Analysis of Unsteady Squeezing Flow Between Two Porous Plates With Variable Magnetic Field.

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Abstract

Analysis will be made for the non-isothermal Newtonian fluid flow between two unsteady squeezing porous plates under the influence of variable magnetic field. The similarity transformations will be used to transform the partial differential equations into nonlinear coupled ordinary differential equations. The modeled nonlinear differential equations representing the flow behavior in the geometry under consideration will be investigated using analytical and numerical method. Comparison of the solutions will be made. Convergence of solution will also be discussed. Flow behavior under the influence of non-dimensional parameters will be discussed with the help of graphical aids.

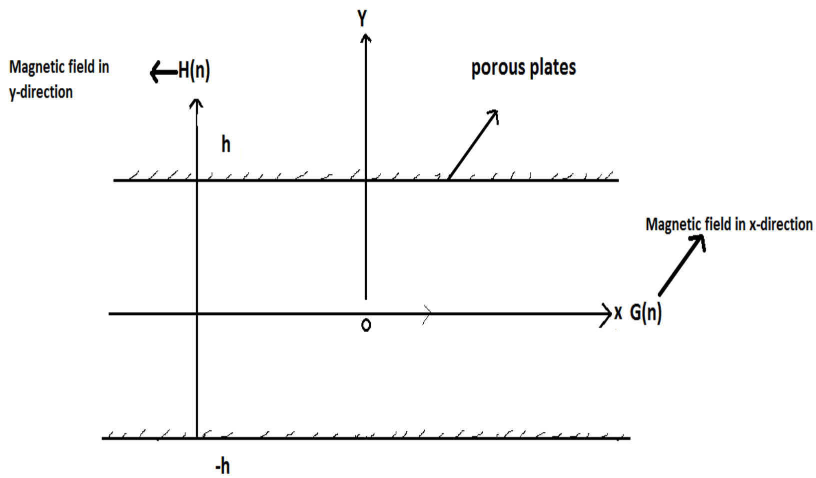
Keywords: porous walls, HAM, Permeation Reynolds number, Non dimensional wall dilation rate, Differential transform method , Magnetic field.

1 Introduction

Many mechanical equipment work under the principle of moving pistons where two plates exhibit the squeezing movement normal to their own surfaces. Electric motors, engines and hydraulic lifters also have this clutching flow in some of their parts. Due to this practical significance squeezing flow between parallel plates has become one of the most active research fields in fluid mechanics. Its biological applications are also of equal importance. Flow inside syringes and nasogastric tubes is also a kind of squeezing flows. Foundational work regarding squeezing flows was performed by Stefan [1] in 1874, who present basic formulation of these types of flows under lubrication assumption. After this numbers of scientists have shown their interests toward squeezing flows and have carried out many scientific studies to understand these flows. Some of selected contributions are mentioned in forthcoming lines.in 1886 Reynolds [2] investigated the squeezing flow between elliptic plates. Later in Archibald [3] 1956 considered the same problem for rectangular plates. After that several attempts have been made by different researchers to understand squeezing flows in a better way. In 1876 R J Grimm discussed The steady axisymmetric squeezing flow of incompressible Newtonian fluid passing through a porous medium. The resulting non-linear boundary value problem is then solved through HPM moreover squeezing flow between parallel disks has also presented in which Homotopy analysis method (HAM) has been employed to obtain analytical solution to the problem [4-9]. Earlier studies on squeezing flows were based on Reynolds equation whose insufficiency for some cases has been shown by Jackson in 1962 and Usha and Sridharan in 1996 [10-11]. More flexible and useful similarity transformation are available due to the efforts of birkhoff [12] he discussed MHD squeezing flow and the effects of involved parameters on velocity with graphical aids. Young et al used investigated the squeezing flow problem for axisymmetric and two-dimensional flows. These similarity transforms reduce the Navier-Stokes equation into a fourth order nonlinear ordinary differential equation and have further been used in some other investigations as well most of the real world problems are inherently in the form of non linearities [15-19]. Rashidi in 2008 studied the problem of laminar, isothermal, incompressible, and viscous ?ow in a rectangular domain bounded by two moving porous walls using the homotopy analysis method [20]. Over the years much attention has been devoted to develop new efficient analytical techniques that can

*The author to whom all the correspondence should be addressed. Email:mmrehan79@yahoo.com cope up with such nonlinearities. Several approximation techniques have been developed to fulfill this purpose [21-30]. Flow of electrically conducting non-Newtonian fluid is a very important phenomenon as in most of the practical situations we have to deal with the flow of conducting fluid which exhibits different behaviors under the influence of magnetic forces. In these cases magneto hydro dynamic (MHD) aspect of the flow is also needed to be considered. Homotopy perturbation solution for Two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al [31]. Domairry and Aziz (2009) [32] investigated the same problem for the flow between parallel disksHe investigate the combined effect of inertia, electromagnetic forces, and suction or injection. With the introduction of a similarity transformation, the continuity and momentum equations governing the squeeze flow are reduced to a single, nonlinear, ordinary differential equation. Recently, Mustafa et al [33] in 2012 examined heat and mass transfer for squeezing flow between parallel plates using the homotopy analysis method (HAM). It is clear from his literature survey that the squeezing flow of a Newtonian fluid between the plates moving normal to their own surface is yet to be inspected. Due to the inherent nonlinearity of the equations governing the fluid flow exact solutions are very rare. Even where they are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays. Rashidi in 2008 studied the problem of laminar, isothermal, incompressible, and viscous ?ow in a rectangular domain bounded by two moving porous walls using the homotopy analysis method [34]. We will extend the work of Rashidi. We will include temperature distribution and variable magnetic field. Constant magnetic field is simple and not so complicated because it only contributed a constant value with velocity equation but the variable magnetic field separately gives two more equations.

2 Formulation and Governing Equations



Consider two dimensional flow of an unsteady viscous incompressible electrically conducting fluid between two parallel plates. The upper plate is kept at temperature T_1 and lower is at T_0. At time $t = 0^+$ the upper plate is squeezing towards the lower one with a velocity a velocity $\frac{dh(t)}{dt}$ where h(t) is the separation between the plates. Coordinate system is selected at the surface of the lower plate in which x-axis is taken along the plane of plates and the y-axis is taken normal to this plane. Assuming the velocity, temperature and magnetic field are of the form $\vec{u}$=$\vec{u}(u(x, y, t), v(x, y, t))$, T=T(x,y,t) and B=B(x,y,t). In this analysis it is assumed that the flow is laminar, non isothermal and the edge effects are negligible.

The governing magnetohydrodynamic (MHD) equations of motion in the x-y plane by making use of assumptions, velocity, temperature and magnetic fields are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) - \frac{\sigma B_0}{\rho}(uB_o + vb) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + \frac{\sigma b}{\rho}(uB_o - vb) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \tag{4}$$

$$\frac{\partial b}{\partial t} - \delta B_0\frac{\partial u}{\partial y} - \delta u\frac{\partial B_o}{\partial y} + \delta b\frac{\partial v}{\partial y} + \delta v\frac{\partial b}{\partial y} - \frac{1}{\delta \mu_e}\frac{\partial^2 b}{\partial y^2} = 0 \tag{5}$$

$$\frac{\partial B_o}{\partial t} - \delta B_0\frac{\partial u}{\partial x} - \delta b\frac{\partial v}{\partial x} = -\frac{1}{\delta \mu_e}\frac{\partial^2 B_0}{\partial y^2} - \delta v\frac{\partial b}{\partial y} \tag{6}$$

where P is the pressure, δ the electrical conductivity, ρ the fluid density, μ_e the permeability of the medium, υ the kinematic viscosity and B_0 is the magnetic field. Boundary conditions are

$$\tilde{u} = 0, \quad \tilde{v} = -V_w = -\frac{\dot{a}}{c}, \ T = 0 \ at \ \tilde{y} = h(t), \tag{7}$$

$$\tilde{u} = 0, \quad \tilde{v} = 0, \ T = T0(1 - e^{-\eta t}) \ at \ y = 0. \tag{8}$$

Introducing the following similarity transformations

$$\theta_\eta = \frac{T - T_o}{T_1 - T_o}, \tag{9}$$

$$b = \frac{\alpha x G_\eta}{2(1 - \alpha t)}, \tag{10}$$

$$B_o = \frac{\alpha l H_\eta}{2(1 - \alpha t)^{\frac{1}{2}}}, \tag{11}$$

$$u = \frac{\alpha x F'_\eta}{2(1 - \alpha t)}, \tag{12}$$

$$v = -\frac{\alpha l F_\eta}{2(1 - \alpha t)^{\frac{1}{2}}}, \tag{13}$$

where

$$\eta = \frac{y}{l(1 - \alpha t)^{\frac{1}{2}}}. \tag{14}$$

Equation (3.1) satisfies identically and the set of equations (3.2-3.6) takes the following form

$$S(F'''\eta + 3F'' + F'F'' - FF''') = A(2F'GH + 2FG^2) + F'''' - A(2F'HH' + H^2F' + F'HG + FHG' + FH'G), \tag{15}$$

$$2\theta'' - F\theta' l^2 - \theta'\eta l^2 = 0, \tag{16}$$

$$G'\eta - 2G - \delta HF'' - \delta F'H' - \delta F'H' - \delta GF' - FG' - G''M = 0, \tag{17}$$

$$H'\eta + H - HF' + H''Q = 0, \tag{18}$$

where $S = \alpha l^2/2v$ is the non-dimensional Squeeze number, $A = \frac{\alpha^2 l^4}{4\nu}$, $M = \frac{2}{\alpha l^2 \delta \mu_e}$ and $Q = \frac{2}{\delta \mu_e \alpha}$ are the non-dimensional parameters. And the boundary condition given in equations (7,8) reduces to

$$F(0) = 0, F(1) = 1, F'(1) = 0, F''(0) = 0, \tag{19}$$

$$\theta(0) = 1, \theta(1) = 0, \tag{20}$$

$$G(0) = 0, G(1) = 1, \tag{21}$$

$$H(0) = 0, H(1) = 1. \tag{22}$$

3 Solution by Homotopy Analysis Method

By the HAM method, the functions $F(z)$, $\theta(z)$, $G(z)$, $H(z)$ are define as

$$F_m(z) = \sum_{n=0}^{\infty}\sum_{n=0}^{\infty} a_{m,n}^k z^k exp(-nz), \tag{23}$$

$$\theta_m(z) = \sum_{n=0}^{\infty}\sum_{n=0}^{\infty} c_{m,n}^k z^k exp(-nz), \tag{24}$$

$$G_m(z) = \sum_{n=0}^{\infty}\sum_{n=0}^{\infty} b_{m,n}^k z^k exp(-nz), \tag{25}$$

$$H_m(z) = \sum_{n=0}^{\infty}\sum_{n=0}^{\infty} d_{m,n}^k z^k exp(-nz), \tag{26}$$

where $a_{m,n}^k z^k$, $c_{m,n}^k z^k$, $b_{m,n}^k z^k$, $d_{m,n}^k z^k$ are the coefficients to be determined. Initial guesses and auxiliary linear operator are chosen as follows

$$F_0(z) = \frac{1}{2}(3z - z^3),\ \ \theta_0(z) = z,\ \ G_0(z) = 1 - z,\ \ H_0(z) = z, \tag{27}$$

$$\begin{aligned} &L_F = \frac{\partial^4 F}{\partial z^4},\ L_G = \frac{\partial^2 G}{\partial z^2},\ L_\theta = \frac{\partial^2 \theta}{\partial z^2}, \\ &L_H = \frac{\partial^2 H}{\partial z^2}\ \ , \end{aligned} \tag{28}$$

the above auxiliary linear operators have the following properties

$$\begin{aligned} &L_F(c_1 + c_2 z + c_3 z^2 + c_4 z^3) = 0, \\ &L_\theta(c_5 + c_6 z^2) = 0, \\ &L_G(c_7 + c_8 z^2) = 0, \\ &L_H(c_9 + c_{10} z^2) = 0, \end{aligned} \tag{29}$$

where $c_i(i = 1 - 14)$ are arbitrary constants.

The zeroth order deformation problems can be obtain as

$$(1-q)L_F[\widehat{F}(z;q) - F_0(z)] = qh_F N_F[\widehat{F}(z;q)], \tag{30}$$

$$(1-q)L_\theta[\widehat{\theta}(z;q) - \theta_0(z)] = qh_\theta N_\theta[\widehat{F}(z;q), \widehat{G}(z;q), \widehat{\theta}(z;q)], \tag{31}$$

$$(1-q)L_G[\widehat{G}(z;q) - G_0(z)] = qh_G N_G[\widehat{F}(z;q), \widehat{G}(z;q)], \tag{32}$$

$$(1-q)L_H[\widehat{H}(z;q) - H_0(z)] = qh_H N_H[\widehat{F}(z;q), \widehat{G}(z;q), \widehat{\theta}(z;q), \widehat{H}(z;q)], \tag{33}$$

$$
\begin{aligned}
N_F[\widehat{F}(z;q)] = {} & S(\eta\frac{\partial^3\widehat{F}(z;q)}{\partial z^3} + 3\frac{\partial^2\widehat{F}(z;q)}{\partial z^2} + \frac{\partial\widehat{F}(z;q)}{\partial z}\frac{\partial^2\widehat{F}(z;q)}{\partial z^2})\widehat{F}(z;q)\frac{\partial^3\widehat{F}(z;q)}{\partial z^3}) \\
& - A(\frac{2\partial\widehat{F}(z;q)}{\partial z}G(z;q)H(z;q) + 2F(z;q)G^2(z;q)) + F(z;q)\frac{\partial^3\widehat{F}(z;q)}{\partial z^3} \\
& - A(2\frac{\partial\widehat{F}(z;q)}{\partial z}G(z;q)H(z;q) + F(z;q)(G(z;q))^2)\frac{\partial^4\widehat{F}(z;q)}{\partial z^4} \\
& + A(2\frac{\partial\widehat{F}(z;q)}{\partial z}H(z;q)\frac{\partial\widehat{H}(z;q)}{\partial z} + H^2(z;q)\frac{\partial\widehat{F}(z;q)}{\partial z} \\
& + \frac{\partial\widehat{F}(z;q)}{\partial z}H(z;q)G(z;q) + F(z;q)H(z;q)\frac{\partial\widehat{G}(z;q)}{\partial z} \\
& + F(z;q)\frac{\partial\widehat{H}(z;q)}{\partial z}G(z;q), \qquad (34)
\end{aligned}
$$

$$
N_\theta[\widehat{\theta}(z;q)] = 2\frac{\partial\widehat{\theta}(z;q)}{\partial z} - F\frac{\partial\widehat{\theta}(z;q)}{\partial z}l^2 - \frac{\partial\widehat{\theta}(z;q)}{\partial z}\eta l^2, \qquad (35)
$$

$$
\begin{aligned}
N_G[\widehat{G}(z;q)] = {} & \frac{\eta\partial\widehat{G}(z;q)}{\partial z} - 2G(z;q) - \delta H(z;q)\frac{\partial^2\widehat{F}(z;q)}{\partial z^2} - 2\delta\frac{\partial\widehat{F}(z;q)}{\partial z}\frac{\partial\widehat{H}(z;q)}{\partial z} \\
& - \delta G\frac{\partial\widehat{F}(z;q)}{\partial z} - F\frac{\partial\widehat{G}(z;q)}{\partial z} - \frac{\partial^2\widehat{G}(z;q)}{\partial z^2}M, \qquad (36)
\end{aligned}
$$

$$
N_H[\widehat{H}(z;q)] = \eta\frac{\partial^2\widehat{H}(z;q)}{\partial z^2} + H - H\frac{\partial\widehat{F}(z;q)}{\partial z} + \frac{\partial^2\widehat{H}(z;q)}{\partial z^2}Q, \qquad (37)
$$

(38)

As we know that the embedding parameter which is increases from 0 to 1. q is an embedding parameter, h_F, h_θ, h_G and h_H are the non-zero auxiliary parameters and N_F and N_θ are nonlinear operators.

For $q = 0$ and $q = 1$ we have

$$
\begin{aligned}
\widehat{F}(z;0) &= F_0(z), \quad \widehat{F}(z;1) = F(z), \\
\widehat{\theta}(z;0) &= \theta_0(z), \quad \widehat{\theta}(z;1) = \theta(z), \\
\widehat{G}(z;0) &= G_0(z), \quad \widehat{G}(z;1) = g(z), \\
\widehat{H}(z;0) &= H_0(z), \quad \widehat{H}(z;1) = H(z),
\end{aligned} \qquad (39)
$$

As the embedding parameter q increases from 0 to 1, $\widehat{F}(z;q)$, $\widehat{\theta}(z;q)$, $\widehat{G}(z;q)$ and $\widehat{H}(z;q)$ varies from their initial guesses F_0, θ_0, G_0 and H_0 to the exact solutions $F(z)$, $\theta(z)$, $G(z)$ and $H(z)$, respectively. Taylor's series expansion of these functions yields

$$
\begin{aligned}
F(z;q) &= F_0(z) + \sum_{p=1}^{\infty} F_p(z)q^p, \\
\theta(z;q) &= \theta_0(z) + \sum_{p=1}^{\infty} \theta_p(z)q^p, \\
G(z;q) &= G_0(z) + \sum_{p=1}^{\infty} G_p(z)q^p, \\
H(z;q) &= H_0(z) + \sum_{p=1}^{\infty} H_p(z)q^p,
\end{aligned} \qquad (40)
$$

where

$$
\begin{aligned}
& F_p = \frac{1}{p!}\frac{\partial^p F(z;q)}{\partial z^p}|_{q=0}, \quad G_p = \frac{1}{p!}\frac{\partial^p G(z;q)}{\partial z^p}|_{q=0}, \quad \theta_p = \frac{1}{p!}\frac{\partial^p \theta(z;q)}{\partial z^p}|_{q=0}, \\
& H_p = \frac{1}{p!}\frac{\partial^p H(z;q)}{\partial z^p}|_{q=0},
\end{aligned} \qquad (41)
$$

Keeping in mind the above series depends on h_F, h_θ, h_G, h_H, h_N, h_P and h_M. On the assumption that the non-zero auxiliary parameters are chosen so that equation (49) converge at $q = 1$.
In infinite series form the dependent variables are

$$\begin{aligned} F(z) &= F_0(z) + \sum_{p=1}^{\infty} F_p(z), \\ \theta(z) &= \theta_0(z) + \sum_{p=1}^{\infty} \theta_p(z), \\ G(z) &= G_0(z) + \sum_{p=1}^{\infty} G_p(z), \\ H(z) &= H_0(z) + \sum_{p=1}^{\infty} H_p(z), \end{aligned} \tag{42}$$

Differentiating m-times the zeroth order deformation equations $(30 - 33)$ one has the mth order deformation equations as

$$L_F[F_p(z) - \chi_p F_{p-1}(z)] = h_F R_{F,p}(z), \tag{43}$$

$$L_\theta[\theta_p(z) - \chi_p \theta_{p-1}(z)] = h_\theta R_{\theta,p}(z), \tag{44}$$

$$L_G[G_p(z) - \chi_p G_{p-1}(z)] = h_G R_{G,p}(z), \tag{45}$$

$$L_H[H_p(z) - \chi_p H_{p-1}(z)] = h_H R_{H,p}(z), \tag{46}$$

where, the boundary conditions (19-22) takes the form

$$\begin{aligned} &F_p(0) = F_p^{'}(0) = F_p^{'}(\infty) = 0, \\ &\theta_p(0) = \theta_p(\infty) = 0, \\ &G_p(0) = G_p(\infty) = 0, \\ &H_p(0) = H_p(\infty) = 0, \end{aligned} \tag{47}$$

$$\begin{aligned} R_{F,p}(z) =& S\Big(\sum_{j=0}^{p-1} \eta_{j-p}(z) \frac{\partial^3 F_{j-p-1}(z)}{\partial z^3} + 3\frac{\partial^2 F_p(z)}{\partial z^2} + \sum_{j=0}^{p-1} \frac{\partial F_{j-p}(z)}{\partial z} \frac{\partial^2 F_{j-p-1}(z)}{\partial z^2} - \sum_{j=0}^{p-1} F_{j-p}(z) \frac{\partial^3 F_{j-p-1}(z)}{\partial z^3}\Big) \\ & - A\Big(2\sum_{j=0}^{p-2} \frac{\partial F_{j-p}(z)}{\partial z} G_{j-p-1}(z) H_{j-p-2}(z) + 2\sum_{j=0}^{p-1} F_{j-p-1}(z)(G_{j-p-1}(z))^2\Big) - \frac{\partial^4 F_{j-p-1}(z)}{\partial z^4} \\ & + A\Big(2\sum_{j=0}^{p-2} \frac{\partial F_{j-p}(z)}{\partial z} H_{j-p-1}(z) \frac{\partial H_{j-p-2}(z)}{\partial z} + \sum_{j=0}^{p-2} (H_{j-p-2}(z))^2 \frac{\partial F_{j-p}(z)}{\partial z} \sum_{j=0}^{p-2} \\ & + \sum_{j=0}^{p-2} \frac{\partial F_{j-p}(z)}{\partial z} H_{j-p-1}(z) G_{j-p-2}(z) + \sum_{j=0}^{p-2} F_{j-p}(z) H_{j-p-1}(z) \frac{\partial G_{j-p-1}(z)}{\partial z} \\ & + \sum_{j=0}^{p-2} F_{j-p}(z) \frac{\partial H_{j-p-1}(z)}{\partial z} G_{j-p-2}(z), \end{aligned} \tag{48}$$

$$R_{\theta,p}(z) = 2\frac{\partial^2 \theta_p(z)}{\partial z^2} - \sum_{j=0}^{p-2} F_{j-p}(z) \frac{\partial \theta_{j-p-1}(z)}{\partial z} (l_{j-p-2}(z))^2 - \sum_{j=0}^{p-2} \frac{\partial \theta_{j-p}(z)}{\partial z} \eta_{j-p-1}(z)(l_{j-p-2}(z))^2, \tag{49}$$

(50)

$$R_{G,p}(z) = \sum_{j=0}^{p-1} \eta_{j-p}(z) \frac{\partial G_{j-p-1}(z)}{\partial z} - 2\ G_{j-p}(z) - \sum_{j=0}^{p-2} \delta_{j-p}(z)\ H_{j-p-1}(z) \frac{\partial^2 F_{j-p-2}(z)}{\partial z^2}$$
$$- 2 \sum_{j=0}^{p-2} \delta_{j-p}(z) \frac{\partial F_{j-p-1}(z)}{\partial z} \frac{\partial H_{j-p-1}(z)}{\partial z} - \sum_{j=0}^{p-2} \delta_{j-p}(z)\ G_{j-p-1}(z) \frac{\partial F_{j-p-2}(z)}{\partial z}$$
$$- \sum_{j=0}^{p-1} F_{j-p}(z) \frac{\partial G_{j-p-1}(z)}{\partial z} - \sum_{j=0}^{p-1} M_{j-p}(z) \frac{\partial^2 G_{j-p-1}(z)}{\partial z^2}, \tag{51}$$

(52)

$$R_{H,p}(z) = \sum_{j=0}^{p-1} \eta_{j-p}(z) \frac{\partial H_{j-p-1}(z)}{\partial z} + H_{j-p}(z) - \sum_{j=0}^{p-1}\ H_{j-p}(z) \frac{\partial F_{j-p-1}(z)}{\partial z}$$
$$+ \sum_{j=0}^{p-1} Q_{j-p}(z) \frac{\partial^2 H_{j-p-1}(z)}{\partial z^2}, \tag{53}$$

$$\begin{cases} 1 & p > 1, \\ 0 & p \leq 1. \end{cases} \tag{54}$$

Finally, the general solution may be written as follows

$$\begin{aligned} F_p(z) &= F_p^* + (c_1 + c_2 z + c_3 z^2 + c_4 z^3), \\ \theta_p(z) &= \theta_p^* + (c_5 + c_6 z^2), \\ G_p(z) &= G_p^* + (c_7 + c_8 z^2), \\ H_p(z) &= H_p^* + (c_9 + c_{10} z^2), \end{aligned} \tag{55}$$

where F_p^*, θ_p^*, G_p^* and H_p^* are the special solutions.

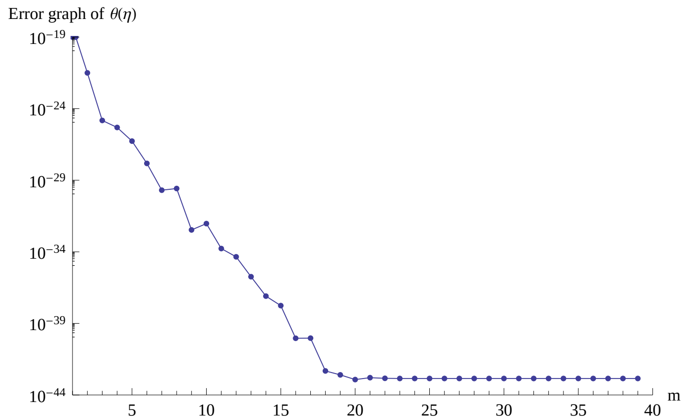


Figure 1: $A1 = 0.0070$, $R = 2.2$, $P2 = 0.0011$, $S3 = 0.0019$, $R4 = 0.0010$, $M2 = 0.9000$ and $Q3 = 0.3000$

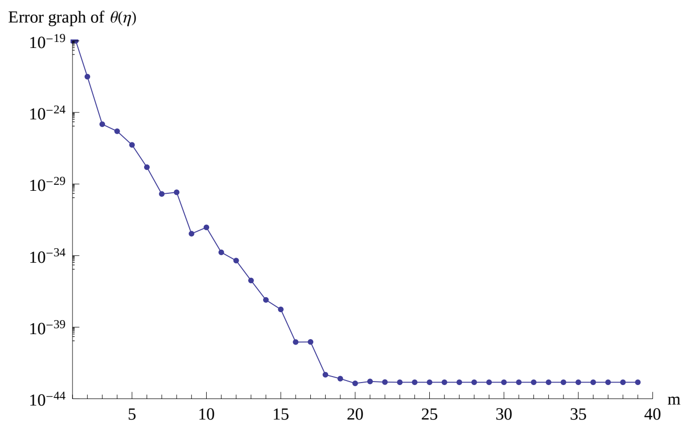


Figure 2: $A1 = 0.0070$,$R = 2.2$, $P2 = 0.0011$, $S3 = 0.0019$, $R4 = 0.0010$, $M2 = 0.9000$ and $Q3 = 0.3000$

3.1 Error Analysis

In this section error analysis are performed for the solution of the set of equations (15-18) by HAM for validity of results. For this purpose Figs. (1-4) are plotted, while tables 1 and 2 are made. Error analysis are performed by fixing residual error 10-40. It is observed in Figs.(1-4) that increasing order of approximation square residual error is decreased for F, θ, G and H which is the evidence of convergence of this method and the authentification of these results.

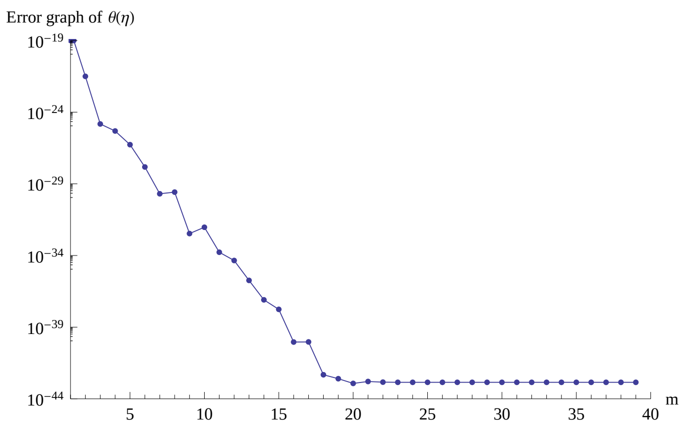


Figure 3: $A1 = 0.0070$,$R = 2.2$, $P2 = 0.0011$, $S3 = 0.0019$, $R4 = 0.0010$, $M2 = 0.9000$ and $Q3 = 0.3000$

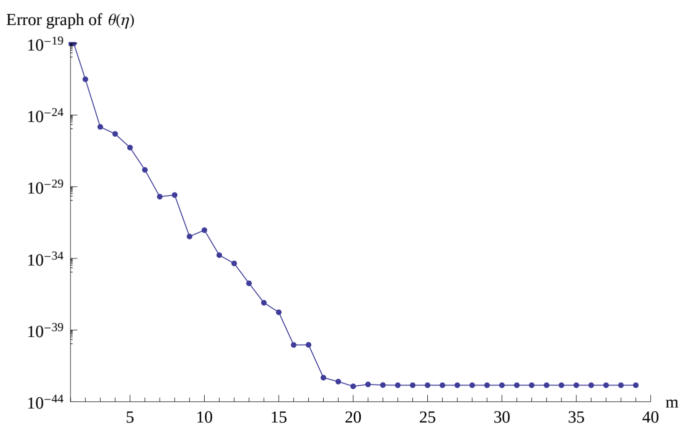


Figure 4: $A1 = 0.0070$, $R = 2.2$, $P2 = 0.0011$, $S3 = 0.0019$, $R4 = 0.0010$, $M2 = 0.9000$ and $Q3 = 0.3000$

Table 1 represent the individual average squared residual error for various order of approximations m. In this table one can clearly observe that the solution for unknown functions F, θ, G and H are convergent and it is accurate up to 10^{-28}, 10^{-33}, 10^{-28} and 10^{-30} for F, θ, G and H respectively taking m=40. Optimal values of convergence central parameter for different order of approximation is tabulated in table 2.

Table 1: Individual averaged squared residual errors using optimal values of auxiliary parameters.

m	$f(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$
2	0.016434×10^{0}	1.439750×10^{-7}	0.012965×10^{0}	0.012676×10^{0}
4	0.000011×10^{0}	1.190760×10^{-11}	0.000011×10^{0}	0.000027×10^{0}
6	5.077940×10^{-8}	8.139630×10^{-15}	6.139820×10^{-8}	5.734100×10^{-8}
8	7.753530×10^{-11}	2.855290×10^{-17}	1.132100×10^{-10}	1.128310×10^{-10}
10	3.121190×10^{-23}	4.319240×10^{-20}	4.708320×10^{-13}	2.353700×10^{-13}
14	1.578870×10^{-18}	1.629190×10^{-25}	5.157600×10^{-18}	1.689000×10^{-18}
18	9.655130×10^{-24}	9.056760×10^{-30}	6.732500×10^{-23}	1.596940×10^{-23}
20	3.733740×10^{-26}	3.241420×10^{-32}	2.449420×10^{-25}	4.262830×10^{-26}
22	1.745230×10^{-28}	1.984350×10^{-34}	8.410520×10^{-28}	1.221130×10^{-28}
26	1.636220×10^{-29}	1.868700×10^{-34}	2.332030×10^{-29}	9.181150×10^{-32}
30	1.634360×10^{-29}	1.868790×10^{-34}	2.335230×10^{-29}	9.325140×10^{-32}
34	1.624360×10^{-29}	1.868790×10^{-34}	3.380290×10^{-29}	9.325140×10^{-32}
36	1.634360×10^{-29}	1.868790×10^{-34}	2.34726×10^{-29}	9.325140×10^{-32}
40	1.653450×10^{-29}	1.868790×10^{-34}	2.347260×10^{-29}	1.538721×10^{-31}

Table 2: Optimal value of convergence control parameters versus different orders of approximation.

$Order of approximation$	$\hbar_f$	$\hbar_\theta$	$\hbar_G$	$\hbar_H$	ε_m^t
1	-0.997950	-0.999944	-0.992811	-1.006690	2.068680×10^{-7}
2	-0.994807	-0.999851	-0.993389	-1.001880	6.296750×10^{-12}
3	-0.997150	-1.000670	-0.993711	-1.000270	1.872500×10^{-16}
4	-0.990989	-0.991294	-0.999643	-1.008390	$-5.030490 \times 10^{-19}$
5	-0.025550	-1.028525	-1.022900	-1.035950	$-8.765390 \times 10^{-16}$
6	-0.937086	-1.056780	-1.054560	-0.942637	1.102300×10^{-15}
7	-0.912960	-1.906420	-1.888486	-1.130030	$-7.579390 \times 10^{-15}$

η	HAM				BVP $4C$			
	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$
0.00000	1.492498	1.000000	0.000000	0.000000	1.492498	1.000000	0.000000	0.000000
0.10060	1.477780	0.899594	0.044262	0.068953	1.477782	0.899593	0.044262	0.068953
0.20120	1.433580	0.799175	0.092933	0.13867	1.433583	0.799175	0.092933	0.138669
0.25120	1.533580	0.899175	0.192933	0.23867	1.533583	0.899175	0.192933	0.238669
0.30030	1.361070	0.700231	0.147622	0.209022	1.361068	0.700230	0.147621	0.209022
0.40090	1.257790	0.59975	0.212737	0.283469	1.257792	0.599750	0.212736	0.283469
0.45090	1.357790	0.69975	0.312737	0.383469	1.357792	0.699750	0.312736	0.383469
0.50000	1.126510	0.500722	0.289403	0.361700	1.126509	0.500721	0.289403	0.361700
0.60060	0.962819	0.400133	0.383518	0.448992	0.962819	0.400133	0.383518	0.448992
0.65060	0.062819	0.500133	0.483518	0.548992	0.062819	0.500133	0.483518	0.548992
0.70120	0.767987	0.29947	0.498072	0.548602	0.767986	0.299470	0.498071	0.548601
0.80030	0.545068	0.20024	0.635108	0.665198	0.545067	0.200240	0.635107	0.665198
0.85030	0.645068	0.30024	0.735108	0.765198	0.645067	0.300240	0.735107	0.765198
0.90090	0.286708	0.09941	0.802966	0.812194	0.286708	0.099414	0.802965	0.812193
1.00000	0.000000	0.000000	1.000000	1.000000	0.000000	0.000000	1.000000	1.000000

Table 3: Computations for $f'(\eta)$, $\theta(\eta)$, $G(\eta)$ and $H(\eta)$ with $A = 0.10, S = 0.10, R = 0.10, M = 2.0, P = 0.10, Q = 2.0$ and various values of η.

η	HAM				BVP $4C$			
	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$
0.00000	1.485126	1.000000	0.000000	0.000000	1.485126	1.0000000	0.000000	0.000000
0.10010	1.571010	0.000672	0.135512	0.164448	1.571007	0.000672	0.135512	0.164448
0.20070	1.528270	0.900798	0.170587	0.230026	1.528268	0.900798	0.177587	0.230025
0.30130	1.456610	0.800825	0.228459	0.297446	1.456605	0.800824	0.228459	0.297445
0.35130	1.356610	0.700825	0.128459	0.197446	1.356605	0.700824	0.128459	0.197445
0.40040	1.357250	0.702199	0.289868	0.367104	1.357253	0.702199	0.289868	0.367104
0.50100	0.126530	0.501886	0.266876	0.343297	0.126534	0.501885	0.266876	0.343296
0.55100	0.226530	0.601886	0.366876	0.443297	0.226534	0.601885	0.366876	0.443296
0.60010	0.967534	0.402836	0.360806	0.426985	0.967533	0.402835	0.360805	0.426985
0.65010	0.067534	0.502836	0.460806	0.526985	0.067533	0.502835	0.460805	0.526985
0.75070	0.874452	0.402006	0.578598	0.625715	0.874452	0.402006	0.578598	0.625715
0.80130	0.548352	0.200856	0.623212	0.646034	0.848351	0.200856	0.623212	0.646033
0.90040	0.298168	0.100869	0.795428	0.797265	0.292167	0.100869	0.795427	0.797264
0.95040	0.398168	0.200869	0.895428	0.897265	0.392167	0.200869	0.895427	0.897264
1.00000	0.000000	0.000000	1.000000	1.000000	0.000000	0.000000	1.000000	1.000000

Table 4: Computations for $f'(\eta)$, $\theta(\eta)$, $G(\eta)$ and $H(\eta)$ with $A = 0.20, S = 0.20, R = 0.20, M = 2.20, P = 0.20, Q = 2.20$ and various values of η.

η	HAM				BVP 4C			
	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$	$f'(\eta)$	$\theta(\eta)$	$G(\eta)$	$H(\eta)$
0.00000	1.471001	1.000000	0.000000	0.000000	1.471001	1.000000	0.000000	0.000000
0.15010	1.557740	0.002971	0.122371	0.159169	0.246805	0.002971	0.122371	0.159169
0.20020	1.417760	0.805751	0.054649	0.119170	1.417760	0.805750	0.054649	0.119170
0.30030	1.350510	0.708148	0.098838	0.181090	1.350507	0.708147	0.098837	0.181090
0.35030	1.250510	0.208148	0.398838	0.381090	1.250507	0.208147	0.398837	0.381090
0.45040	1.355040	0.709973	0.257512	0.346533	1.355037	0.709972	0.257511	0.346532
0.50050	0.130070	0.511040	0.233873	0.317956	0.130067	0.511039	0.233872	0.317956
0.60060	0.973982	0.411680	0.331699	0.399169	0.973981	0.411168	0.331698	0.399168
0.65060	0.273982	0.211680	0.231699	0.299169	0.273981	0.211168	0.231698	0.299168
0.70070	0.784866	0.310183	0.455057	0.496080	0.784865	0.310182	0.455057	0.496080
0.80080	0.560552	0.207917	0.607578	0.617899	0.560552	0.207917	0.607578	0.617898
0.85080	0.060552	0.007917	0.007578	0.017899	0.060552	0.007917	0.007578	0.017898
0.90090	0.298684	0.104216	0.790959	0.779086	0.298684	0.104216	0.790958	0.779086
0.95090	0.798684	0.704216	0.790959	0.779086	0.798684	0.704216	0.790958	0.779086
1.00000	0.000000	0.000000	1.000000	1.000000	0.000000	0.000000	1.000000	1.000000

Table 5: Computations for $f'(\eta)$, $\theta(\eta)$, $G(\eta)$ and $H(\eta)$ with $A = 0.40, S = 0.40, R = 0.40, M = 2.40, P = 0.40, Q = 2.40$ and various values of η.

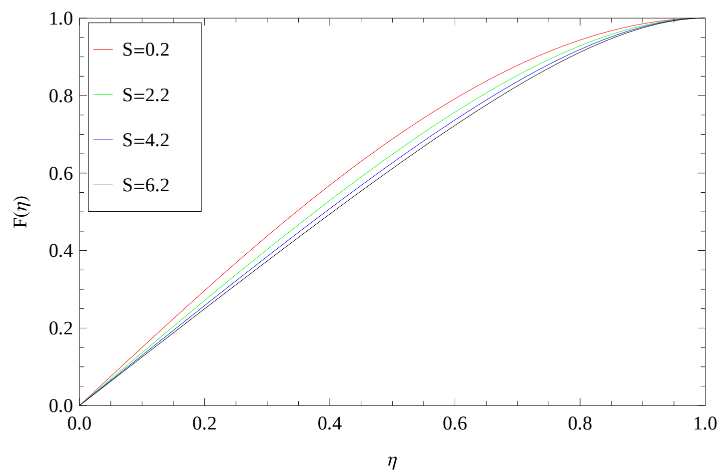


Figure 5: The effect of S on velocity profile with $A = 0.2$, $R = 2.2$, $P = 4.2$, $M = 4.2$ and $Q = 4.2$

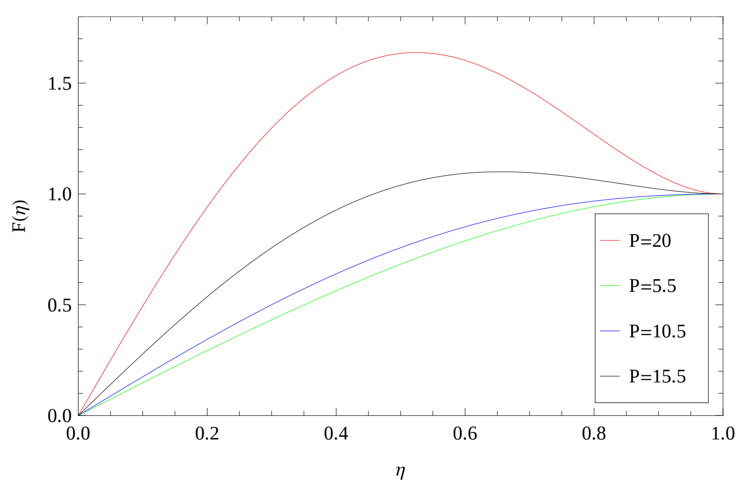


Figure 6: The effect of P on velocity profile with $A = 0.2$, $S = 2.2$, $R = 2.2$, $M = 4.2$ and $Q = 4.2$

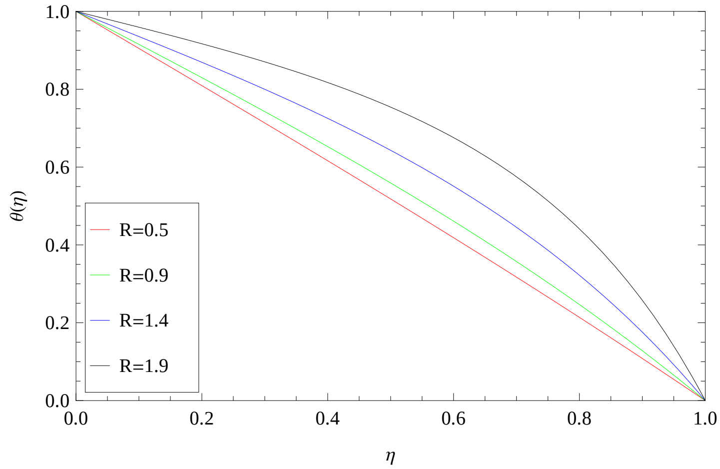


Figure 7: The effect of R on temperature profile with $A = 0.2$, $S = 2.2$, $P = 4.2$, $M = 4.2$ and $Q = 4.2$

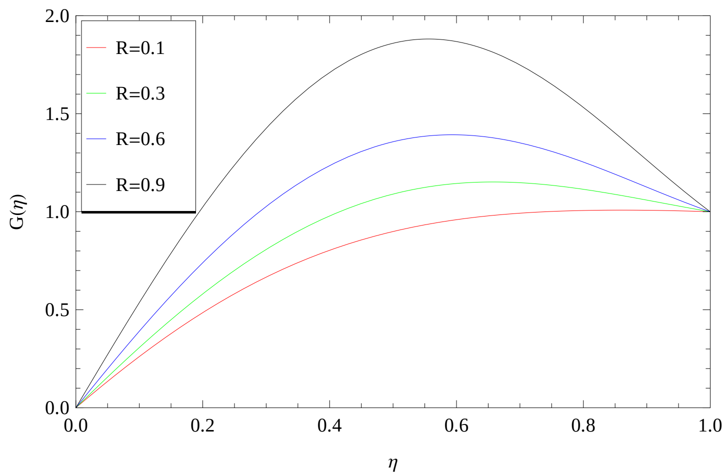


Figure 8: The effect of R on magnetic field in x-component with $A = 0.5$, $S = 0.2$, $R = 0.2$, $M = 0.34$ and $Q = 0.42$

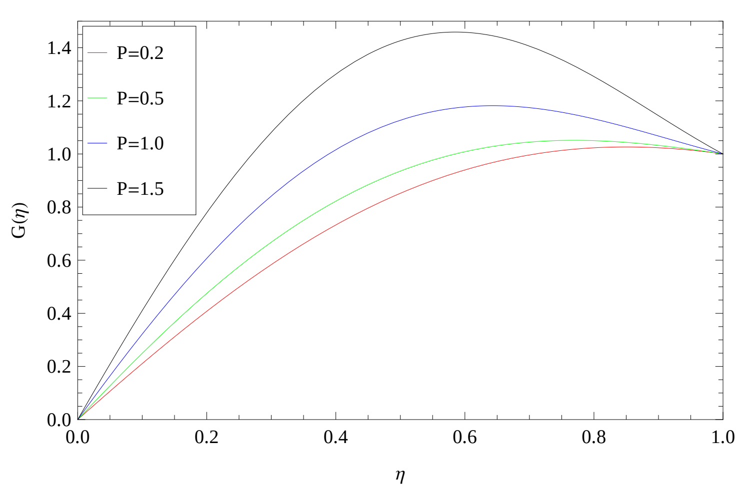


Figure 9: Effect of P on magnetic field in x-component with $A = 0.5$, $S = 0.2$, $P = 0.5$, $M = 0.34$ and $Q = 0.42$

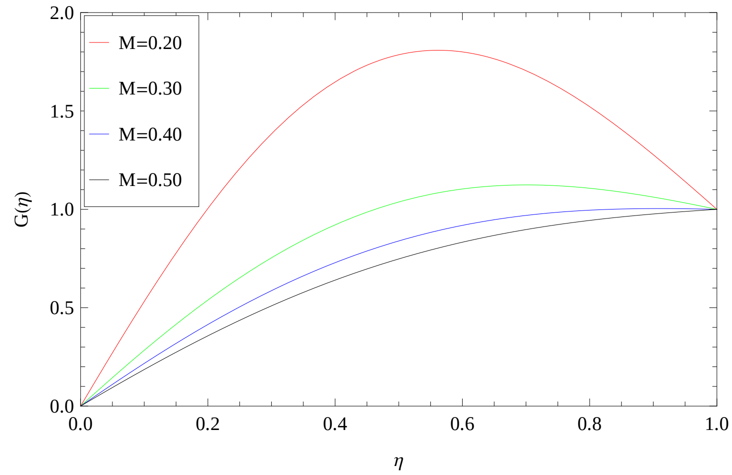


Figure 10: Effect of M on magnetic field in x-component $A = 0.5$, $S = 0.2$, $R = 0.2$, $M = 0.34$ and $Q = 0.42$

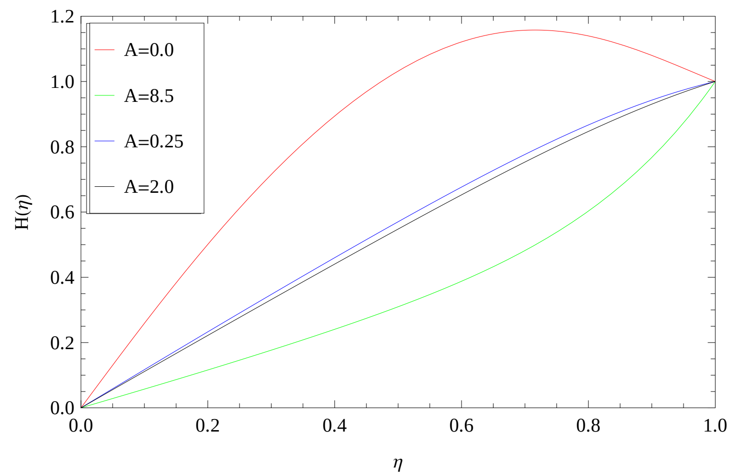


Figure 11: Effect of A on magnetic field in y-component $A = 0.5$, $S = 0.2$, $R = 0.2$, $P = 0.5$ and $Q = 0.42$

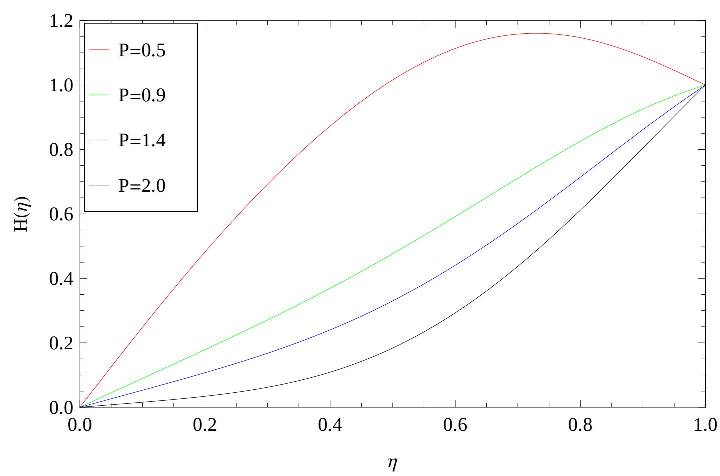


Figure 12: Effect of P on magnetic field in y-component $S = 0.2$, $R = 0.2$, $P = 0.5$, $M = 0.34$ and $Q = 0.42$

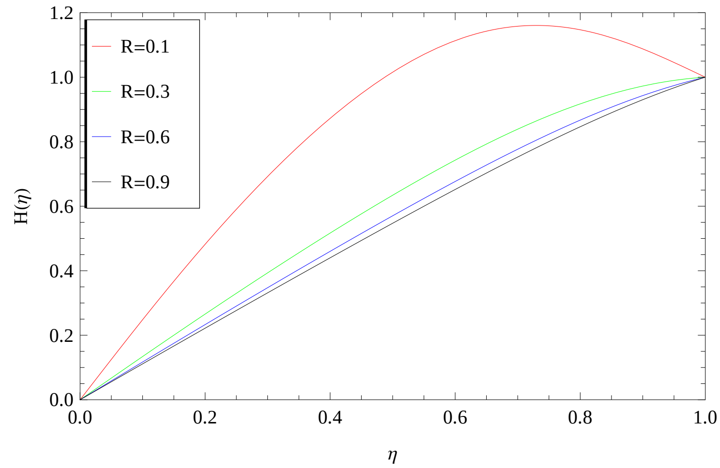


Figure 13: Effect of R on magnetic field in y-component $A = 0.5$, $S = 0.2$, $R = 0.2$, $M = 0.34$ and $Q = 0.42$

3.2 Results and discussions

Liquid flow between two squeezing porous plates in the presence of two dimensional magnetic field is captured in the form of non-linear system of ordinary differential equations reported inequations (15-18) with boundary conditions given in equations (19-22). Resulting system is solved by means of HAM and by numerical Package BVP4c. For the comparison of non-dimensional velocity, temperature and magnetic field tables (3-5) are made for various values of involved parameters. Also, important results of our interest are plotted in Figs. (5-13). It can be seen in tables (3-5) that the results of HAM are in good agreement with the results obtain by numerical BVP4c package.

Influence of squeezing parameter 'S' over axial velocity is shown in fig(1). which shows that axial velocity increases by increasing squeezing parameter 'S'. Effect of 'P' on axial velocity is shown in fig(2). we see in fig(2) that for p=20.5 it gives us a parabolic curve. fig3 and fig4 shows the effect of 'R' and 'p' on temperature which shows that increasing 'R' and 'p' the inter molecular forces increases and as a result the temperature effect increases. Fig5 and Fig6 shows the effect of 'R' and 'P' on magnetic field in x-direction. we see in fig 5,6 that by increasing 'R' and 'P' the magnetic field effect in x-direction increases respectively. fig7 shows the effect of 'M' on magnetic field in x-direction. fig7 shows that by increasing 'P' the magnetic field effect reduces and by decreasing 'M' the magnetic field effect increases in x-direction. fig8 and fig9 shows the effect of 'A' and 'P' on magnetic field in y-direction which shows that by increasing 'A' and 'P' the magnetic field effect increases in y-direction respectively. fig10 shows the effect of 'R' on magnetic field of 'R' on magnetic field in y-direction. it is clear from fig10 that by reducing 'R' the magnetic field effect increasing and by increasing 'R' the magnetic field effect in y-direction decreases. fig10 also shows that by increasing 'R' the linearity increases.

3.3 Conclusions

squeezing flow of a non-newtonian fluid is considered between two porous parallel plates. By using some transformation the naiver-stroke equation is transformed to the non linear system of differential equations with the set of bouldery conditions. This system of equations is highly coupled and non linear.For solution purpose we used HAM as analytical method and for comparison purpose we use RK4 method.The validity of our analytical solution is verified by the numerical results. Results have been compared in table 3,4 and 5 which shows that both the results are correct up to four decimal places. The effect of different parameters on axial velocity, Temperature, and magnetic field in x and y direction are shown by different graphs.

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