Electronic Journal of Graph Theory and Applications **6** (1) (2018), 201–207

Electronic Journal of Graph Theory and Applications

# Computing the edge irregularity strengths of chain graphs and the join of two graphs

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**Abstract**

In computer science, graphs are used in variety of applications directly or indirectly. Especially quantitative labeled graphs have played a vital role in computational linguistics, decision making software tools, coding theory and path determination in networks. For a graph $G(V, E)$ with the vertex set $V$ and the edge set $E$, a vertex $k$-labeling $\phi : V \to \{1, 2, \dots, k\}$ is defined to be an *edge irregular $k$-labeling* of the graph $G$ if for every two different edges $e$ and $f$ their $w_\phi(e) \neq w_\phi(f)$, where the weight of an edge $e = xy \in E(G)$ is $w_\phi(xy) = \phi(x) + \phi(y)$. The minimum $k$ for which the graph $G$ has an edge irregular $k$-labeling is called the *edge irregularity strength* of $G$, denoted by $es(G)$. In this paper, we determine the edge irregularity strengths of some chain graphs and the join of two graphs. We introduce a conjecture and open problems for researchers for further research.

*Keywords:* edge irregularity strength, blocks, chain graphs, join of graphs
Mathematics Subject Classification : 05C78, 05C12
DOI: 10.5614/ejgta.2018.6.1.15

## 1. Introduction

A graph $G(V, E)$ with the vertex set $V$ and the edge set $E$ is connected if for any pair of vertices in $G$ there exists a path connecting them. For a graph $G$, the degree of a vertex $v$ is the number of

Received: 25 September 2017, Revised: 10 March 2018, Accepted: 31 March 2018.

edges incident to $v$ and denoted by $d(v)$. Two vertices are adjacent if and only if there is an edge between them.

A graph labeling is an assignment of integers to the vertices or edges or both with subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a *vertex labeling* (or an *edge labeling*). If the domain is $V(G) \cup E(G)$ then we call the labeling a *total labeling*. Thus, for an edge $k$-labeling $\phi : E(G) \to \{1, 2, \ldots, k\}$ the associated weight of a vertex $x \in V(G)$ is

$$w_\phi(x) = \sum \phi(xy),$$

where the sum is over all vertices $y$ adjacent to $x$.

Chartrand *et al.* [9] introduced edge $k$-labeling $\phi$ of a graph $G$ such that $w_\phi(x) \neq w_\phi(y)$ for all vertices $x, y \in V(G)$ with $x \neq y$. Such labelings were called *irregular assignments* and the *irregularity strength* $s(G)$ of a graph $G$ is known as the minimum $k$ for which $G$ has an irregular assignment using labels at most $k$. This parameter has attracted much attention [5, 6, 8, 11].

In 2007, Bača *et al.* [7] investigated two modifications of the irregularity strength of graphs, namely a *total edge irregularity strength*, denoted by $tes(G)$ and a *total vertex irregularity strength*, denoted by $tvs(G)$. Some results on total edge irregularity strength and total vertex irregularity strength can be found in [1, 2, 3, 12, 13].

Motivated by these papers, Ahmad *et al.* [4] introduced the following irregular labeling: A vertex $k$-labeling $\phi : V \to \{1, 2, \ldots, k\}$ is defined to be an *edge irregular $k$-labeling* of the graph $G$ if for every two different edges $e$ and $f$ their $w_\phi(e) \neq w_\phi(f)$, where the weight of an edge $e = xy \in E(G)$ is $w_\phi(xy) = \phi(x) + \phi(y)$. The minimum $k$ for which the graph $G$ has an edge irregular $k$-labeling is called the *edge irregularity strength* of $G$ denoted by $es(G)$.

The following theorem that is proved in [4], establishes a lower bound for the edge irregularity strength of a graph $G$.

**Theorem 1.1.** *[4] Let $G = (V, E)$ be a simple graph with maximum degree $\Delta = \Delta(G)$. Then,*

$$es(G) \geq \max\left\{ \left\lceil \frac{|E(G)| + 1}{2} \right\rceil, \Delta(G) \right\}.$$

In [4] it is shown that for a path $P_n, n \geq 2, es(P_n) = \lceil \frac{n}{2} \rceil$, for a star $K_{1,n}, n \geq 1, es(K_{1,n}) = n$, for a double star $S_{m,n}, 3 \leq m \leq n, es(S_{m,n}) = n$ and for the Cartesian product of two paths $P_n$ and $P_m, m, n \geq 2, es(P_n \Box P_m) = \lceil \frac{2mn-m-n+1}{2} \rceil$. I. Tarawneh *et al.* [14, 15] determined the edge irregularity strength of the corona product of graphs with paths and cycle with isolated vertices.

## 2. Edge irregularity strength of chain graphs

A *chain graph* is a graph with blocks $B_1, B_2, \ldots, B_n$ such that for every $i$, $B_i$ and $B_{i+1}$ have a common vertex in such a way that the block-cut- vertex graph is a path. We will denote the chain graph with $n$ blocks $B_1, B_2, \ldots, B_n$ by $C[B_1, B_2, \ldots, B_n]$. If $B_1 = B_2 = \cdots = B_n = B$. we will write $C[B_1, B_2, \ldots, B_n]$ as $C[B^{(n)}]$. Suppose that $c_1, c_2, \ldots, c_{n-1}$ are the consecutive cut vertices of $C[B_1, B_2, \ldots, B_n]$. In the next theorem, we study the edge irregularity strength of chain graphs whose blocks are combination of $C_4$.

**Theorem 2.1.** *For $n \geq 2$, the edge irregularity strength of $C[C_4^{(n)}]$ is $2n+1$.*

*Proof.* Let us consider the vertex set and the edge set of $C[C_4^{(n)}]$ are

$$V(C[C_4^{(n)}]) = \{x_0, y_0\} \cup \{x_1^i, x_2^i : 1 \leq i \leq n\} \cup \{c_1, c_2, \dots, c_{n-1}\}$$
$$E(C[C_4^{(n)}]) = \{c_i x_1^i, c_i x_2^i, c_i x_1^{i+1}, c_i x_2^{i+1} : 1 \leq i \leq n-1\} \cup \{x_0 x_1^1, x_0 x_2^1, y_0 x_1^n, y_0 x_2^n\}.$$

According to the Theorem 1.1, $es(C[C_4^{(n)}]) \geq \max\left\{\left\lceil \frac{4n+1}{2} \right\rceil, 4\right\} = 2n+1$, for $n \geq 2$. For the converse, we define a vertex labeling $\phi$ as follows:

$$\phi(x_0) = 1, \phi(y_0) = 2n+1, \phi(c_i) = 2i+1, \text{ for } 1 \leq i \leq n-1,$$
$$\phi(x_1^i) = 2i-1, \phi(x_2^i) = 2i, \text{ for } 1 \leq i \leq n.$$

Since $w_\phi(x_0 x_1^1) = 2, w_\phi(x_0 x_2^1) = 3, w_\phi(y_0 x_1^n) = 4n, w_\phi(y_0 x_2^n) = 4n+1$ and $w_\phi(c_i x_1^i) = 4i, w_\phi(c_i x_2^i) = 4i+1, w_\phi(c_i x_1^{i+1}) = 4i+2, w_\phi(c_i x_2^{i+1}) = 4i+3$, for $1 \leq i \leq n-1$. It is a routine matter to verify that all vertex labels are at most $2n+1$, and the edge weights form the set of different integers, namely $\{2, 3, 4, \dots, 4n+1\}$. Thus, the labeling $\phi$ is the desired edge irregular $(2n+1)$-labeling. This completes the proof. □

From the Theorem 2.1, we proposed the following conjecture:

**Conjecture 1.** *For $n \geq 2, m \geq 5$, the edge irregularity strength of $C[C_m^{(n)}]$ is $\left\lceil \frac{nm+1}{2} \right\rceil$.*

We denote by $mK_n$-path a chain graph with $m$ blocks where each block is identical and isomorphic to the complete graph $K_n$. We consider edge irregular $k$-labelings of $mK_n$-paths for $n = 2, 3$ and $4$. If $n = 2$ then $mK_2$-path $\cong P_m + 1$. It is well known that $P_n$ has an edge irregular $\lceil \frac{n}{2} \rceil$-labeling. Consequently, $es(P_n) = \lceil \frac{n}{2} \rceil$. If $n = 3$, then $mK_3$-path$\cong C[C_3^{(n)}]$. In the next theorem, we determine the bounds for edge irregularity strength of $mK_3$-path.

**Theorem 2.2.** *If $H_m$ is a $mK_3$-path, then $\left\lceil \frac{3m+3}{2} \right\rceil \leq es(H_m) \leq 2m+1$.*

*Proof.* Let us consider the vertex set and the edge set of $H_m(\cong mK_3$-path$)$:

$$V(H_m) = \{x_i : 1 \leq i \leq m+1\} \cup \{y_i : 1 \leq i \leq m\},$$
$$E(H_m) = \{x_i x_{i+1}, x_i y_i, y_i x_{i+1} : 1 \leq i \leq m\}.$$

Observe that the graph $H_m$ has $2m+1$ vertices, $3m$ edges and $\Delta(H_m) = 4$, for $m \geq 2$. According to the Theorem 1.1, $es(H_m) \geq \max\left\{\left\lceil \frac{3m+1}{2} \right\rceil, 4\right\} = \left\lceil \frac{3m+1}{2} \right\rceil$, for $m \geq 2$. Since every block is a complete graph $K_3$, therefore under every edge irregular labeling no couple of adjacent vertices can be assigned by the same label. This implies that the smallest edge weight 2 is not possible. So if the smallest edge weight is 3 then the largest edge weight will be at least $3m+2$. Since each edge weight is a sum of two labels, at least one label is at least $\left\lceil \frac{3m+2}{2} \right\rceil$, as $3m+2$ is divisible by 2, when $m$ is even, therefore the label of both end vertices of largest edge weight will be $\left\lceil \frac{3m+2}{2} \right\rceil$, which is not possible because no couple of adjacent vertices can be assigned by the same label. Therefore, at least one label is at least $\left\lceil \frac{3m+3}{2} \right\rceil$, for $m$ even. For $m$ odd, $\left\lceil \frac{3m+2}{2} \right\rceil = \left\lceil \frac{3m+3}{2} \right\rceil$. Hence, $es(H_m) \geq \left\lceil \frac{3m+3}{2} \right\rceil$.

For the upper bound, we define a vertex labeling $\phi_1$ as follows:

$$\phi_1(x_1) = 1, \phi(x_i) = 2i-2, \text{ for } 2 \leq i \leq m+1 \text{ and } \phi_1(y_i) = 2i+1, \text{ for } 1 \leq i \leq m.$$

Since $w_{\phi_1}(x_1x_2) = 3, w_{\phi_1}(x_1y_1) = 4, w_{\phi_1}(x_ix_{i+1}) = 4i-2, w_{\phi_1}(x_iy_i) = 4i-1$, for $2 \leq i \leq m$ and $w_{\phi_1}(y_ix_{i+1}) = 4i+1$, for $1 \leq i \leq m$, so the edge weights are distinct for all pairs of distinct edges. Thus, the vertex labeling $\phi_1$ is an optimal edge irregular $(2m+1)$-labeling i. e. $es(H_m) \leq 2m+1$. This completes the proof. □

**Open Problem 1.** *Determine the edge irregularity strength of a $mK_3$-path for $m \geq 2$.*

**Theorem 2.3.** *If $G$ is a $mK_4$-path, then the edge irregularity strength of $G$ is $3m+2$.*

*Proof.* Let us consider the vertex set and the edge set of $G(\cong mK_4$-path):

$$V(G) = \{x_i : 1 \leq i \leq m+1\} \cup \{y_i, z_i : 1 \leq i \leq m\}$$
$$E(G) = \{x_ix_{i+1}, x_iy_i, x_iz_i, y_iz_i, y_ix_{i+1}, z_ix_{i+1} : 1 \leq i \leq m\}.$$

Observe that the graph $G$ has $3m+1$ vertices, $6m$ edges and $\Delta(G) = 6$. According to the Theorem 1.1, $es(G) \geq \max\left\{\left\lceil \frac{6m+1}{2} \right\rceil, 6\right\} = 3m+1$, for $m \geq 2$. Since every block is a complete graph $K_4$, therefore under every edge irregular labeling no two adjacent vertices can be assigned by the same label. This implies that the smallest edge weight 2 is not possible. So if the smallest edge weight is 3 then the largest edge weight will be at least $6m+2$. Since each edge weight is a sum of two labels, at least one label is at least $\left\lceil \frac{6m+2}{2} \right\rceil = 3m+1$, as $6m+2$ is divisible by 2, therefore the label of both end vertices of largest edge weight will be $3m+1$, which is not possible because no two adjacent vertices can be assigned by the same label. Therefore, at least one label is at least $3m+2$. For the converse, we define the vertex labeling $\phi_2$ as follows:

$$\phi_2(x_i) = 3i-1, \text{ for } 1 \leq i \leq m+1 \text{ and } \phi_2(y_i) = 3i, \phi_2(z_i) = 3i-2 \text{ for } 1 \leq i \leq m.$$

Since $w_{\phi_2}(x_ix_{i+1}) = 6i+1, w_{\phi_2}(x_iy_i) = 6i-1, w_{\phi_2}(x_iz_i) = 6i-3, w_{\phi_2}(y_iz_i) = 6i-2, w_{\phi_2}(y_ix_{i+1}) = 6i+2$, and $w_{\phi_2}(z_ix_{i+1}) = 6i$, for $1 \leq i \leq m$. It is a routine matter to verify that all vertex labels are at most $3m+2$. and the edge weights form the set of different integers, namely $\{3, 4, 5, \ldots, 6m+2\}$. This implies that $es(G) \leq 3m+2$, for $m \geq 2$. This completes the proof. □

**Open Problem 2.** *Determine the edge irregularity strength of a $mK_n$-path for $m \geq 2$ and $n \geq 5$.*

## 3. Edge irregularity strength of join of two graphs

There are several ways to produce a new graph from a given pair of graphs. For two vertex-disjoint graphs $G$ and $H$, $G \cup H$ is disconnected graph with the vertex set $V(G) \cup V(H)$ and the edge set $E(G) \cup E(H)$. The join $G + H$ consists of $G \cup H$ and all the edges joining a vertex of $G$ and a vertex of $H$. For detail see [10].

**Theorem 3.1.** *For $n \geq 3$, the edge irregularity strength of $G = K_{1,n} + \overline{K_1}$ is $n+2$.*

*Proof.* Let $G = K_{1,n} + \overline{K_1}$ be a graph with the vertex set $V(G) = \{x, y\} \cup \{x_i : 1 \leq i \leq n\}$ and the edge set $E(G) = \{xy, xx_i, yx_i : 1 \leq i \leq n\}$. Then $|V(G)| = n + 2$, $|E(G)| = 2n + 1$ and $\Delta(G) = n + 1$. According to the Theorem 1.1 $es(G) \geq \max\left\{\left\lceil\frac{2n+2}{2}\right\rceil, n+1\right\} = n + 1$. Since each two adjacent vertices in $G$ are a part of a complete graph $K_3$, therefore under every edge irregular labeling the all vertices in $G$ must contain different labels. Since there are $n + 2$ vertices in $G$, then the maximum vertex label is at least $n + 2$. Therefore $es(G) \geq n + 2$. To prove the equality, it suffices to prove the existence of an optimal edge irregular $(n + 2)$-labeling. Let $\phi : V(G) \to \{1, 2, \dots, n + 2\}$ be a vertex labeling such that

$$\phi(x) = 1, \phi(y) = n + 2, \phi(x_i) = i + 1, \text{ for } 1 \leq i \leq n.$$

Since $w_\phi(xy) = \phi(x) + \phi(y) = n + 3$ and $w_\phi(xx_i) = \phi(x) + \phi(x_i) = i + 2, w_\phi(yx_i) = \phi(y) + \phi(x_i) = n + 3 + i$, for $1 \leq i \leq n$, so the edge weights are distinct for all edges. Thus, the vertex labeling $\phi$ is an optimal edge irregular $(n + 2)$-labeling. This completes the proof. □

When $m = 1$ and $n \geq 1$, $P_m + \overline{K_n}$ is a star $K_{1,n}$, the edge irregularity strength of star is determined in [4] i. e $es(K_{1,n}) = n$. When $m = 2$ and $n \geq 1$, $P_m + \overline{K_n}$ is isomorphic to $K_{1,n} + \overline{K_1}$, the edge irregularity strength of $K_{1,n} + \overline{K_1}$ is determined in Theorem 3.1. Therefore $es(P_m + \overline{K_n}) = n + 2$, for $m = 2$. In the next theorem, we determine the bounds of the edge irregularity strength of $P_m + \overline{K_n}$ for $m \leq 6$ and $n \geq 3$.

**Theorem 3.2.** *For* $3 \leq m \leq 6$ *and* $n \geq 3$,

$$\left\lceil\frac{m(n+1)}{2}\right\rceil \leq es(P_m + \overline{K_n}) \leq \begin{cases} 2n + 2, & \text{for } m = 3, \\ 3n + 3, & \text{for } m = 4, \\ 4n + 3, & \text{for } m = 5, \\ 5n + 4, & \text{for } m = 6. \end{cases}$$

*Proof.* Let us consider the path $P_m$ with $V(P_m) = \{x_1, x_2, \dots, x_m\}$, $E(P_m) = \{x_i x_{i+1} : i \in [1, m-1]\}$. Then the vertex set and the edge set of $P_m + \overline{K_n}$ are

$$V(P_m + \overline{K_n}) = \{y_1, y_2, \dots, y_n\} \cup \{x_1, x_2, \dots, x_m\},$$
$$E(P_m + \overline{K_n}) = \{x_i x_{i+1} : i \in [1, m-1]\} \cup \{x_i y_j : i \in [1, m],\ j \in [1, n]\}.$$

According to the Theorem 1.1 $es(P_m + \overline{K_n}) \geq \max\left\{\left\lceil\frac{m(n+1)}{2}\right\rceil, n+2\right\} = \lceil\frac{m(n+1)}{2}\rceil$, for $m \geq 3$. Since each two adjacent vertices in $P_m + \overline{K_n}$ are a part of complete graph $K_3$, therefore under every edge irregular labeling the all vertices in $P_m + \overline{K_n}$ must contain different labels. For $m = 2$, $P_m + \overline{K_n} \cong K_{1,n} + \overline{K_1}$, therefore the edge irregular labeling $\phi$ of $P_2 + \overline{K_n}$ is already defined in Theorem 3.1. Let us define $\phi(x_3) = 2n + 2, \phi(x_4) = 3n + 3, \phi(x_5) = 4n + 3$ and $\phi(x_6) = 5n + 4$. By using Theorem 3.1 and the labels of $x_3, x_4, x_5, x_6$. We obtain the vertex labeling $\phi$ of $P_m + \overline{K_n}$, for $2 \leq m \leq 6$. Since $w_\phi(x_1x_2) = n + 3, w_\phi(x_2x_3) = 3n + 4, w_\phi(x_3x_4) = 5n + 5, w_\phi(x_4x_5) = 7n + 6, w_\phi(x_5x_6) = 9n + 7$ and $w_\phi(x_1y_j) = j + 2, w_\phi(x_2y_j) = n + 3 + j, w_\phi(x_3y_j) = 2n + 3 + j, w_\phi(x_4y_j) = 3n + 4 + j, w_\phi(x_5y_j) = 4n + 4 + j, w_\phi(x_6y_j) = 5n + 5 + j$, for $1 \leq j \leq n$, so the edge weights are distinct for all edges. Thus, the vertex labeling $\phi$ is the required edge irregular labeling, which shows that

$$es(P_m + \overline{K_n}) \leq \begin{cases} 2n+2, & \text{for } m = 3, \\ 3n+3, & \text{for } m = 4, \\ 4n+3, & \text{for } m = 5, \\ 5n+4, & \text{for } m = 6. \end{cases}$$

This completes the proof. □

**Open Problem 3.** *Find the edge irregularity strength of $P_m + \overline{K_n}$ for any $n \geq 1$ and $m \geq 7$.*

**Theorem 3.3.** *Let $H_1 = K_{1,m}$ and $H_2 = K_{1,n}$. Let $V(H_1) = \{x, x_1, x_2, \ldots, x_m\}$ and $V(H_2) = \{y, y_1, y_2, \ldots, y_n\}$ with $d(x) = m, d(y) = n$. Then the graph $G$ obtained by joining $x$ to all vertices of $H_2$ and $y$ to all vertices of $H_1$ has the edge irregularity strength $m + n + 2$.*

*Proof.* Let us consider the vertex set $V(G) = \{x, y, x_i, y_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and the edge set $E(G) = \{xy, xx_i, xy_j, yy_j, yx_i : 1 \leq i \leq m, 1 \leq j \leq n\}$. Suppose that $m \leq n$. This implies that the maximum degree $\Delta(G) = n + 1$. According to the Theorem 1.1, $es(G) \geq \max\left\{\left\lceil \frac{2m+2n+2}{2} \right\rceil, n+1\right\} = m + n + 1$. Since each two adjacent vertices in $G$ are a part of complete graph $K_3$, in this way under every edge irregular labeling the smallest edge weight has to be at least 3 and the largest edge weight has to be at least $2m + 2n + 3$. Since the edge weight $2m + 2n + 3$ is the sum of two labels, so at least one label is at least $\left\lceil \frac{2m+2n+3}{2} \right\rceil = m + n + 2$. Therefore $es(G) \geq m + n + 2$.

To prove the equality, it suffices to prove the existence of an optimal edge irregular $(m+n+2)$-labeling. Let $\phi : V(G) \to \{1, 2, \ldots, m + n + 2\}$ be a vertex labeling such that

$$\phi(x) = 1, \phi(y) = m + n + 2,$$
$$\phi(x_i) = i + 1, \text{ for } 1 \leq i \leq m,$$
$$\phi(y_j) = m + 1 + j, \text{ for } 1 \leq j \leq n.$$

Since $w_\phi(xy) = \phi(x) + \phi(y) = m + n + 3$, $w_\phi(xx_i) = \phi(x) + \phi(x_i) = i + 2$, $w_\phi(yx_i) = \phi(y) + \phi(x_i) = m + n + 3 + i$, for $1 \leq i \leq m$ and $w_\phi(yy_i) = \phi(y) + \phi(y_j) = 2m + n + 3 + j$, $w_\phi(xy_j) = \phi(x) + \phi(y_j) = m + 2 + j$, for $1 \leq j \leq n$, so the edge weights are distinct for all edges. Thus, the vertex labeling $\phi$ is an optimal edge irregular $(m + n + 2)$-labeling. This completes the proof. □

**Acknowledgement**

The authors are grateful to the anonymous referee for useful comments. We thank the Deanship of Research, Jazan University for supporting this project under the Grant No. 000114/7/37.

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