A Two-Warehouse Model for Deteriorating Items with Holding Cost under Inflation and Soft Computing Techniques

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Abstract—A deterministic inventory model has been developed for deteriorating items and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy having a ramp type demands with the effects of inflation with two-warehouse facilities. The owned warehouse (OW) has a fixed capacity of W units; the rented warehouse (RW) has unlimited capacity. Here, we assumed that the inventory holding cost in RW is higher than those in OW. Shortages in inventory are allowed and partially backlogged and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy it is assumed that the inventory deteriorates over time at a variable deterioration rate. The effect of inflation has also been considered for various costs associated with the inventory system and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy. Numerical example is also used to study the behaviour of the model. Cost minimization technique is used to get the expressions for total cost and other parameters.

Keywords—Two warehouses, deterioration, Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy.

I. INTRODUCTION

Many researchers extended the EOQ model to time-varying demand patterns. Some researchers discussed of inventory models with linear trend in demand. The main limitations in linear-time varying demand rate is that it implies a uniform change in the demand rate per unit time. This rarely happens in the case of any commodity in the market. In recent years, some models have been developed with a demand rate that changes exponentially with time. For seasonal products like clothes, Air conditions etc. at the end of their seasons the demand of these items is observed to be exponentially decreasing for some initial period. Afterwards, the demand for the products becomes steady rather than decreasing exponentially. It is believed that such type of demand is quite realistic. Such type situation can be represented by ramp type demand rate.

An important issue in the inventory theory is related to how to deal with the unfulfilled demands which occur during shortages or stock outs. In most of the developed models researchers assumed that the shortages are either completely backlogged or completely lost. The first case, known as backordered or backlogging case, represent a situation where the unfulfilled demand is completely back ordered. In the second case, also known as lost sale case, we assume that the unfulfilled demand is completely lost.

II. GENETIC ALGORITHM

The principles of Genetic Algorithms (GA) and the mathematical framework underlying it were developed in the late 1960s (Holland, 1962; Kristinson and Dumont, 1992; Koppen et al., 2006). GA is normally discussed in the context of Evolutionary Computing (EC). The core methodologies of EC are Genetic Algorithms (GA), Evolutionary Programming (EP), Evolution Strategies (ES) and Genetic Programming (GP) (Oduguwa et al., 2005). In GA, attempt is made to model the processes underlying population genetic theory by using random search. GAs uses the survival-of-the-fittest strategy, where stronger individuals in a population have a higher chance of creating an offspring. To achieve this, the current input (population) is used to create a new and better population based on specified constraints. The inputs are normally represented as string and they model chromosome in human genetics. In materials engineering, for example, the input string will represent some properties of materials that are of interest.

One iteration of the algorithm is referred to as a generation. The basic GA is very generic and there are many aspects that can be implemented differently according to the problem (for instance, representation of solution or chromosomes, type of encoding, selection strategy, type of crossover and mutation operators, etc.) in practice, GA are implemented by having arrays of bits.
or characters to represent the chromosomes. The individuals in the population then go through a process of simulated evolution. Simple bit manipulation operations allow the implementation of crossover, mutation and other operations. The number of bits for every gene (parameter) and the decimal range in which they decode are usually the same but precludes the utilization of a different number of bits or range.

Fig: Flow chart of basic genetic algorithm

III. PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy introduced by Kennedy and Eberhart in 1995 is a population based evolutionary computation technique. It has been developed by simulating bird flocking fish schooling or sociological behaviour of a group of people artificially. Here the population of solution is called swarm which is composed of a number of agents known as particles. Each particle is treated as point in d-dimensional search space which modifies its position according to its own flying experience and that of other particles present in the swarm. The algorithm starts with a population (swarm) of random solutions (particles). Each particle is assigned a random velocity and allowed to move in the problem space. The particles have memory and each of them keeps track of its previous (local) best position.

Concept of modification of a searching point by PSO.

IV. RELATED WORK

Buzacott (1975) developed the first EOQ model taking inflationary effects into account. In this model, a uniform inflation was assumed for all the associated costs and an expression for the EOQ was derived by minimizing the average annual cost. Misra (1975, 1979) investigated inventory systems under the effects of inflation. Bierman and Thomas (1977) suggested the inventory decision policy under inflationary conditions. An economic order quantity inventory model for deteriorating items was developed by Bose et al. (1995). Authors developed inventory model with linear trend in demand allowing inventory shortages and backlogging. The effects of inflation and time-value of money were incorporated into the model. Hariga and Ben-Daya (1996) then discussed the inventory replenishment problem over a fixed planning horizon for items with linearly time-varying demand under inflationary conditions. Ray and Chaudhuri (1997) developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on-hand inventory at that instant. The effects of inflation and time value of money were taken into account. The effects of inflation and time-value of money on an economic order quantity model have been discussed by Moon and Lee (2000). The two-warehouse inventory models for deteriorating items with constant demand rate under inflation were developed by Yang (2004). The shortages were allowed and fully backlogged in the models. Some numerical examples for illustration were provided. Models for ameliorating / deteriorating items with time-varying demand pattern over a finite planning horizon were proposed by Moon et al. (2005). The effects of inflation and time value of money on an economic order quantity model have been discussed by Moon et al. (2005). The effects of inflation and time value of money were taken into account. An inventory model for deteriorating items with stock-dependent consumption rate with shortages was produced by Hou (2006). Model was developed under the effects of inflation and time discounting over a finite planning horizon. Jaggi et al. (2007) presented the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon. Shortages in inventory were allowed and completely backlogged and demand rate was assumed to be a function of inflation. Two stage inventory problems over...
finite time horizon under inflation and time value of money was discussed by Dey et al. (2008). The concept of soft computing techniques (fuzzy logic) first introduced by Zadeh (1965). The invention of soft computing techniques (fuzzy set theory or fuzzy logic) by the need to represent and capture the real world problem with its fuzzy data due to uncertainty. Instead of ignoring or avoiding uncertainty, Zadeh developed a set theory to remove this uncertainty. It is to use hybrid intelligent methods to quickly achieve an inexact solution rather than use an exact optimal solution via a big search. Since Genetic Algorithms are good for adaptive studies and fuzzy logic can be used to solve complex problems using linguistic rule-based techniques. Silver and Peterson (1985) discussed on decision systems for inventory management and production planning. Zimmermann (1985) gives a review on fuzzy set theory and its applications. Bard and Moore (1990) discussed a model for production planning with variable demand. Avraham (1999) presented a review on enterprise resource planning (ERP).


V. ASSUMPTION AND NOTATIONS

The mathematical model of two warehouse inventory model for deteriorating items is based on the following notation and assumptions

Notations:

- $O_i$: Cost of Ordering per Order
- $\phi$: Capacity of OW.
- $K$: The length of replenishment cycle.
- $M$: Maximum inventory level per cycle to be ordered.
At $k=0$, conditions as follows:

- $k_1$: The time up to which product has no deterioration.
- $k_2$: The time at which inventory level reaches to zero in RW.
- $k_3$: The time at which inventory level reaches to zero in OW.

- $H^{RW}_1$: The holding cost per unit time in OW i.e. $H^{RW}_1=(u+1)_1$; where $(u+1)_1$ is a positive constant.
- $H^{RW}_2$: The holding cost per unit time in RW i.e. $H^{RW}_2=(u+1)_2$, where $(u+1)_2>0$ and $H^{RW}_1>H^{RW}_2$.
- $S_2$: The shortages cost per unit per unit time.

- $\Psi^{RW}(k)$: The level of inventory in RW at time $[0 \ k_1]$ in which the product has no deterioration.
- $\Psi^{OW}(k)$: The level of inventory in OW at time $[0 \ k_1]$ in which the product has no deterioration.
- $\Psi^{RW}(k)$: The level of inventory in OW at time $[k_1 \ k_2]$ in which the product has no deterioration.
- $\Psi^{RW}(k)$: The level of inventory in OW at time $[k_1 \ k_2]$ in which only Deterioration takes place.
- $\Psi^{RW}(k)$: The level of inventory in OW at time $[k_1 \ k_2]$ in which only Deterioration takes place.
- $\Psi^1(k)$: Determine the inventory level at time $k$ in which the product has shortages.
- $\Psi^2(k)$: Determine the inventory level at time $k$ in which the product has shortages.

\[ (\nu - 1) : \text{Deterioration rate in RW Such that } 0 < (\nu-1) < 1; \]
\[ (\omega + 1) : \text{Deterioration rate in OW such that } 0 < (\omega+1) < 1; \]

$R_d$: Deterioration cost per unit in RW. $k$

$O_d$: Deterioration cost per unit in OW.

### VI. MATHEMATICAL FORMULATION OF MODEL AND ANALYSIS

In the beginning of the cycle at $k=0$ a lot size of $M$ units of Inventory enters in to the system in which backlogged (M-R) units are cleared and the remaining units $R$ is kept in to two storage as $W$ units in OW and RW units in RW.

\[
dv^{RW}(k) = -(ab+u)k ; \quad 0 \leq k \leq k_1
\]

\[
dv^{RW}(k) = -(\nu-1)\Psi^{RW}(k) - (ab+u)k ; \quad k_1 \leq k \leq k_2
\]

\[
dv^{RW}(k) = 0 ; \quad 0 \leq k \leq k_1
\]

\[
dv^{RW}(k) = -(\omega+1)\Psi^{RW}(k) ; \quad k_2 \leq k \leq k_2
\]

\[
dv^{RW}(k) = -(ab+u)k ; \quad k_2 \leq k \leq K
\]

Now Inventory level at different time intervals is given by solving the above differential equations (1) to (6) with boundary conditions as follows:

At $k=0$,

\[ \Psi^{RW}(k)=R-W \]

There fore Differential eq. (1) gives

\[ \Psi^{RW}(k)=R-W - \frac{(ab+u)k^2}{2} ; \quad 0 \leq k \leq k_1 \]

Differential eq. (2) is solved at $k=k_1$ and boundary condition $\Psi^{RW}(k_1)=0$, which yields

\[ \Psi^{RW}(k)=\frac{(ab+u)}{2}\left(\frac{(v-1)k_2-1}{(v-1)(k_2-k)}-(v-1)(k_2-1)\right) ; \quad k_1 \leq k \leq k_2 \]

Solution of differential eq. (3) with boundary condition at $k=0$ and $\Psi^{OW}(0)=W$

\[ \Psi^{OW}(k) = \varphi ; \quad 0 \leq k \leq k_1 \]

Differential eq. (4) yields at $k=k_1$, and $\Psi^{OW}(k_1)=\varphi$

\[ \Psi^{OW}(k) = \varphi e^{(\alpha+1)(k_1-k)} ; \quad k_1 \leq k \leq k_2 \]

$K_{PC}(k_2,k_3,K)$: The total relevant inventory cost per unit time of inventory system.

Assumption

1. Replenishment rate is infinite and lead time is negligible i.e. zero.
2. Holding cost is variable and is linear function of time.
3. The time horizon of the inventory system is infinite.
4. Goods of OW are consumed only after the consumption of goods kept in RW due to the more holding cost in RW than in OW.
5. The OW has the limited capacity of storage and RW has unlimited capacity.
6. Demand vary with time and is linear function of time and given by $D(k)=(ab+u)k$; where $(ab+u)>0$
7. For deteriorating items a fraction of on hand inventory deteriorates per unit time in both the warehouse with different rate of Deterioration.
8. Shortages are allowed and demand is fully backlogged at the beginning of next replenishment.
9. The unit inventory cost (Holding cost + Deterioration cost) in RW>OW.
Solution of eq. (5) at k= k₁ and Ψ_{1OW}(k₁) = 0 gives
\[ Ψ_{1OW}(k) = \frac{(ab+u)}{(ω+1)} \sum_{k=0}^{k-1} k (k+1)(k+2) \]
\[ k₁ ≤ k ≤ K \]  
(11)
Lastly the solution of eq. (6) at k=k₃ and Ψ_{1IS}(k₃)=0, is given as
\[ Ψ_{1IS}(k) = \frac{(ab+u)}{2} \left( k^{2} - k^₁ \right) ; \]
\[ k₁ ≤ k ≤ K \]  
(12)
Now considering the continuity of Ψ_{1R}(k₁)=Ψ_{2R}(k₁), at k=k₁ from eq. (7) & (8) we have
\[ R = \frac{(ab+u)}{ω} \left( k^{2} - k^₁ \right) + \frac{u}{(ω+1)} \left( k^{2} - k^₁ \right) \]
\[ k₁ ≤ k ≤ K \]  
(13)
Substituting eq. (13) in to eq. (7) we have
\[ Ψ_{1RW}(k) = \frac{(ab+u)}{(ω+1)} \left( k^{2} - k^₁ \right) + \frac{u}{(ω+1)} \left( k^{2} - k^₁ \right) \]
\[ k₁ ≤ k ≤ K \]  
(14)
Cost of Inventory deteriorated in RW is given by
\[ C_{Ψ_{1RW}} = \sum_{k=0}^{k₁} R \left( k^{2} - k^₁ \right) \]  
(15)
The maximum Inventory to be ordered is
\[ M = R + IS \]  
(16)
Next the total relevant Inventory Cost per cycle consists of the following elements:

1. Cost of ordering
\[ C_{O} = \frac{(ab+u)}{2} \left( k^{2} - k^₁ \right) \]  
(17)

2. Inventory holding Cost in RW
\[ C_{Ψ_{1RW}} = \int_{0}^{k₁} \left( \frac{(ab+u)}{(ω+1)} \left( k^{2} - k^₁ \right) + \frac{u}{(ω+1)} \left( k^{2} - k^₁ \right) \right) dk \]  
(18)

3. Inventory holding Cost in OW
\[ C_{Ψ_{1OW}} = \int_{0}^{k₁} \left( \frac{(ab+u)}{(ω+1)} \left( k^{2} - k^₁ \right) + \frac{u}{(ω+1)} \left( k^{2} - k^₁ \right) \right) dk \]  
(19)

4. Cost of Inventory deteriorated in RW
\[ C_{Ψ_{1RW}} = \int_{0}^{k₁} R \left( k^{2} - k^₁ \right) dk \]  
(20)

5. Cost of Inventory deteriorated in OW
\[ C_{Ψ_{1OW}} = \int_{0}^{k₁} R \left( k^{2} - k^₁ \right) dk \]  
(21)

6. Inventory Shortages Cost
\[ C_{IS} = \sum_{k=0}^{k₁} \left( \frac{(ab+u)}{6} \left( k^{2} - k^₂ \right) \right) K \]  
(22)

K^C(k₁, k₃, K) = \frac{1}{K} \left[ \text{Ordering cost + Inventory holding cost per cycle in RW + Inventory holding cost per cycle in OW + Deterioration cost per cycle in RW + Deterioration cost per cycle in OW + Shortage cost} \right]  
(23)

K^C(k₁, k₃, K) = \frac{1}{K} \left[ OC + \frac{b²}{8} k₃ + \frac{(ab+u)(u+1)}{(ω+1)^2} - \frac{2(ω+1)}{(ω+1)^2} \right]  
(24)
VII. FUZZY MODEL:

Triangular fuzzy number: A triangular fuzzy number is specified by the triplet (a, b, c) where a ≤ b ≤ c and defined by its continuous membership function λ_A:

\[ \lambda_A(y) = \begin{cases} \frac{y-a}{b-a} & \text{if } a \leq y \leq b \\ \frac{c-y}{c-b} & \text{if } c \leq y \leq c \\ 0 & \text{otherwise} \end{cases} \]

The above developed model is a crisp model in which all assumed parameters are fixed but in fuzzy environment, the parameter’s values are not fixed and fluctuated around a fixed point in some interval.

Using equation (24) and fuzzy parameters

\[ H^0 = (h_1), \]
\[ H^R = (h_s), \]
\[ S_c = (S_c^2), \]
\[ P_c = (P_c^2) \]

And

\[ k_1 = (d^-), \] we have,

VIII. PARTICLE SWARM OPTIMIZATION (PSO)

| Z^v | particle’s velocity |
| S^v | particle’s position |
| ν | number of elements in a particle, |
| q | inertia weight of the particle, |
| m | generation number, |
| U_1, U_2 | acceleration constants, |
| e | and random value between 0 and 1 |
| P^v_{best} | local best position of the particle, |
| R^v_{best} | global best position of particle in the swarm |

Cost function \( T^C(k_2,k_3,K) \)

\begin{align*}
| & \text{Cost function} & T^C(k_2,k_3,K) \\
| & k_2 & 1.12455 \\
| & k_3 & 2.14757 \\
| & K & 17.1285 \\
| & \text{Total relevant cost} & 1011.15 \\
| & \text{Particle Swarm Optimization (PSO)} & 534.51 \\
| & \text{Genetic Algorithms (GA)} & 533.21 \\
| & \text{Fuzzy} & 532.11 \\
\end{align*}

IX. GENETIC ALGORITHMS MODEL

When compared to other evolutionary algorithms one of the most important GA feature is its focus on fixed length character strings although variable length strings and other structures have been used.

Step 1: Start:- “Randomly generate population of n chromosomes as per population size.”

Step 2: Fitness:- “Evaluate the fitness f(y) of each chromosome y in the population”

Step 3: New population:- “Create new population by repeating following steps until the new population is complete.”

a. Selection: - “Select two parent chromosomes from a population.”

b. Crossover: - “With a crossover probability, crossover the parents to form a new offspring. If no crossover was performed, offspring is the exact copy of parents.”

c. Mutation: - “With a mutation probability, mutate new offspring at each locus.”

d. Accepting: - “Place new offspring in the new population.”

Step 4: Replace: - “Use new generated population for a future run of the algorithm.”

Step 5: Test: - “If the end condition is satisfied, stop, and return the best solution in current population.”

Step 6: Loop: - “Go to step 2”.

Step 7: Stop: - “Stop when the fittest value is obtained.”

We are using those basic steps for finding the optimal resources for an organization in Medium range prospective using MATLAB software package10.

Numerical Example:

In order to illustrate the above solution procedure, consider an inventory system with the following data in appropriate units: C=100, φ =75, (ab+u)=5, (u+1)1=1.5, (u+1)2=1.2, t_1=0.3, (v-1)=0.65, (o+1)=0.23, C_0=3.4, and C_0=3.2. The values of decision variables are computed for the model and also for the models of special cases. The computational optimal solutions of the models are shown in Table-1.

<table>
<thead>
<tr>
<th>Cost function</th>
<th>T^C(k_2,k_3,K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_2</td>
<td>1.12455</td>
</tr>
<tr>
<td>k_3</td>
<td>2.14757</td>
</tr>
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</tr>
<tr>
<td>Fuzzy</td>
<td>532.11</td>
</tr>
</tbody>
</table>

X. CONCLUSION

In this study, we have future a deterministic two-warehouse facilities inventory model for two-warehouse inventory model deteriorating items time varying demand and two-warehouse inventory model holding cost varying with respect to ordering cycle length with the objective of minimizing the total inventory cost and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy. Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy Two-warehouse inventory model shortages are allowed and two-warehouse inventory model partially backlogged and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy. Furthermore the proposed model is very useful for
two-warehouse inventory model deteriorating items. This model can be further extended by incorporation with other deterioration rate probabilistic demand pattern and Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and Fuzzy.

REFERENCES