Decision Making in Optimizing a Product of a Small Scale Industry: A Bayesian Analysis Approach

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Abstract—*This paper intends to find Expected monetary value (EMV), Expected opportunity loss (EOL) and conditional profit of the main product (Mukta) of a small scale industry–"ORGAMAN" situated at Jorhat District of Assam. To meet the above specific objectives, the method of Bayesian Analysis has been adopted. The data used in this endeavor is secondary in nature, collected by direct personal investigation. As per prior information, the target of the industry is to produce a minimum of 50 MT (low production) of product and a maximum of 350 MT (high production) of the same per month. The prior analysis reveals that the expected monetary value and expected opportunity loss are optimum against high production. Based on both the prior analysis and posterior analysis, it is observed that the profit for the product of the industry is maximum against high production of 350 MT per month. Although, the profit based on posterior analysis is slightly high, it seems that the additional amount of money has to be spend to collect additional information for posterior analysis.*

Keywords— EMV, EOL, Posterior Analysis, Prior Analysis.

I. INTRODUCTION

Decision theory is concerned with the process of making decisions. The term statistical decision theory pertains to decision making in the presence of statistical knowledge, by shedding light on some of the uncertainties involved in the problem. Decision theory focuses on how we use our freedom. In the situations treated by decision theorists, there are options to choose between and we choose in a non-random way. Our choices, in these situations, are goal-directed activities. Hence, decision theory is concerned with goal-directed behavior in the presence of options. The decision making is based on two situations, decision making under certainty and decision making under uncertainty. In the history of almost any activity, there are periods in which most of the decision-making is made and other periods in which most of the implementation takes place. Modern decision theory has developed since the middle of the 20th century through contributions from several academic disciplines. Since 90's researchers [Epstein (1962), Pauker and Kassirer (1975), Simon (1979), Almeida and Bohoris (1995), Myung *et al.* (2005), Broniatowski *et al.* (2009), Cukierman (2010), Reedy (2010), Fenton and Neil (2011)] have applied decision making theory in various fields. The basic ideas of decision theory and of decision theoretic methods lend themselves to a variety of applications, computational and analytic advances in various fields.

As mentioned above, there are two types of decision-making situation which is based upon the knowledge that the decision maker has about the states of nature. Most situations in which the decision theory approach is applied involve decision making under uncertainty. There are various decision theoretic approaches for decision making under uncertainty. But in most cases, bayesian analysis is seen to be widely applied.

The term 'Bayesian' refers to Thomas Bayes (1702-1761), who proved a special case in a paper titled-"An Essay towards solving a Problem in the Doctrine of Chances" which is now called Bayes' Theorem. Then Pierre-Simon Laplace (1749-1827), introduced a general version of the Bayes' theorem and applied in celestial mechanics, medical statistics, reliability, and jurisprudence. By definition, bayesian analysis is a statistical procedure which endeavors to estimate parameters of an underlying distribution based on the observed distribution. Begin with a prior distribution which may be based on anything, is assume to be a uniform distribution over the appropriate range of values. Then calculate the likelihood of the observed distribution as a function of parameter values, multiply this likelihood function by the prior distribution, and normalize to obtain a unit probability over all possible values. This is called the posterior distribution. The mode of the distribution is then the parameter estimation and "probability intervals" can be calculated using the standard procedure. Bayesian analysis is controversial in the sense that the validity of the result depends on how valid the prior distribution is, and this cannot be assesses statistically.

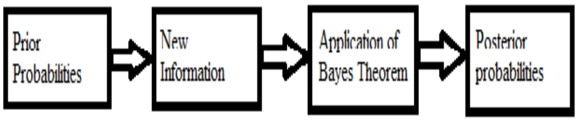

Fig. 1: Probability revision using Bayes theorem

Bayes' theorem is stated as follows-

If $E_1, E_2 \ldots E_n$ are mutually disjoint events with $P(E_i) \neq 0$, (i= 1, 2,…,n) then for any arbitrary event A which is subset of $\bigcup_{i=1}^{n} E_i$ such that P(A)> 0, we have

$$P(E_i \mid A) = \frac{P(E_i)\, P(A \mid E_i)}{\sum_{i=1}^{n} P(E_i)\, P(A \mid E_i)} = \frac{P(E_i)\, P(A \mid E_i)}{P(A)};$$

i = 1, 2 … n

[Parmigiani and Inoue (2009); Winn and Johnson (1978)]

This work intends to study a decision problem of an industry. The specific objectives of this study are:

- to find Expected Monetary Value (EMV) of the product of the industry- ORGAMAN
- to find Expected Opportunity Loss (EOL) of the product of the industry- ORGAMAN
- to find Conditional Profit (Loss) of the product of the industry-ORGAMAN

In this paper, section-2 describes the steps that have been undertaken while applying bayesian analysis. Section-3, gives us idea about the product as well as the industry. Section-4, illustrates the findings in brief. And last section-5, concludes the findings of the respective study.

II. METHODOLOGY

The objective of this study is to apply Bayesian Analysis in Decision Making under uncertainty. In our present study, the following steps have been undertaken [Parmigiani and Inoue (2009)].

Step 1: In this analysis, first of all prepare a payoff table based on our objective where combination of decision alternative and states of nature are present.

Step 2: Next, probabilities are assigned to the events about which uncertainties exist. The probabilities are known as Prior Probabilities.

Step 3: Using Baye's Criteria, Expected Monetary Value (EMV) have to be calculated for each action and then find the optimal action. The optimal value of EMV is the highest value. The EMV is given as follows-

$EMV(A_i)=\sum P(S_j)V(A_i, S_j)$; i=1,2,…n; j=1,2,…m

Where, V (A_i, S_j) denotes the monetary value for decision alternative A_i and state of nature S_j.

Step 4: Expected Opportunity Loss (EOL) have to be calculated and then find the optimal action. The Optimal Value of EOL is the lowest Value. The EOL is given as follows-

$EOL(A_i)=\sum P(S_j)R(A_i, S_j)$; i=1,2,…n; j=1,2,…m

Where, R (A_i, Sj) denotes the opportunity loss for decision alternative A_i and state of nature S_j.

Step5: Find Expected value of Perfect information (EVPI). EVPI is given as follows-

EVPI = Expected value under perfect information – EMV with prior probabilities

Step 6: Next calculate Conditional Probabilities.

$P(X|S) \rightarrow$ Probability of X when S is the case (S is the state of nature)

Step 7: Then calculate the Joint and Marginal Probabilities. Joint Probability is given as follows-

Joint probability = (conditional probability * prior probability)

Step 8: With the help of Joint and Marginal Probability, find the Posterior Probabilities or Revised Prior Probabilities. Posterior Probability is given by-

$P(S|X) = P(X \text{ and } S)/P(X)$

Step 9: Next calculation have to be done for Revised EMV with Marginal Probability $P(X_1)$ and $P(X_2)$.

Step 10: After step 9, calculation have been done for gross value and EMV under imperfect information.

III. SOURCE OF DATA

This paper is primarily based on secondary data i.e. the data have been collected from the concerned Industry "ORGAMAN".

The Industry ORGAMAN wants to determine how much of the Product "MUKTA" should produce for the month of May to get the maximum possible profit based on the expected demand for the month of May. "MUKTA", which is organic manure, has been serving for last 37 years since 1978. But many competitors with "MUKTA" gradually have been arising in the market. Hence the risk of loss also increases day by day for "MUKTA". So, there is a situation arise for the Manager to take decision at every step of production of "MUKTA". For this, the Manager could have come to a conclusion on production to get maximum possible profit based on the expected demand. With the help of Bayesian Analysis discussed, a statistical analysis is done for the above problem. The data used in the study are their monthly records of production of their product "MUKTA". According to the Industry the product MUKTA has 80% demands in the market.

IV. FINDINGS

Based on the objectives, first of all we prepare the pay-off table for the production against the demand of the product MUKTA. The payoff table is as follows-

Table.4.1: Payoff table (in weight)

ACTIONS	STATES OF NATURE	
	High demand(S_1)	Low demand(S_2)
Low production(A_1)	300 MT	50 MT
High production(A_2)	350 MT	-50 MT
Prior Probabilities	.8	.2

From the production cost and market selling price of the product, we get the conditional payoff table for profit as follows-

Table.4.2: Conditional payoff table for profit

ACTIONS	STATES OF NATURE	
	High demand(S_1)	Low demand(S_2)
Low production(A_1)	Rs.750000	Rs.125000
High production(A_2)	Rs.875000	-Rs.125000

Then the process starts with finding Expected Monetary Value (EMV) and Expected Opportunity Loss (EOL) using prior probabilities. In Bayesian theory this is known as Prior Analysis. The EMV of a decision alternative is the sum of weighted payoffs for the alternative. The optimal value of EMV is the highest value. Here it has been found that, the high production (A_2) has greater EMV of Rs. 675000 than the low production (A_1) i.e. Rs.625000, if the decision is made to make 350 MT .Next, EOL has been calculated. EOL for low production (A_1) is Rs.100000 and for high production (A_2) is Rs.50000. Since the optimal value of EOL is the lowest one, so EOL for A_2 is the optimal decision. Here decision is also the production of 350 MT.

Table.4.3: Payoff table for EMV and EOL

ACTIONS	STATES OF NATURE			
	High demand (S_1) (in Rs.)	Low demand (S_2) (in Rs.)	EMV (in Rs.)	EOL (in Rs.)
Low production(A_1)	750000	125000	625000	100000
High production(A_2)	875000	-125000	675000	50000

Under Perfect Information Expected Monetary Value is same as Expected Opportunity Loss which indicates that the Industry will have to consider paying for better information, but not more than the Rs. 50000.

Second part of Bayesian Analysis is Preposterior analysis. For this, calculation of the conditional probability of the indicators S_i given A_i ($S_i|A_i$), (i=1, 2) has been done. With the help of these probabilities first join probabilities and then marginal and revised (posterior) probabilities has been calculated. From these results prior information have been revised. Then revised expected monitory value has been calculated.

Table.4.4: Posterior or Revised prior probabilities

ACTIONS	STATES OF NATURE		
	High demand(S_1)	Low demand(S_2)	Marginal Probability (X_i)
Low Production(A_1) $P(S_i\|A_1)$	0.368/0.468 =0.786	0.1/0.468 =.214	0.368+ 0.1 = 0.468
High Production(A_2) $P(S_i\|A_2)$	0.432/0.532 =0.813	0.1/0.532 =.187	0.432 +0 .1 = 0.532

Each probability indicates the probability that S_i will occur if a particular A_i will occur. The figures in this table will now permit the manager to evaluate the value of collecting more information from an experiment. All these calculations that the manager have been conducting are done before any sample data are collected.

Final revised EMV with Revised Prior Probability $P(X_1)$ and Revised Prior Probability $P(X_2)$ is as follows-

Table.4.5: Revised EMV with probabilities

ADVICE	OPTIMAL ACTION	PROBABILITIES	EMV*
X	A*	P(X)	Expected profit
X_1	A_2	0.468	Rs.661000
X_2	A_2	0.532	Rs.688000

Next we calculate the Expected Value of Sampling Information (EVSI) by deducting EMV with prior probabilities from EMV with posterior and marginal probabilities. This is equal to Rs. 364.

Rs. 364 is the gross value. This is called a gross result since it does not contain the costs of conducting the experiments.

As we work with the decision Tree (Fig.2), we compute from left to right, a process called backward Induction. Then calculation is done to find gross value .The gross value of the experiment can be determined by taking the difference between the highest experiment payoff and the highest prior analysis payoff.

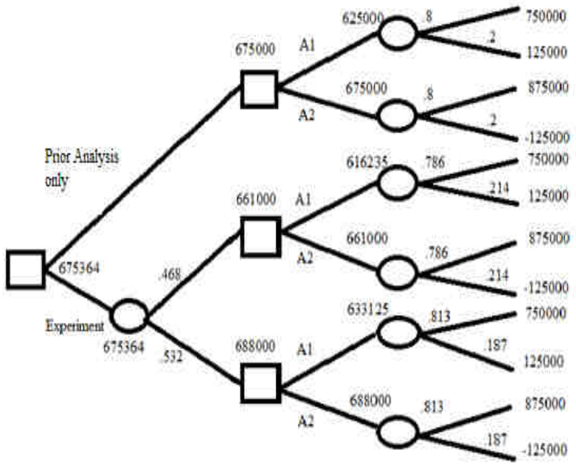

Fig.2: Decision Tree

V. CONCLUSION

In the present study, the strategy is to select the branch that yields the maximum gain or payoff. After the calculations using Prior Analysis, it has been noticed that the Expected Monetary Value and Expected Opportunity Loss for the states of nature A_2 is optimum. So, according to decision theory we have to work with states of nature A_2, the high production, which decision is a beneficial one, for the Industry ORGAMAN. Here the state of nature A_2 implies that the 350 MT production of MUKTA will give highest profit of Rs.675000. But if the industry would like to collect new information using experiment then the profit is Rs.675364. Hence the gross value which does not consider the cost of conducting the experiment is Rs.364. This difference is very meager amount. In order to apply this strategy, the firm has to collect additional information for which the firm has to spend additional amount of money which is obviously more than Rs.364. So, it would be better to select the prior analysis strategy and anticipate the resulting expected payoff.

REFERENCES

[1] Almeida,A.T.de,Bohoris,G.A.(1995),"Decision theory in maintenance decision making", Journal of Quality in Maintenance Engineering, Vol.1, Issue.1,pp. 39-45
[2] Anderson, D.R., Sweeney, D.J., Williams, T. A. (1984), "Statistics for Business and Economics", 2nd edition, West publishing company.
[3] Broniatowski, D. A., Magee, C. L., Coughlin, J., and Yang, M. C. (2009), "Bayesian Analysis of Decision Making in Technical Expert Committees", Paper presented in 7th Annual Conference on Systems Engineering Research (CSER), USA.
[4] Cukierman, A. (2010), "The Effects of Uncertainty on Investment under Risk Neutrality with Endogenous Information", the Journal of Political Economy, Vol. 88, No. 3, pp. 462-475.
[5] Epstein, E.S. (1962), "A Bayesian Approach to Decision Making in Applied Meteorology", Publication No.54 from the Meteorological laboratories, The University of Michigan.
[6] Fenton, N., and Neil, M.(2011), "The use of Bayes and causal modeling in decision making, uncertainty and risk", Queen Mary University of London.
[7] Myung, J. I., Karabatsos, G., Iverson, G.J.(2005), "A Bayesian approach to testing decision making axioms", Journal of Mathematical Psychology 49, 205–225, USA.
[8] Olivia, M., Fontes, D.B.M .M., Pereira,T. (2013), "Multicriteria Decision Making: A case study in the Automobile Industry", FEP Economic and Management, N.483, ISSN:0870-8541
[9] Parmigiani, G., Inoue, L. Y. T. (2009), "Decision Theory: Principles and Approaches", John Wiley & Sons, Ltd
[10] Pauker, S.G., M.D., and Kassirer, J. P., M.D. (1975), "Therapeutic Decision Making: A Cost-Benefit Analysis", New England Journal of Medicine, 293:229-234.
[11] Reedy, E. (2010), "Bayesian Analysis on Quantitative Decision-Making", School of Doctoral Studies (European Union) Journal, Belgium.
[12] Simon,H.A.(1979), "Rational decision making in Bussiness organizations", The American Economic Review,Vol.69,No.4,pp. 493-513.
[13] Winn, P. R., Johnson, R. H. (1978), "Business Statistics", Macmillan Publishing Co., Inc, Collier Macmillan Publishers.