An Inventory Model with Three Rates of Production Rate under Stock and Time Dependent Demand for Time Varying Deterioration Rate with Shortages

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Abstract— *An inventory model with three different rates of production rate and stock dependent demand is considered. Deterioration plays a vital role in present environment of market. In this paper, a two parameter Weibull distribution has been used to represent the deterioration rate. The objective is to determine the optimal total cost and the optimal time schedule of the plan for the proposed model. Some numerical examples have been carried out to illustrate the developed model.*

Keywords—Deterioration, Shortages, Stock-Time dependent demand, Weibull distribution.
Subject Classification Code: 90B05

I. INTRODUCTION

Economic order quantity(EOQ) inventory models have been attracting consider able amount of research attention. For the last fifteen years, researchers in this area have extended investigation into various models with considerations of item shortage, deterioration, demand patterns, items order cycles and their combinations. The control and maintenance of production inventories of deteriorating items with shortages have received much attention of several researchers in the recent years because most physical goods deteriorate over time. Deterioration is defined a decay, damage, spoilage evaporation and loss of unity of the product. Deterioration in inventory is a realistic feature and need to consideration it. Often we encounter products such as fruits, milk, drug, vegetables and photographic films etc that have a defined period of life time. Such items are referred as deterioration items. The loss due to deterioration cannot be avoided. Due to deterioration, inventory system faces the problem of shortages and loss of good will or loss of profit. [11]Singh and Singh developed a model to determine an optimal order quantity by using calculus technique of maxima and minima.Numerical solution of the suggested model had also been proposed.[15]Tripathi developed EOQ model for deteriorating items with shortages under inflation considering demand as a function of time.[9]Raman developed a two warehouse inventory model for deteriorating items having linear demand with inflation and permissible delay in payments. Also sensitivity with respect to the parameters have been carried out.[12]Sharmila and Uthayakumar examined the partial trade credit financing in a supply chain by EOQ-based model for decomposing items together with shortages.

[5]Lakshmidevi discussed a continuous production inventory model for time dependent deteriorating items with shortages in which three different rates of production and quadratic demand rate. [3]Jhuma and Samanta assumed that the demand and production rates are constant and the distribution of the time to deterioration of an item follows the exponential distribution. Also results are illustrated by numerical examples.[4]Krishna and Bani developed a mathematical model of an inventory system in which demand depending upon stock level and time with various degrees , gave more flexibility of the demand pattern and more general to the study dine so far with the condition to minimize the total average cost of the system.

[14]Sushil assumed that the retailer pays to the supplier as soon as he received the items and in such cases the supplier offers a cash discount or credit period (permissible delay) to the retailer.[2]Debashis and Pavan proposed a deteriorating inventory model with time-dependent demand rate and varying holding cost at partial backlogging. Also shortages and partial backlogging were allowed.[16]Vinod et al developed a deterministic inventory model with time-dependent demand and time-varying holding cost where deterioration is time proportional.[6]Mehdi presented a practical solution for producer of breakable goods like ceramic, gypsum boards, glasses, etc. to make better decision at a situation when increase the number of inventory in hand cause more damaging.[17]Vinod reduced the deterioration rate during deterioration period of non-instantaneous deteriorating items inventory, we use the preservation technology, and a solution procedure to determine an optimal

replenishment cycle, shortage period, order quantity and preservation technology cost so that the total inventory cost per unit time is minimum.[13]Sharmila and Uthayakumar presented fuzzy inventory model for deteriorating items with shortages under fully backlogged condition.

[10]Sarbjit formulated for the perishable items provides retailers a mechanism to decide their economic order quantity and the cycle time for the items having variable holding cost.[7]Mohd considered an integrated supplier-production-inventory economic lot-sizing model for maximizing the total joint port of the supplier, vendor and buyer. [8]Mohammad et al. developed a fuzzy inventory model for deteriorating items with price and time dependent demand considering inflation effect on the system. Also shortages if any are allowed and partially backlogged with a variable rate dependent on the duration of waiting time up to the arrival of next lot.[1]Chun et al. presented the inventory models for deterioration items when the demand is a function of the selling price and stock on display.

II. NOTATIONS AND ASSUMPTIONS

2.1 Notations

$Q(t)$	is the inventory level at time $t(>0)$
T	the time length of the plan
p_1	the production rate in $[0,t_1]$
p_2	the production rate in $[t_1,t_2]$
p_3	the production rate in $[t_4,T]$
h	holding cost per unit per time
s	the shortage cost per unit per unit time
p	the production cost per unit per unit time
TC	the total cost per unit time
$\theta(t)$	the deterioration rate at time t.

2.2 Assumption

1. The demand rate $D(q,t)=a+bt^{\beta-1}q(t)$.
2. The deterioration rate $\theta(t)=\alpha\beta t^{\beta-1}$, where $0<\alpha<1$.
3. The production rate $p_3>p_2>p_1$.
4. The shortages are allowed.
5. There is no replacement or repair of deteriorated items during the cycle under consideration.
6. The items considered in this model are deteriorating with time t.
7. The time for allowing shortages is same as back order time.

III. PROPOSED MODEL

The production of the item is started initially at t=0 at the rate p_1. When $t=t_1$ the rate of production is transformation to $p_2(>p_1)$ and the production is stopped at time t_2 and the inventory depleted at a rate $D(q,t)$. The inventory level reaches zero at t_3. It is decided to backlog demand upto $t=t_4$ which occur during stock - out line. The production is started at a faster rate $p_3(>p_2>p_1)$ so as to clear the backlog and when the inventory level reaches zero, that is the backlog cleared, the next production cycle starts. At the time duration $[0,t_1]$, the production is at the rate p_1 and the consumption by demand. At the time duration $[t_1,t_2]$, the production is at the rate p_2 and the consumption by demand. At the time duration $[t_2,t_3]$, there is no production, but only consumption by demand. At the time duration $[t_3,t_4]$, the shortages are allowed.$[t_4,T]$ in this duration of time to backlog at production rate p_3. The cycle then repeats itself after time T. The deterioration is instantaneous with the rate □t in the duration $[0,t_3]$.

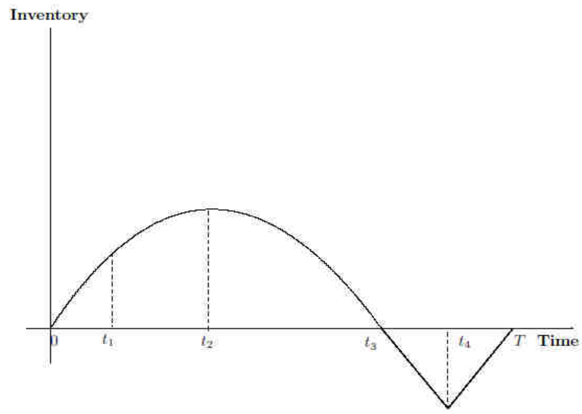

Fig.1: A Production Inventory Model

IV. MATHEMATICAL FORMULATION

The change of inventory level can be described as follows

$$\frac{dQ(t)}{dt}+(\alpha\beta+b)t^{\beta-1}Q(t)=p_1-a, 0\le t\le t_1 \quad (1)$$

$$\frac{dQ(t)}{dt}+(\alpha\beta+b)t^{\beta-1}Q(t)=p_2-a, t_1\le t\le t_2 \quad (2)$$

$$\frac{dQ(t)}{dt}+(\alpha\beta+b)t^{\beta-1}Q(t)=-a, t_2\le t\le t_3 \quad (3)$$

$$\frac{dQ(t)}{dt}+bt^{\beta-1}Q(t)=-a, t_3\le t\le t_4 \quad (4)$$

with boundary conditions $Q(0)=0, Q(t_1)=Q_1, (Qt_2)=S, Q(t_3)=0, Q(t_4)=-Q_2, Q(T)=0$

V. SOLUTION PROCEDURE

Solution of the above differential equations (1),(2),(3)and (4) are given by,

$$Q(t)=\begin{cases} (p_1-a)e^{-kt^\beta}\left(t_1+\frac{kt^{\beta+1}}{\beta+1}\right) & 0\le t\le t_1 \\ (p_2-a)e^{kt^\beta}\left[\left(t+\frac{kt^{\beta+1}}{\beta+1}\right)+Q(t_1)e^{kt_1^\beta}\left(t_1+\frac{kt_1^{\beta+1}}{\beta+1}\right)\right] & t_1\le t\le t_2 \\ ae^{-kt^\beta}\left[-\left(t+\frac{kt^{\beta+1}}{\beta+1}\right)+t_3+\frac{kt_3^{\beta+1}}{\beta+1}\right] & t_2\le t\le t_3 \\ e^{-k_1t_4^\beta}\left[-a\left(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1}\right)+Q(t_4)e^{k_1t_4^\beta}+a\left(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1}\right)\right] & t_3\le t\le t_4 \\ e^{-k_1t^\beta}(p_3-a)\left[t+\frac{k_1t^{\beta+1}}{\beta+1}+T+\frac{k_1T^{\beta+1}}{\beta+1}\right] & t_4\le t\le T \end{cases} \tag{5}$$

where $k=\frac{ab+b}{b}$, $k_1=\frac{b}{b}$

Total inventory carried over the period [0,T]

$$HC=h\left[\int_0^{t_1}Q(t)dt+\int_{t_1}^{t_2}Q(t)dt+\int_{t_2}^{t_3}Q(t)dt\right]$$

$$\begin{aligned} HC = {} & h\Bigg[(p_1-a)\left[\frac{t_1^2}{2}+\frac{kt_1^{\beta+2}}{(\beta+1)(\beta+2)}+k\left[\frac{t_1^{\beta+2}}{\beta+2}+\frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}\right]\right]+ \\ & (p_2-a)\Bigg[\Bigg(\left[\frac{t_2^2}{2}+\frac{kt_2^{\beta+2}}{(\beta+1)(\beta+2)}+Q(t_1)e^{kt_1^\beta}\left(t_1+\frac{kt_1^{\beta+1}}{\beta+1}\right)t_2\right] \\ & -k\left[\frac{t_2^{\beta+2}}{\beta+2}+\frac{kt_2^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{t_2^{\beta+1}}{\beta+1}\left(Q(t_1)e^{kt_1^\beta}\left(t_1+\frac{kt_1^{\beta+1}}{\beta+1}\right)\right)Bigg]\right)- \\ & \left(\left[\frac{t_1^2}{2}+\frac{kt_1^{\beta+2}}{(\beta+1)(\beta+2)}+Q(t_1)e^{kt_1^\beta}\left(t_1^2+\frac{kt_1^{\beta+2}}{\beta+2}\right)\right]-\right. \\ & k\left[\frac{t_1^{\beta+2}}{\beta+2}+\frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{t_1^{\beta+1}}{\beta+1}\left(Q(t_1)e^{-kt_1^\beta}\left(t_1+\frac{kt_1^{\beta+1}}{\beta+1}\right)\right)\right]\Bigg)\Bigg] \\ & +a\Bigg(\Bigg[-\left(\frac{t_3^2}{2}+\frac{kt_3^{\beta+2}}{(\beta+1)(\beta+2)}\right)+\left(t_3^2+\frac{kt_3^{\beta+2}}{\beta+1}\right)-k\Bigg(\Bigg[-\left(\frac{t_3^{\beta+2}}{\beta+2}+\frac{kt_3^{\beta+3}}{(\beta+1)(\beta+3)}\right)+ \\ & \frac{t_3^{\beta+2}}{\beta+1}+\frac{kt_3^{(\beta+1)(\beta+1)}}{(\beta+1)^2}\Bigg]\Bigg)\Bigg]-\left[-\left(\frac{t_2^2}{2}+\frac{kt_2^{\beta+2}}{(\beta+1)(\beta+2)}\right)+\left(t_3+\frac{kt_3^{\beta+1}}{\beta+1}\right)t_2\right] \\ & -k\left(-\left(\frac{t_2^{\beta+2}}{\beta+2}+\frac{kt_2^{\beta+3}}{\beta}\right)+\frac{t_2^{\beta+1}}{\beta+1}\left(t_3+\frac{kt_3^{\beta+1}}{\beta+1}\right)\right)\Bigg)\Bigg] \end{aligned} \tag{6}$$

Total number of deteriorating items in $[0,t_3]$

$$DC=d\left[p_1t_1+p_2(t_2-t_1)-\int_0^{t_3}D(q,t)dt\right]$$

$$DC=d\left[p_1t_1+p_2(t_2-t_1)-\left[a\left(\frac{kt_3^{\beta+1}}{\beta+1}\right)-b\left(\frac{t_3^{\beta+1}}{\beta(\beta+1)}\right)\right]\right]$$

Total Shortage occurred in $[t_3, T]$

$$SC=s\left[-\int_{t_3}^{t_4}Q(t)dt-\int_{t_4}^{T}Q(t)dt\right]$$

$$\begin{aligned} SC = {} & s\left[-\left[e^{-kt_4^\beta}-a\left(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1}\right)+Q(t_4)e^{k_1t_4^\beta}+a\left(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1}(t_4-t_3)\right)\right]\right. \quad (7) \\ & -\Bigg((p_3-a)\Bigg[\Bigg(\left[\frac{t_4^2}{2}+\frac{k_1t_4^{\beta+2}}{(\beta+1)(\beta+2)}+Tt_4+\frac{k_1T^{\beta+1}t_4}{\beta+1}\right] \\ & -k\left[\frac{t_4^{\beta+2}}{\beta+2}+\frac{k_1t_4^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{Tt_4^{\beta+1}}{\beta+1}+\frac{kT^{\beta+1}t_4^{\beta+1}}{(\beta+1)^2}\right]\Bigg)\Bigg] \\ & \left[\left(\frac{T^2}{2}+\frac{k_1T^{\beta+2}}{(\beta+1)(\beta+2)}+T^2+\frac{k_1T^{\beta+1}T}{\beta+1}\right)-\right. \\ & k\left(\frac{T^{\beta+2}}{\beta+2}+\frac{k_1T^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{TT^{\beta+1}}{\beta+1}+\frac{kT^{\beta+1}T^{\beta+1}}{(\beta+1)^2}\right)\Bigg]\Bigg)\Bigg] \end{aligned}$$

Total number of units produced in [0,T]

$$PC=p(p_1t_1+p_2(t_2-t_1)+p_3(T-t_4)) \tag{8}$$

Total Cost

$$TC=HC+DC-SC+PC \tag{9}$$

The time for allowing shortages is same as back order time

i.e. $t_4-t_3=T-t_4$

Therefore $t_3=2*t_4-T$

The necessary conditions for $TC(t_1,t_2,t_4,T)$ to be minimum are

$$\frac{\partial TC}{\partial t_1}=0;\ \frac{\partial TC}{\partial t_2}=0\ \frac{\partial TC}{\partial t_4}=0;\ \frac{\partial TC}{\partial T}=0 \tag{10}$$

Solving those equation we get the optimal values t_1^*, t_2^*, t_4^*, T^* which minimize total cost provided they satisfy the following sufficient condition

H=The Hessian Matrix of TC

$$=\begin{pmatrix} \frac{\partial^2TC}{\partial t_1^2} & \frac{\partial^2TC}{\partial t_1\partial t_2} & \frac{\partial TC}{\partial t_1\partial t_4} & \frac{\partial^2TC}{\partial t_1\partial T} \\ \frac{\partial^2TC}{\partial t_2\partial t_1} & \frac{\partial^2TC}{\partial t_2^2} & \frac{\partial TC}{\partial t_2\partial t_4} & \frac{\partial^2TC}{\partial t_2\partial T} \\ \frac{\partial^2TC}{\partial t_4\partial t_1} & \frac{\partial^2TC}{\partial t_4\partial t_2} & \frac{\partial TC}{\partial t_4^2} & \frac{\partial^2TC}{\partial t_2\partial T} \\ \frac{\partial^2TC}{\partial T\partial t_1} & \frac{\partial^2TC}{\partial T\partial t_2} & \frac{\partial TC}{\partial T\partial t_4} & \frac{\partial^2TC}{\partial T^2} \end{pmatrix} \text{ is positive definite} \tag{11}$$

If the solutions obtained from (10) do not satisfy the sufficient condition (11), then no feasible solution will be optimal for the set of parameter values taken to solve equations (10) is considered. Such a situation will imply that the parameter values are inconsistent and there is some error in their estimation.

VI. NUMERICAL RESULTS AND DISCUSSION

Example 1

Let $a = 100, b = 0.30, c = 0.002, t = 2, T = 3.8815, \alpha_1 = 0.02, t_1 = 1.3496, t_2 = 3.0748, t_3 = 3.1419, t_4 = 3.5117, p_1 = 2, p_4, p_6, \beta = 0.7$

The total cost TC=Rs.5312.5

The production quantity PC=11.8188

Example 2

Let $a = 100, b = 0.30, c = 0.002, t = 2, T = 3.8815, t_1 = 1.3496, t_2 = 3.0748, t_3 = 3.1419, t_4 = 3.5117, \beta = 0.7$

Example 3

Let $a = 100, b = 0.30, c = 0.002, t = 2, T = 3.8815, t_1 = 1.3496, t_2 = 3.0748, t_3 = 3.1419, t_4 = 3.5117$

Example 4

Let $a = 100, b = 0.30, c = 0.002, t = 2, T = 3.8815, t_1 = 1.3496, t_2 = 3.0748, t_3 = 3.1419, t_4 = 3.5117$

Table.1: Example 2

various value of α, p_1,p_2 & p_3

α	p_1	p_2	p_3	*PC*	*TC*
0.05	3	5	7	15.2634	8006.0
0.09	4	6	8	18.7080	8840.4
0.13	5	6	9	22.1526	9708.3
0.17	6	8	10	25.5972	10609
0.21	7	9	11	29.0419	11540
0.25	8	9	12	32.4864	12501
0.30	9	11	13	35.9310	13766
0.40	10	12	14	39.3756	16559
0.50	11	13	15	42.8202	19587

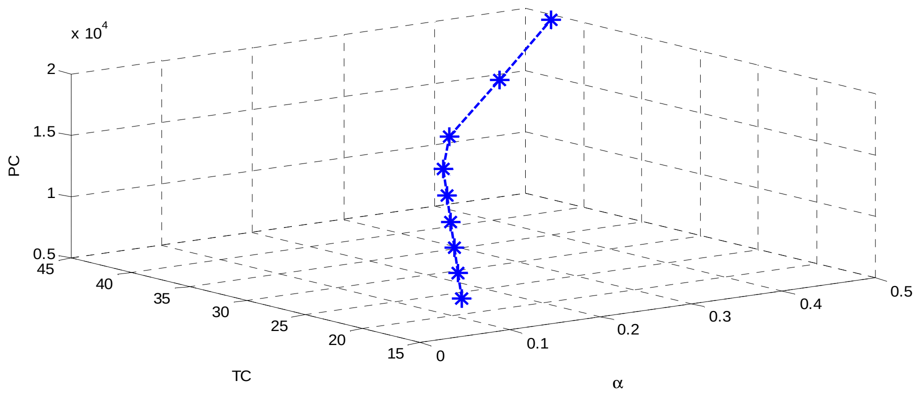

Fig.2: (Example 2)Variation of α and p_1,p_2 & p_3

Table.2: Example 3

various value of α, β,p_1,p_2 & p_3

α	β	p_1	p_2	p_3	*PC*	*TC*
0.05	0.2	3	5	7	15.2634	29988
0.09	0.3	4	6	8	18.7080	16760
0.13	0.4	5	6	9	22.1526	12335
0.17	0.5	6	8	10	25.5972	10609
0.21	0.6	7	9	11	29.0418	10076
0.25	0.7	8	9	12	32.4864	10237
0.30	0.8	9	11	13	35.9310	11189
0.40	0.9	10	12	14	39.3756	14294

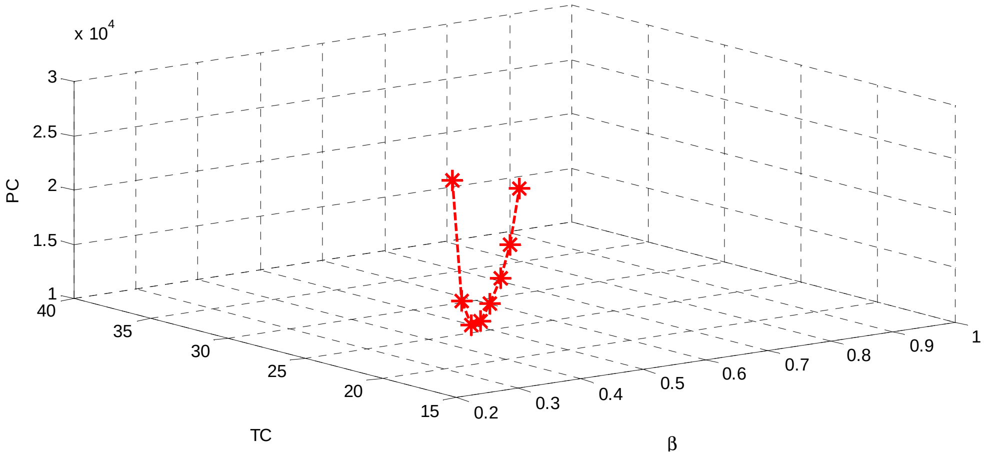

Fig.3: (Example 3) Variation of β and p_1,p_2 & p_3

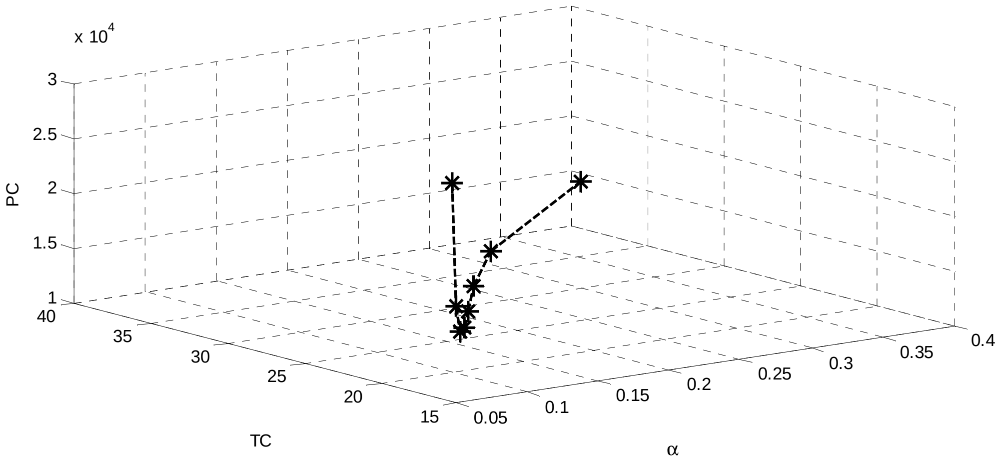

Fig.4: (Example 3) Variation of α and p_1,p_2 & p_3

Table 3: Example 4
various value of α, β,p_1,p_2 & p_3

α	β	p_1	p_2	p_3	*PC*	*TC*
0.10	0.2	1	2	3	5.9094	32644
0.20	0.3	2	3	4	9.3540	20553
0.30	0.4	3	4	5	12.7986	17455
0.40	0.5	4	5	6	16.2432	17394
0.50	0.6	5	6	7	19.6878	18996
0.60	0.7	6	7	8	23.1324	21906
0.70	0.8	7	8	9	26.5770	26120
0.80	0.9	8	9	10	30.0216	31833

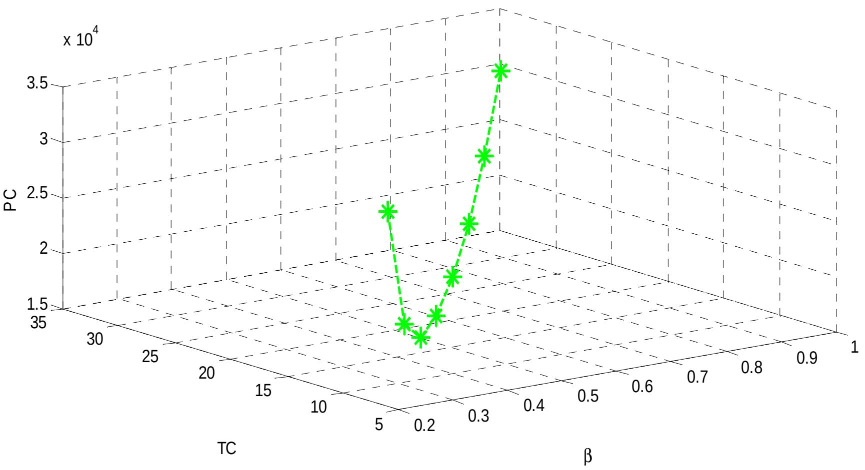

Fig.5: (Example 4 Variation of β and p_1,p_2 & p_3

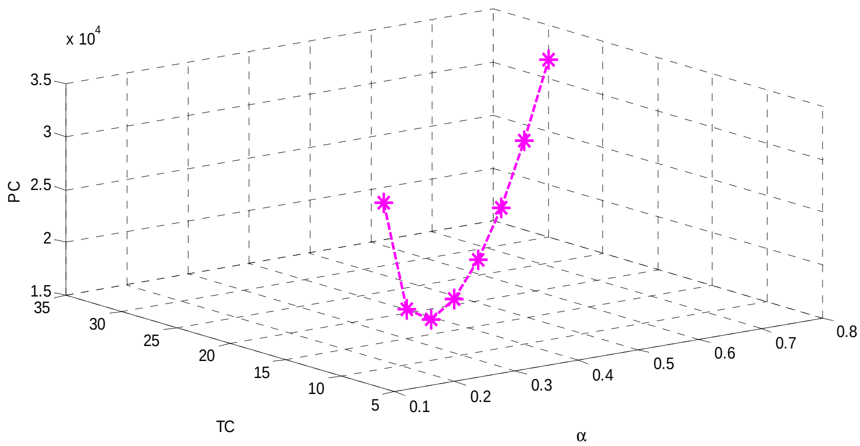

Fig.6: (Example 4) Variation of β and p_1,p_2 & p_3

VII. OBSERVATION

On the basis of numerical experiment performed, it has been observed

- From table 1, if the values of α, p_1,p_2 and p_3 increases the production quantity and total cost also increases.

- From table 2, If the values of α,β,p_1,p_2 and p_3 increases the production quantity increases and the total cost increases till $\alpha=0.17,\beta=0.5$. At $\alpha=0.21,\beta=0.6$ the total cost decreases. Further increasing value of α and β the total cost increases.
- From table 3, If the values of α,β, p_1,p_2 and p_3 increases the production quantity and total cost increases.

VIII. CONCLUSION

The present paper develops a mathematical model of an inventory system in which demand depending upon stock level and time with various degree β, gives more flexibility of the demand pattern and more general to the study done so far with the condition to minimize the total average cost of the system. It is observed that rise in the stock level of the product usually has a positive impact on its demand of the product. It has been perceived that failure rate and life expectancy of many items can be expressed in terms of Weibull distribution. Moreover, Weibull distribution is capable of representing constant, increasing and decreasing rate of deteriorations, as a consequence it has a wide range of application to model the deteriorating inventory.

A continuous inventory model with three rates of production rate under stock and time dependent demand for time varying deterioration rate with shortages is considered. The case of change of production is very useful in practical situations. The variation in production rate provides a way resulting consumer satisfaction and earning potential profit.

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Appendix I

$$TC=h[(p_1-a)[\frac{t_1^2}{2}+\frac{kt_1^{\beta+2}}{(\beta+1)(\beta+2)}+k[\frac{t_1^{\beta+2}}{\beta+2}+\frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}]]+(p_2-a)[([\frac{t_2^2}{2}+\frac{kt_2^{\beta+2}}{(\beta+1)(\beta+2)}+Q(t_1)e^{kt_1^{\beta}}(t_1+\frac{kt_1^{\beta+1}}{\beta+1})t_2]$$

$$-k[\frac{t_2^{\beta+2}}{\beta+2}+\frac{kt_2^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{t_2^{\beta+1}}{\beta+1}(Q(t_1)e^{kt_1^{\beta}}(t_1+\frac{kt_1^{\beta+1}}{\beta+1}))Bigg])-$$

$$([\frac{t_1^2}{2}+\frac{kt_1^{\beta+2}}{(\beta+1)(\beta+2)}+Q(t_1)e^{kt_1^{\beta}}(t_1^2+\frac{kt_1^{\beta+2}}{\beta+2})]-$$

$$k[\frac{t_1^{\beta+2}}{\beta+2}+\frac{kt_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{t_1^{\beta+1}}{\beta+1}(Q(t_1)e^{-kt_1^{\beta}}(t_1+\frac{kt_1^{\beta+1}}{\beta+1}))])]+a([-(\frac{t_3^2}{2}+\frac{kt_3^{\beta+2}}{(\beta+1)(\beta+2)})+(t_3^2+\frac{kt_3^{\beta+2}}{\beta+1})-k([-(\frac{t_3^{\beta+2}}{\beta+2}+\frac{kt_3^{\beta+3}}{(\beta+1)(\beta+3)})+$$

$$\frac{t_3^{\beta+2}}{\beta+1}+\frac{kt_3^{(\beta+1)(\beta+1)}}{(\beta+1)^2}])]-[-(\frac{t_2^2}{2}+\frac{kt_2^{\beta+2}}{(\beta+1)(\beta+2)})+(t_3+\frac{kt_3^{\beta+1}}{\beta+1})t_2]-k(-(\frac{t_2^{\beta+2}}{\beta+2}+\frac{kt_2^{\beta+3}}{\beta})+\frac{t_2^{\beta+1}}{\beta+1}(t_3+\frac{kt_3^{\beta+1}}{\beta+1})))]+$$

$$d[p_1t_1+p_2(t_2-t_1)-[a(\frac{kt_3^{\beta+1}}{\beta+1})-b(\frac{t_3^{\beta+1}}{\beta(\beta+1)})]]-s[-[e^{-kt_4^{\beta}}-a(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1})+Q(t_4)e^{k_1t_4^{\beta}}+a(t_4+\frac{k_1t_4^{\beta+1}}{\beta+1}(t_4-t_3))]$$

$$-((p_3-a)[([\frac{t_4^2}{2}+\frac{k_1t_4^{\beta+2}}{(\beta+1)(\beta+2)}+Tt_4+\frac{k_1T^{\beta+1}t_4}{\beta+1}]-k[\frac{t_4^{\beta+2}}{\beta+2}+\frac{k_1t_4^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{Tt_4^{\beta+1}}{\beta+1}+\frac{kT^{\beta+1}t_4^{\beta+1}}{(\beta+1)^2}])]$$

$$[(\frac{T^2}{2}+\frac{k_1T^{\beta+2}}{(\beta+1)(\beta+2)}+T^2+\frac{k_1T^{\beta+1}T}{\beta+1})-k(\frac{T^{\beta+2}}{\beta+2}+\frac{k_1T^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{TT^{\beta+1}}{\beta+1}+\frac{kT^{\beta+1}T^{\beta+1}}{(\beta+1)^2})])]+p(p_1t_1+p_2(t_2-t_1)+p_3(T-t_4)$$

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