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Fuzzy Graphs With Equal Fuzzy Domination and Independent Domination Numbers

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Abstract—**The basic definitions of fuzzy independent set, fuzzy dominating set,and fuzzy independent dominating sets are discussed. The aim of this paper is to find on what conditions the fuzzy graph has equal domination number and independent domination number. It is discussed briefly and also when the fuzzy graph is domination perfect is proved. Finally, the independent domination number for a connected fuzzy graph is obtained.**

Keywords: **fuzzy dominating set, fuzzy independent dominating set, fuzzy domination and independent domination perfect, connected fuzzy graph.AMS Subject Classification 2010: 03E72, 05C69, 05C72**

I. INTRODUCTION

Rosenfeld [11] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Nagoorgani and Chandrasekaran [8] discussed domination in fuzzy graph using strong arcs. Nagoorgani and Vadivel [9] discussed fuzzy independent dominating sets.In this paper we discuss when the fuzzy graph has equal domination and independent domination number and when it is domination perfect. The necessary definitions are given and explained with examples. Some fuzzy graphs also compared with the crisp case.

1. PRELIMINARIES

A fuzzy subset of a nonempty set V is a mapping $\sigma : V \to [0,1]$. A fuzzy relation on V is a fuzzy subset of V x V. A fuzzy graph $G = (\sigma,\mu)$ is a pair of function $\sigma: V\to [0,1]$ and $\mu : V \times V \to[0,1]$, where $\mu (u, v) \leq\sigma(u) \wedge\sigma(v)$ for all $u, v \in V$. The underlying crisp graph of $G = (\sigma,\mu)$ is denoted by $G^*= (V,E)$, where $V= \{u\in V: \sigma(u) > 0\}$ and $E = \{(u, v) \in V \times V: \mu(u, v) >0\}$. The order p and size q of fuzzy graph $G=(\sigma,\mu)$ are defined by $p = \sum_{v\in V}\sigma(v)$ and $q = \sum_{(u,v)\in E} \mu(u, v)$.The graph $G = (\sigma,\mu)$ is denoted by G , if unless otherwise mentioned.

The strength of connectedness between two nodes u, v in a fuzzy graph G is $\mu^\infty (u, v) = \sup \{\mu^k(u, v) : k= 1,2,3,\ldots\}$ where $\{\mu^k(u, v) = \sup \{\mu(u,u_1) \wedge\mu(u_1, u_2) \wedge\ldots \wedge\mu(u_{k-1}, v)\}$. An arc (u, v) is said to be a strong arc if $\mu (u, v) \geq\mu^\infty (u, v)$ and the node v is said to be a strong neighbor of u. If $\mu (u, v) = 0$ for every $v \in V$, then u is called isolated node.

Let u be a node in fuzzy graph G then $N (u) = \{v: (u, v)$ is a strong arc$\}$ is called neighborhood of u and $N[u] =N(u)\cup\{u\}$ is called closed neighborhood of u. Neighborhood degree of the node is defined by the sum of the weights of the strong neighbor node of u and is denoted by $d_N(u)=\sum_{v\in V}\sigma(v)$.Minimum neighborhood degree of a fuzzy graph G is defined by $\delta_N(G)=\min\{d_N(u): u\in V(G)\}$and maximum neighborhood degree of G is by $\Delta_N(G) = \max\{d_N(u): u\in V(G)\}$

II. FUZZY INDEPENDENT SET

Definition 3.1: Let $G = (\sigma,\mu)$ be a fuzzy graph. Two nodes in a fuzzy graph G are said to be fuzzy independent if there is no strong arc between them. A subset S of V is said to be fuzzy independent set for G if every two nodes of S are fuzzy independent.

Definition 3.2: Let $G = (\sigma,\mu)$ be a fuzzy graph. A fuzzy independent set S of G is said to be maximal fuzzy independent set if there is no fuzzy independent set whose cardinality is greater than the cardinality of S.

The maximum cardinality among all maximal fuzzy independent set is called fuzzy independence number of G and it is denoted by $\beta(G)$.

2. FUZZY DOMINATING SET

Definition 4.1: Let $G = (\sigma,\mu)$ be a fuzzy graph. A subset D of V is said to be a dominating set of G if for every $v \in V- D$, there exists a $u \in D$ such that u dominates v.

Definition4.2: A dominating set D of a fuzzy graph G is called minimal dominating set of G if there does not exist any dominating set of G, whose cardinality is less than the cardinality of D.

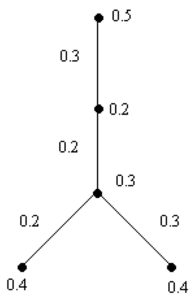
Minimum cardinality among all minimum dominating set in G is called domination number of G is denoted by $\gamma(G)$. The smallest cardinality of all independent fuzzy dominating set of G is called independent fuzzy domination number of G and is denoted by i(G).

Definition4.3: Let G = (σ,μ) be a fuzzy graph such that its crisp graph is a cycle, then G is called a fuzzy cycle if there does not exists a unique arc (x, y) such that $\mu(x,y) = \Lambda \{\mu(u,v); (u,v) > 0\}$.

3. EQUAL FUZZY DOMINATION AND INDEPENDENT DOMINATION NUMBERS

Proposition5.1: Let G = (σ, μ) be a fuzzy graph. Let D be a dominating set with the domination number $\gamma(G)$ and i(G) denotes the independent domination number. Then clearly $\gamma(G) \leq i(G)$.

Example 5.2:



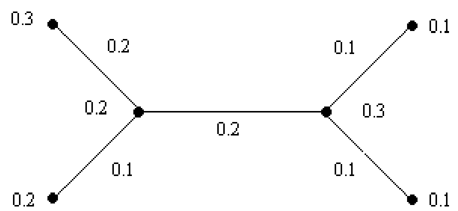
Here $\gamma(G) = 0.3 + 0.2 = 0.5$; $i (G) = 0.3 + 0.5 = 0.8$ and $\gamma(G) \leq i (G)$.

Theorem 5.3: Let G = (σ, μ) be a fuzzy graph. Let D be a minimum dominating set with the domination number $\gamma(G)$.The subgraph<D> induced by D has isolated nodes (i.e) $\mu(u, v) = 0$ for all u,v ε D then $\gamma(G) = i(G)$ where i(G) denotes the independent domination number.

Proof: It is clear from the definition that the minimum dominating set D is the smallest dominating set among all minimal dominating sets.Since the subgraph induced with the nodes of D are isolated implies that they are independent. Hence $\gamma(G) = i(G)$.

In comparing to the crisp case,$\gamma(G) = i(G)$ ifthe graph G is claw free but that is not required for fuzzy graph. Explain this concept in the example given below.

Example5.4:

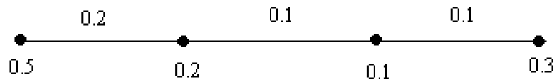


Here $\gamma(G) = 0.2 + 0.1 + 0.1 = 0.4 = i (G)$. But in crisp case $\gamma(G) = 2$ and $i (G) =3$

Corollary 5.5: Let G = (σ, μ) be a fuzzy line graph. If the subgraph induced by D has isolated nodes then $\gamma(L(G)) = i(L(G))$.

In the crisp case $\gamma(L(G)) = i(L(G))$ for any graph G.

Example 5.6:



Here $\gamma(L(G)) =0.2 +0.1 = 0.3$; $i(L(G)) = 0.2 +0.3 = 0.5$. Hence $\gamma(L(G)) \neq i(L(G))$.

Corollary 5.7: If G = (σ, μ) is a complete fuzzy graph then $i(G) < \gamma(G)$.

Proof:Since G is a complete fuzzy graph every arc in G is a strong arc. Hence i(G) =0 and
$\gamma(G) = \Lambda\{\sigma(v)$; for all v ε V$\}$ and i(G) =0 . It is clear that $i(G) < \gamma(G)$.

Theorem5.8 : Let G = (σ, μ) be a fuzzy graph. Let D be a fuzzy dominating set with domination number $\gamma(G)$ and i(G) is the independent dominating set. If the subgraph<D> induced by D has some arcs between the nodes in D. Then $\gamma(G) = i(G)$ if the arcs between any two nodes in D must have a fuzzy cycle with nodes in V-D.

Proof: Given that all the nodes in the induced subgraph<D> are not isolated nodes. Some nodes in D are connected. Assume the contrary that if the arcs between any two nodes in D does not have a fuzzy cycle with nodes in V-D. Then the arcs between those two nodes will be a strong arc and they cannot be independent. Therefore we get $\gamma(G) \neq i(G)$. Hence the theorem.

Definition 5.9: Let G = (σ, μ) be a fuzzy graph. The fuzzy graph G is fuzzy domination perfect if $\gamma(H) = i(H)$ for every induced subgraph H of G.

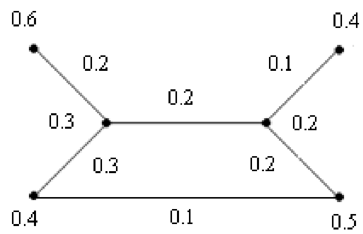
Theorem 5.10: Let G = (σ, μ) be fuzzy graph . Let D be be a dominating set with domination number $\gamma(G)$ and i(G) is the independent dominating set. Then G is said to be fuzzy domination perfect if it satisfies the following conditions

(i) The subgraph<D> induced by D has isolated nodes (i.e) $\mu(u, v) = 0$ for all u,v ε D.

(ii) G does not have a fuzzy cycle in it.

Proof:If the condition (i) is true then by theorem 5.3 we know $\gamma(G) = i(G)$ (i.e) the domination number and independent domination number are equal. To prove G is fuzzy domination perfect we have to prove the condition (ii).Assume the contrary that G has a fuzzy cycle in it. Then $\gamma(G)$ and i(G) differs for the induced subgraph containing the fuzzy cycle. Hence it cannot be domination perfect. So G cannot have a fuzzy cycle.

Example 5.11:



In this example $\gamma(G) = 0.3 + 0.2 = 0.5$; $i(G) = 0.3 + 0.2 = 0.5$. Since the arc between 0.3 and 0.2 is not strong arc

Definition 5.12: A fuzzy graph $G = (\sigma, \mu)$ is said to be connected if there exists a strongest path between any two nodes of G.

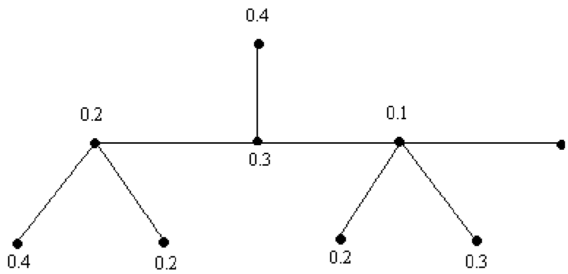
Definition 5.13: Let u be a node in fuzzy graph G then $N(u) = \{v : (u, v)$ is a strong arc$\}$ is called neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u. Neighborhood degree of the node is defined by the sum of the weights of the strong neighbor node of u and is denoted by $d_N(u) = \sum_{v \in N(u)} \sigma(v)$

Theorem 5.14:Let $G = (\sigma, \mu)$ be connected fuzzy graph which does not have an induced fuzzy cycle subgraph.Let D be a minimum dominating set with domination number $\gamma(G)$ .Then $i(G) = \beta(Y) + \sum_{u \varepsilon D-Y} d(N(U))$ where Y is maximal independent set of D.

Proof: The nodes in the dominating set D is also connected since the fuzzy graph G is connected. It shows that $\gamma(G) \neq i(G)$. Let Y denote the maximal independent set in D and its cardinality is $\beta(Y)$.

The nodes in V-D are independent if not they would have an induced fuzzy cycle which contradicts our assumption. Since the nodes in D-Y are not independent its corresponding neighbors are independent. Hence $i(G) = \beta(Y) + \sum_{u \varepsilon D-Y} d(N(U))$.

Example 5.15:



In this example $\gamma(G) = 0.2 + 0.3 + 0.1 = 0.6$; $i(G) = 0.2 + 0.1 + 0.4 = 0.7$.

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