we define the dimensionless quantum versions of the Qs by

$$\hbar L_n \equiv Q_{\zeta_n} + \frac{3J}{2}\delta_n, \tag{31}$$

together with the usual rule when we go from classical to quantum physics, the Dirac brackets change to commutators as $\{.,.\}_{D.B.} \to -\frac{i}{\hbar}[.,.]$. The quantum version of charge algebra can be written as

$$[L_m, L_n] = (m - n) L_{m+n} + \frac{J}{\hbar} m(m^2 - 1)\delta_{m+n,0}. \tag{32}$$

Hence, the central charge for extreme Kerr black holes can be read off as

$$c_L = \frac{12J}{\hbar} \ . \tag{33}$$

For GRS 1915+105[3], this gives $c_L = (2 \pm 1) \times 10^{79}$, with some uncertainties from the measured black hole mass uncertainty [1].

1.6. Temperature

Unlike the Schwarzschild black hole, Kerr black hole doesn't have everywhere timelike Killing vector. Consequently we cannot employ Hartle-Hawking vacuum [12] to define the corresponding vacuum near to Kerr black hole's horizon. In regard to this problem, Frolov and Throne [13] defined a suitable vacuum for near horizon of Kerr black holes by using the Killing vector $\partial_{\hat{t}} - \Omega_H \partial_{\hat{\phi}}$.

To construct the the Frolov-Thorne vacuum for generic Kerr starts by expanding the quantum fields in eigenmodes of the asymptotic energy ω and angular momentum m. As an example we could write an expansion for scalar field Φ as

$$\Phi = \sum_{\omega,m,l} \phi_{\omega m l} e^{-i\omega\hat{t}+im\hat{\phi}} f_l(r,\theta). \tag{34}$$

After we trace over the region inside of the horizon, the vacuum is a diagonal density matrix in the energy-angular momentum eigenbasis with a Boltzmann weighting factor

$$e^{-\hbar\frac{\omega-\Omega_H m}{T_H}}. \tag{35}$$

In the non-rotating case, $\Omega_H = 0$, (35) reduces to the Hartle-Hawking vacuum.

A procedure to take the limit of near horizon region and near extremal black hole [1] allows us to have

$$e^{-i\omega\hat{t}+im\hat{\phi}} = e^{-\frac{i}{\lambda}(2M\omega-m)t+im\phi} = e^{-in_R t+in_L\phi}, \tag{36}$$

where

$$n_L \equiv m, \qquad n_R \equiv \frac{1}{\lambda}(2M\omega - m) \tag{37}$$

are the left and right charges associated to ∂_ϕ and ∂_t in the near-horizon region. In terms of these variables the Boltzmann factor (35) is

$$e^{-\hbar\frac{\omega-\Omega_H m}{T_H}} = e^{-\frac{n_L}{T_L}-\frac{n_R}{T_R}}, \tag{38}$$

where the dimensionless left and right temperatures are

$$T_L = \frac{r_+ - M}{2\pi(r_+ - a)}, \qquad T_R = \frac{r_+ - M}{2\pi\lambda r_+}. \tag{39}$$

In the case of extremal limit $a \to M$, (39) reduce to

$$T_L = \frac{1}{2\pi}, \qquad T_R = 0. \tag{40}$$

[3]GRS 1915+105 or V1487 Aquilae is an X-ray binary star system which shows a regular star and a black hole.

1.7. Microscopic origin of the Bekenstein-Hawking-Kerr entropy

In the previous subsection, only one copy of the tempertures which is non-zero, i.e. T_L. Correspondingly, the associate central charge c_L would be $12J$. To get the corresponding black hole entropy via CFT description, we employ the famous Cardy formula

$$S_{CFT} = 2\pi\sqrt{\frac{c_L E}{6}}, \tag{41}$$

where E is the energy. The first law of thermodynamics dictates that $dE = TdS$, so we could have

$$dS_{CFT} = 2\pi\sqrt{\frac{c_L}{6}}\frac{dE}{\sqrt{E}} = 2\pi\sqrt{\frac{c_L}{6}}\frac{T}{2\sqrt{E}}dS_{CFT} \tag{42}$$

which provides us

$$E = \frac{c_L}{6}\pi^2 T^2. \tag{43}$$

Hence we can write the alternative form of Cardy formula as

$$S_{CFT} = \frac{1}{3}\pi^2 c_L T_L, \tag{44}$$

after we plug the corresponding T_L rahter than T in the last expression. Having in our hand $T_L = 1/2\pi$ then (44) gives us

$$S_{CFT} = 2\pi J \tag{45}$$

which is exactly what we have for Bekenstein-Hawking entropy of Kerr black holes[].

2. Non-Extremal Kerr/CFT

2.1. Introduction

In the previous section, the Kerr/CFT correspondence is the one which applies for extremal case only. That approach does not work to a general Kerr black hole. The near horizon geometry of a non-extremal Kerr black hole is Rindler space, which is not known associated to a CFT. The work by Castro, Maloney, and Strominger [14] reveals the $SL(2,\mathbb{R})$ symmetry of the solution space for the Klein-Gordon equation in Kerr bacground in some physical limits. This kind of Kerr/CFT correspondence had also been applied for the more general rotating black holes [15, 16] or even the non rotating case [17].

2.2. Wave equation and $SL(2,\mathbb{R})$ Casimir

We start by writing a general form of our scalar wave solution which can be decomposed as

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t + im\phi} R(r) S(\theta). \tag{46}$$

Plugging this ansatz into Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right)=0 \tag{47}$$

gives us the angular part of the equation

$$\left[\frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta)-\frac{m^2}{\sin^2\theta}+\omega^2a^2\cos^2\theta\right]S(\theta)=-K_lS(\theta), \tag{48}$$

and the radial one

$$\left[\partial_r(\Delta\partial_r)+\frac{(2Mr_+\omega-am)^2}{(r-r_+)(r_+-r_-)}-\frac{(2Mr_-\omega-am)^2}{(r-r_-)(r_+-r_-)}+(r^2+2M(r+2M))\omega^2\right]R(r)=K_lR(r). \tag{49}$$

Both of these equations above can be solved by Heun functions.

However we can restrict our discussion to the low frequencies only, i.e. $M\omega\ll 1$, hence the last term in the square bracket of (49) can be neglected. Then the spacetime in our discussion can be divided into 2 regions,

$$\text{“Near”}:\quad r\ll\frac{1}{\omega}, \tag{50}$$

$$\text{“Far”}:\quad r\gg M. \tag{51}$$

Expression (50) means that the wavelength of our test particle is very large compared to the radius of curvature, and (51) indicates the very far region from black hole horizon, i.e. a large number multiple of M. This two regions overlap in the matching region

$$M\ll r\ll\omega^{-1}. \tag{52}$$

The wave equation (49) can be solved both in the near and far regions by using some special functions. However, to get a full solution, one need to match the obtained solutions in the near and far regions along a surface in the matching region (52).

In the near region, the angular equation (48) for the low frequency of scalar field reduces to

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right)-\frac{m^2}{\sin^2\theta}\right]S\left(\theta\right)=-K_lS\left(\theta\right)\quad,\quad l=-m,...,+m \tag{53}$$

This is just the standard Laplacian on 2-sphere, where the separation constant is $K_l=l\left(l+1\right)$. Moreover the radial one (49) becomes

$$\left[\partial_r(\Delta\partial_r)+\frac{(2Mr_+\omega-am)^2}{(r-r_+)(r_+-r_-)}-\frac{(2Mr_-\omega-am)^2}{(r-r_-)(r_+-r_-)}\right]R(r)=l(l+1)\,R(r), \tag{54}$$

which can be solved analytically and the solutions are hypergeometric functions.

2.3. Hidden Conformal Symmetry

To show the $SL(2,\mathbb{R})$ symmetry of the solution space for radial equation (54), first we introduce the following coordinates

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_R \phi}\ ,\ w^- = \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_L \phi - t/2M}\ ,\ y = \sqrt{\frac{r-r_+}{r-r_-}}\, e^{\pi(T_R+T_L)\phi - t/4M} \quad (55)$$

where

$$T_L = \frac{r_+ + r_-}{4\pi a}, \quad T_R = \frac{r_+ - r_-}{4\pi a}. \quad (56)$$

Accordingly, we can define the vector fields

$$\begin{aligned} H_1 &= i\partial_+ & \bar{H}_1 &= i\partial_- \\ H_0 &= i\left(w^+\partial_+ + \frac{1}{2}y\partial_y\right) & \bar{H}_0 &= i\left(w^-\partial_- + \frac{1}{2}y\partial_y\right) \\ H_{-1} &= i\left(\left(w^+\right)^2\partial_+ + w^+ y\partial_y - y^2\partial_-\right) & \bar{H}_{-1} &= i\left(\left(w^-\right)^2\partial_- + w^- y\partial_y - y^2\partial_+\right) \end{aligned} \quad (57)$$

which obey the algebra $SL(2,\mathbb{R})$ group

$$[H_0, H_{\pm 1}] = \mp i H_{\pm 1}, \quad [H_{-1}, H_1] = -2iH_0, \quad (58)$$

and similarly for $\bar{H}_0$ and $\bar{H}_{\pm 1}$. In (t,r,θ,ϕ) coordinates, the generators of $SL(2,\mathbb{R})$ above can be written as

$$\begin{aligned} H_{\pm 1} &= ie^{\mp 2\pi T_R \phi}\left(\pm\Delta^{1/2}\partial_r + \frac{1}{2\pi T_R}\left(\frac{r-M}{\Delta^{1/2}}\right)\partial_\phi + \frac{2T_L}{T_R}\left(\frac{Mr-a^2}{\Delta^{1/2}}\right)\partial_t\right) \\ H_0 &= \frac{i}{2\pi T_R}\partial_\phi + 2iM\frac{T_L}{T_R}\partial_t \end{aligned} \quad (59)$$

and

$$\begin{aligned} \bar{H}_{\pm 1} &= ie^{\mp 2\pi T_L \phi + \frac{t}{2M}}\left(\pm\Delta^{1/2}\partial_r - \frac{a}{\Delta^{1/2}}\partial_\phi - 2M\frac{r}{\Delta^{1/2}}\partial_t\right) \\ \bar{H}_0 &= 2iM\partial_t \end{aligned} \quad (60)$$

In group theory we know about the Casimir operator, i.e. the operator that commutes with all generators of the group. For this $SL(2,\mathbb{R})$ algebra, the quadratic Casimir in (t,r,θ,ϕ) variables by using (59) to (60) is

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = \partial_r(\Delta\partial_r) + \frac{(2Mr_+\omega - am)^2}{(r-r_+)(r_+-r_-)} - \frac{(2Mr_-\omega - am)^2}{(r-r_-)(r_+-r_-)}\,. \quad (61)$$

It is interesting to notice that equation (54) for $\Phi(r)$ in the near region can be written as

$$\mathcal{H}^2\Phi = \bar{\mathcal{H}}^2\Phi = l(l+1)\Phi. \quad (62)$$

Under the periodic identification

$$\phi \sim \phi + 2\pi. \quad (63)$$

the conformal coordinates behave as

$$w^+ \sim e^{4\pi^2 T_R} w^+ \quad , \quad w^- \sim e^{4\pi^2 T_L} w^- \quad , \quad y \sim e^{2\pi^2 (T_R + T_L)} y. \tag{64}$$

This identification is generated by the element of $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ group

$$g = e^{-4\pi^2 i T_R H_0 - 4\pi^2 i T_L \bar{H}_0} \tag{65}$$

which is exactly the form of the identification for a CFT partition function at finite temperature (T_L, T_R). So the $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ symmetry is spontaneously broken into $U(1)_L \times U(1)_R$ subgroup generated by $(\bar{H}_0, H_0)$ for $\phi \sim \phi + 2\pi$. We note that the symmetry group $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ is acting on the solution space of the wave equation, not in the background geometry.

2.4. CFT temperature and entropy

At fixed radius, from (55) we can write the relation between the conformal coordinates (w^+, w^-) and Boyer-Lindquist coordinates (ϕ, t) is

$$w^\pm = e^{\pm t^\pm}, \tag{66}$$

with

$$\begin{aligned} t^+ &= 2\pi T_R \phi \\ t^- &= \frac{t}{2M} - 2\pi T_L \phi \end{aligned} \tag{67}$$

This is precisely the relation between Minkowski $(w^\pm)$ and Rindler $(t^\pm)$ coordinates.

Under the periodic identification of $\phi \sim \phi + 2\pi$, the Rindler coordinates will have the identifications

$$t^+ \sim t^+ + 4\pi^2 T_R \quad , \quad t^- \sim t^- - 4\pi^2 T_L. \tag{68}$$

Observing from Minkowski vacuum by tracing over the quantum state, we will get a thermal density matrix at temperature (T_L, T_R). Hence Kerr black holes should be dual to a finite temperature (T_L, T_R) mixed state in the dual CFT.

From the extrmal Kerr discussion, we have obtain the value of central charge is $12J$. Assuming this central charge can also be used in general Kerr black holes, so we have $c_L = c_R = 12J$, thus by using Cardy formula

$$S_{CFT} = \frac{\pi^2}{3} \left(c_L T_L + c_R T_R \right), \tag{69}$$

together with the CFT left and right temperatures (56), the CFT entropy is

$$S_{CFT} = 2\pi M r_+ = S_{BH}. \tag{70}$$

3. Summary

In this review article, we have described the idea of Kerr/CFT correspondence, both for the extremal and non extremal cases. Some basic materials on Kerr black holes in general relativity, as well as the corresponding thermodynamical aspects, and also some concepts in conformal field theory are provided in the earlier sections. This is done to give the reader some background materials that are needed to grasp the idea of Kerr/CFT correspondence. In order to motivate the reader about the importance of duality[4] nowadays, we also provide a very brief introduction to the famous AdS/CFT correspondence or duality. The duality between CFT and extremal Kerr in section (1.) is shown by following the ASG method proposed in [11]. The geometry of NHEK has $SL(2,\mathbb{R})$ symmetry that points to conformal symmetry. By using the ASG method, we obtain the corresponding central charge for NHEK geometry which is $12J$. By using Cardy formula for entropy in CFT, we can recover the Bekenstein-Hawking entropy for Kerr black holes.

Unlike the extremal case, the conformal symmetry in general (non-extremal) Kerr geometry is found from the solution space of the corresponding Klein-Gordon equation. It was shown in section (2.) that the radial equation in some limits is just the squared Casimir of $SL(2,\mathbb{R})$ working on the wave function with separation constant $l(l+1)$ as the eigen value. The central charge can't be derived in this geometry, but by assuming that it has no difference with the one from extremal case, again by using Cardy formula we can also recover the Bekenstein-Hawking entropy for Kerr black hole. Moreover, we can also recover the absorption cross section of a near region scalar field in Kerr background from CFT point of view which matches with the classical analysis which was done by Starobinsky [18] long time ago.

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