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On classes of neighborhood resolving sets of a graph

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Abstract

Let $G = (V, E)$ be a simple connected graph. A subset S of V is called a neighbourhood set of G if $G = \bigcup_{s \in S} \langle N[s] \rangle$, where $N[v]$ denotes the closed neighbourhood of the vertex v in G. Further for each ordered subset $S = \{s_1, s_2, \ldots, s_k\}$ of V and a vertex $u \in V$, we associate a vector $\Gamma(u/S) = (d(u, s_1), d(u, s_2), \ldots, d(u, s_k))$ with respect to S, where $d(u, v)$ denote the distance between u and v in G. A subset S is said to be resolving set of G if $\Gamma(u/S) \neq \Gamma(v/S)$ for all $u, v \in V - S$. A neighbouring set of G which is also a resolving set for G is called a neighbourhood resolving set (nr-set). The purpose of this paper is to introduce various types of nr-sets and compute minimum cardinality of each set, in possible cases, particularly for paths and cycles.

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1. Introduction

All the graphs considered in this paper are connected, simple, undirected, and finite. Let p_1 be a graph property satisfied by at least one subset of vertices of G. Then such subsets S which satisfies the property p_1 are called p_1-sets of G. A p_1-set S of G is called a P_1-set if $\bar{S}$ is not a p_1-set of G. A $p_1^\star$-set of G is a set S such that both S and $\bar{S}$ are p_1-sets of G. A $P_1^\star$-set of G is a

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set S such that both S and $\bar{S}$ are not p_1-sets of G. If p_2 is another graph property satisfied by any subset of vertices of G, then a set S which satisfies both the property p_1 and p_2 is called a p_1p_2-set. If S is a p_1-set and also a $p_2^\star$-set, then we say S is a $p_1p_2^\star$-set. Similarly, $p_1p_2p_3$-sets, $p_1P_2^\star p_3$-sets, $p_1P_2P_3^\star$-sets, etc., are defined.

A pq-set is said to be a minimal pq-set of G if none of its proper subsets are pq-set of G. The minimum cardinality of a minimal pq-set of G is called lower pq number of G and is denoted by $l_{pq}(G)$.

Let G be a graph and v be a vertex of G. Let $N(v)$ be the set of vertices adjacent to v in G and $N[v] = N(v) \cup \{v\}$. A subset S of vertex set of G is called a neighbourhood set or an n-set of G if $G = \bigcup_{v \in S} \langle N[v] \rangle$, where $\langle N(v) \rangle$ is the subgraph of G induced by the set S. Further a subset S of a vertex set of G is called a resolving set or an r-set of G if for each pair $u, v \notin S$ there is a vertex $w \in S$ with the property that $d(v, w) \neq d(u, w)$.

The metric dimension of G, denoted by $\beta(G)$, is the minimum cardinality of all the resolving sets of G. A resolving set with minimum cardinality is called a *metric basis*. The concept of Metric dimension was introduced by F. Harary and R.A. Melter [3] and independently by P.J. Slater [13] under the term locating set. For more works on metric dimension, we refer [2, 5, 6, 7, 10, 11, 12, 14, 15].

The neighbourhood number of a graph was introduced by E. Sampathkumar et al. in [8] and studied the relationship of $l_n(G)$ (denoted by n_0) with some other known graph parameters.

If S is both neighbourhood and resolving, then in the above notation we write S as an nr-set. The terms not defined here may found in [1]. Throughout this paper P_k denotes a path on k vertices with a vertex set $V = \{v_i : 1 \leq i \leq k\}$ and an edge set $E = \{v_iv_{i+1} : 1 \leq i \leq k-1\}$. Similarly, C_k denotes a cycle on k vertices with a vertex set $V = \{v_i : 1 \leq i \leq k\}$ and an edge set $E = \{v_iv_{i+1}\} \bigcup \{v_1v_k\}$.

Remark 1.1. From the definition of a resolving set, it is clear that any 2-element subset of vertices of a path P_k is always an r-set of P_k. In fact, if $S = \{a, b\}$ and u, v be arbitrary vertices of P_k such that $d(u, a) = d(v, a)$, then a is the central vertex of the uv-path in P_k, but then exactly one of the paths, ub-path or vb-path, in P_k contains the vertex a and hence $d(u, b) \neq d(v, b)$.

Remark 1.2. A singleton set $S = \{v\}$ is a resolving set of a path P if and only if v is an end vertex of P_k.

Remark 1.3. A subset of vertices of P_k containing an end vertex is always a resolving set of P_k.

Remark 1.4. For a connected graph G of order k, every subset of cardinality at least $k-1$ is always an n-set.

Remark 1.5. Since a superset of any r-set of a graph G is also an r-set of the graph G, it follows from Remark 1.1 that every i-element subset of the vertex set of a path P_k is always an r-set of P_k, for every i, $2 \leq i \leq k$.

Observation 1.1. *Every n-set of a path P_k has at least 2 elements, whenever $k \geq 4$.*

Observation 1.2. *Every r-set of a path P_k, $2 \leq k \leq 3$, contains a pendent vertex.*

We recall the following for immediate reference;

Theorem 1.1 (S. Khuller, B. Raghavachari, and A. Rosenfeld [6]). *For a simple connected graph G, $\beta(G) = 1$ if and only if $G \cong P_k$.*

Theorem 1.2 (F. Harary and R.A.Melter [3]). *For any integer $k \geq 3$, the metric dimension of a cycle on k vertices is 2.*

Theorem 1.3 (B. Sooryanarayana [14]). *A graph G with $\beta(G) = k$, cannot contain $k_{2^k+1} - (2^{k-1} - 1)e$ as a subgraph.*

Theorem 1.4 (E. Sampathkumar and Prabha S. Neeralagi [9]). *For a path P_k on k vertices, the lower neighbourhood number $l_n(P_k) = \lfloor \frac{k}{2} \rfloor$.*

Theorem 1.5 (E. Sampathkumar and Prabha S. Neeralagi [8]). *For a cycle C_k of length $k \geq 4$, the lower neighbourhood number $l_n(C_k) = \lceil \frac{k}{2} \rceil$.*

Theorem 1.6 (E. Sampathkumar and Prabha S. Neeralagi [8]). *A set S of vertices of a graph G is an n-set if and only if every line of $\langle V(G) - S \rangle$ belongs to a triangle one of whose vertices belong to S.*

2. nr-sets and Dimensions of a Path

Theorem 2.1. *For any integer $k \geq 1$, $l_{nr}(P_k) = \begin{cases} \lceil \frac{k}{2} \rceil, & \text{for } k \leq 3, \\ \lfloor \frac{k}{2} \rfloor, & \text{for } k \geq 4. \end{cases}$*

Proof. For the case $k = 1, 2$, it is easy to see that any singleton subset of $V(P_k)$ is always an nr-set. For $k = 3$, a singleton subset containing an end vertex is not an n-set and a singleton subset containing the central vertex is not an r-set of P_3. Therefore, every nr-set should have at least two elements. Further, as any subset $S \subseteq V(P_3)$ with $|S| = 2$ is an nr-set for $P_3, l_{nr}(P_3) = 2$. Now for $k \geq 4$, any subset $S \subseteq V(P_k)$ containing two or more elements is always an r-set (by Remark 1.5). Therefore, as $l_n(P_k) \geq 2$ for all $k \geq 4$, it follows that $l_{nr}(P_k) = l_n(P_k) = \lfloor \frac{k}{2} \rfloor$(by Theorem 1.4). □

Theorem 2.2. *For any integer $k \geq 1$, $l_{nR}(P_k) = \begin{cases} k, & \text{for } k = 1, 2, \\ k - 1, & \text{for } k \geq 3. \end{cases}$*

Proof. Let S be an nR-set of a path P_k. Then S is an r-set and $\bar{S}$ is not an r-set. So, by Remark 1.1 and Remark 1.3, it follows that a minimal R-set S should contain both the end vertices and is of cardinality at least $k - 1$ whenever $k \geq 3$ or exactly k if $k \leq 2$. But then, by Remark 1.4, S is an n-set of P_k. Hence $l_{nR} = k - 1$ if $k \geq 3$ or $l_{nR} = k$ if $k \leq 2$. □

Theorem 2.3. *For any integer $k \geq 1$, $l_{NR}(P_k) = \begin{cases} k, & \text{for } k \leq 2, \\ k - 1, & \text{for } k \geq 3. \end{cases}$*

Proof. Follows by the proof of the previous Theorem 2.2, as each nR-set S of P_k is also an NR-set of P_k (Since the set $\bar{S}$ contains at most one element which is non-end vertex and hence by Observation 1.1 and Observation 1.2, $\bar{S}$ is not an n-set if $k \neq 3$ and not an r-set if $k = 3$). □

Lemma 2.1. *Any independent set S of vertices of a path P_k contains more than $\frac{k}{2}$ vertices is always an n-set.*

Proof. Let S be an independent set of the path P_k contains more than $\frac{k}{2}$ vertices. Then k is odd, $S = \{v_1, v_3, v_5, \ldots, v_{k-2}, v_k\}$, and $\bigcup_{v \in S} N[v] = V(P_k)$. Let $e_i = v_i v_{i+1}$ be an edge of P_k, $1 \leq i \leq k-1$. Then e_i is an edge of either $\langle N[v_i] \rangle$ or $\langle N[v_{i+1}] \rangle$ depending upon whether i is odd or even. Hence for each i, the edge $e_i \in \langle N[v_j] \rangle$ for some odd j. Therefore, $\bigcup_{v_i \in S} \langle N[v_i] \rangle = G$. □

Similarly, we prove:

Lemma 2.2. *Any independent set S of vertices of a path P_{2k} contain (at least) k vertices is always an n-set of P_{2k}.*

Lemma 2.3. *If S is an n-set of the graph G, then $\bar{S}$ is independent.*

Proof. If not, suppose that $\bar{S}$ contains two adjacent vertices say x and y, then the edge xy is not in the graph $\bigcup_{v \in S} \langle N[v] \rangle = G$, a contradiction to the fact that S is an n-set. □

Theorem 2.4. *For any integer,* $l_{Nr}(P_k) = \begin{cases} k, & \textit{for} \quad k = 1, 2, \\ \lceil \frac{k}{2} \rceil, & \textit{for} \quad k \geq 3. \end{cases}$

Proof. The result is obvious for $k \leq 4$. Consider the case $k \geq 5$, let S be an N-set of P_k. Then S is an n-set, so by Theorem 1.4, $|S| \geq \lfloor \frac{k}{2} \rfloor \geq 2$ vertices and hence by Remark 1.5, S is also an r-set. If k is odd and $|S| = \lfloor \frac{k}{2} \rfloor$, then $|\bar{S}| \geq \lfloor \frac{k}{2} \rfloor$, so by Lemma 2.3 and Lemma 2.1 the subset $\bar{S}$ is an n-set, a contradiction to the fact that S is an N-set. Therefore, $|S| \geq \lceil \frac{k}{2} \rceil$ for all k implies that $l_{Nr}(P_k) \geq \lceil \frac{k}{2} \rceil$. On the other hand, it is easy to see that the set $S = \{v_{2\lfloor \frac{k}{4} \rfloor}, v_{2\lfloor \frac{k}{4} \rfloor - 2}, \ldots, v_2\} \bigcup \{v_P\} \bigcup \{v_{\lfloor \frac{k}{2} \rfloor + 1}, v_{\lfloor \frac{k}{2} \rfloor + 3}, \ldots, v_{k-1}\}$ is an Nr-set of P_k with $|S|=\lceil \frac{k}{2} \rceil$ where $p = 2$, if k is even and $p = 1$, if k is odd. Thus, $l_{Nr}(P_k) \leq \lceil \frac{k}{2} \rceil$. □

Theorem 2.5. *For any positive integer k, $k \neq 1, 3$, $l_{n^\star r}(P_k) = l_{nr^\star}(P_k) = l_{n^\star r^\star}(P_k) = \lfloor \frac{k}{2} \rfloor$.*

Proof. The result is obvious for $k = 2$. Now for the case $k \geq 4$, as every $n^\star$-set S is also an n-set, we have $|S| \geq \lfloor \frac{k}{2} \rfloor$ (by Theorem 1.4) and hence $l_{n^\star r^\star}(P_k),\ l_{n^\star r}(P_k),\ l_{nr^\star}(P_k) \geq \lfloor \frac{k}{2} \rfloor$. On the other hand, we see that the set $S = \{v_2, v_4, \ldots, v_{2\lfloor \frac{k}{2} \rfloor}\}$ is an n-set of P_k. So, by Lemma 2.1 or Lemma 2.2 respectively when k is odd or even, the set $\bar{S}$ is an n-set. Since $k \geq 4$, both S and $\bar{S}$ have at least two elements and hence each of them will resolve P_k. Hence S is an $n^\star r$-set as well as $nr^\star$-set and $n^\star r^\star$-set with $|S| = \lfloor \frac{k}{2} \rfloor$. Therefore, $l_{n^\star r}(P_k) \leq \lfloor \frac{k}{2} \rfloor, l_{nr^\star}(P_k) \leq \lfloor \frac{k}{2} \rfloor$, and $l_{n^\star r^\star}(P_k) \leq \lfloor \frac{k}{2} \rfloor$. □

Remark 2.1. When $k = 1$, $\bar{S}$ is empty. Hence $n^\star$-set as well as $r^\star$-set are not defined. But when $k = 3$, it is easy to see that $l_{n^\star r}(P_3) = l_{nr^\star}(P_3) = 2$. However, P_3 has no $n^\star r^\star$-set S and hence $l_{n^\star r^\star}(P_3)$ is not defined.

Theorem 2.6. *For any integer $k \geq 4$, $l_{N^\star r}(P_k) = l_{N^\star r^\star}(P_k) = 2$.*

Proof. Let S be an $N^{\star}r$-set of P_k. Then S is not an n-set, $\bar{S}$ is not an n-sets, and S is an r-set. Now, if $|S| = 1$, then S contains only an end vertex of P_k (by Remark 1.2) and hence $|\bar{S}| = k - 1$. But then, $\bar{S}$ is an n-set (by Remark 1.4), a contradiction. Thus, $2 \leq |S| \leq k - 2$. Hence $l_{N^{\star}r}(P_k) \geq 2$ and $l_{N^{\star}r^{\star}}(P_k) \geq 2$. On the other hand, take $S' = \{v_1, v_2\}$. The set S' as well as $\bar{S}'$ are not n-sets (since the edge v_1v_2 is not an edge of $\bigcup_{v\in\bar{S}'}\langle N[v]\rangle$). But S' is an r-set (and $\bar{S}'$ is also an r-set), whenever $k \geq 4$ (since $|S'| = 2$ and $|\bar{S}'| \geq 2$ and by Remark 1.5). Hence $l_{N^{\star}r}(P_k) \leq 2$ and $l_{N^{\star}r^{\star}}(P_k) \leq 2$. □

Remark 2.2. If $k \leq 3$, for every subset S of $V(P_k)$, either S or $\overline{S}$ is an n-set. Hence no $N^{\star}$-set exists.

We end up this section with the following theorem, whose proof follows similar to the proof of Theorem 2.4.

Theorem 2.7. *For any integer $k \geq 3$, $l_{Nr^{\star}}(P_k) = \lceil \frac{k}{2} \rceil$.*

When $k = 1$, no $r^{\star}$-set exists and when $k = 2$, no N-set exists. It is easy to see that the other sets like $nR^{\star}$-set, $n^{\star}R^{\star}$-set, $NR^{\star}$-set, and $N^{\star}R^{\star}$-set are not exists in any path due to the non-existence of $R^{\star}$-sets. Finally, the non-existence of $N^{\star}R$-set is due to the fact that if S is any such set, then its complement should contains exactly one vertex other than the end vertex to become an R-set implies that the set S is an n-set (so not an $N^{\star}$-set).

3. nr-sets and Dimensions of a Cycle

We first restate the consequences of Theorem 1.6 as;

Lemma 3.1. *Let $e = xy$ be an edge of a graph G such that e is not an edge of a triangle in G and S be an n-set of G. Then $x, y \in N[v]$ for some $v \in S$ if and only if $x = v$ or $y = v$.*

Lemma 3.2. *If S is an n-set of a graph G, then for each edge $e = xy$ there exists a vertex v in S such that both $x, y \in N[v]$.*

Theorem 3.1. *For each integer $i \geq 3$, every i-element subset S of vertices of a cycle C_k is always an r-set.*

Proof. Let S be a subset of the vertices of C_k with cardinality at least 3. Let $a, b, c \in S$ and x, y be any two vertices of cycle C_k for $k \geq 3$. If possible, let $d(a, x) = d(a, y)$ and $d(b, x) = d(b, y)$. Then a and b lie in distinct xy-paths in C_k and C_k is an even cycle. In case if c lies between a and x, then $d(c, x) < d(c, y)$ and hence c resolves the pair x, y. Similarly, other cases follows by symmetry. □

Remark 3.1. A set containing two adjacent vertices of a cycle C_k is always an r-set of C_k for each $k \geq 3$.

Theorem 3.2. *For any integer $k \geq 3$,* $l_{nr}(C_k) = \begin{cases} 3, & \text{for } k = 4, \\ \lceil \frac{k}{2} \rceil, & otherwise. \end{cases}$

Proof. In the case $k = 4$, it follows by Theorem 1.4 that $|S| \geq 2$. If $|S| = 2$, then S contains two adjacent vertices (else it is not an r-set). But then, $\langle V(C_4) - S\rangle$ contains an edge and hence by Theorem 1.6, C_k should contain a triangle, a contradiction. Hence every nr-set should have at least 3 elements. For the case $k \geq 5$, it is easy to see from Theorem 1.5 and Theorem 1.6 that the set $S = \{v_1, v_3, v_5, \dots, v_{2\lceil \frac{k}{2}\rceil -1}\}$ is an n-set and hence by Theorem 3.1, it follows that $l_{nr}(C_k) = |S| = \lceil \frac{k}{2}\rceil$. □

Theorem 3.3. *For any integer $k \geq 4$, $l_{N^\star r}(C_k) = l_{N^\star r^\star}(C_k) = 2$*

Proof. Let $e = xy$ be an edge of C_k and $S = \{x, y\}$. Then S is a resolving set for C_k. Now as $k \geq 4$, there is an edge $e_1 = uv$ not adjacent to e. So, by Lemma 3.2, S is not an n-set (Since C_k has no triangle and $u, v \notin S$). Hence S is an $N^\star r$-set. Further as $\beta(C_k) = 2$, there are no singleton r-sets implies that the above set S is a minimal $N^\star r$-set, $l_{N^\star r}(C_k) = 2$. Also, $\bar{S}$ contains at least 3 vertices if $k > 4$ and 2 adjacent vertices if $k = 4$. So, by Theorem 3.1 and Remark 3.1, $\bar{S}$ is an r-set. Therefore, S is also an $N^\star r^\star$-set of minimum cardinality, so $l_{N^\star r^\star}(C_k) = 2$ for all $k \geq 4$. □

Lemma 3.3. *Let S be a minimal n-set of a graph G with $\Delta(G)$=2 and H=$\langle S\rangle$. Then $\Delta(H) < 2$.*

Proof. If possible, let S be a minimal n-set of G and $\Delta(H) = 2$. Then there exists $a, b, c \in S$, Such that $ab, bc \in E(G)$. Consider the set $S' = S - \{b\}$. Since $\Delta(G) = 2$, we have $deg_G(b) = 2$ and hence b is adjacent to only a and c. Therefore, S' covers all the edges of G incident with b as well as other edges of G (Since other edges covered by S). This shows that S' is an n-set, a contradiction to the minimality of S. □

Theorem 3.4. *For any integer $k > 4, l_{Nr}(C_k) = l_{Nr^\star}(C_k) = \lceil \frac{k+1}{2}\rceil$. Also, $l_{Nr}(C_4) = 3$.*

Proof. Let S be a minimal Nr-set of cycle C_k, $k > 4$. Then S is an n-set, therefore by Theorem 1.5, $|S| \geq \lceil \frac{k}{2}\rceil$ and by Lemma 3.3 the induced subgraph $\langle S\rangle$ has no two adjacent edges of G (i.e $deg_{\langle S\rangle}(v) \leq 1, \forall v \in S$). So, if k is even and $|S| = \lceil \frac{k}{2}\rceil$, then in the view of Lemma 3.2, we have, $\bar{S}$ is an n-set, a contradiction to the fact that S is an N-set. Thus, $|S| \geq \lceil \frac{k+1}{2}\rceil$ implies that $l_{Nr}(C_k) \geq \lceil \frac{k+1}{2}\rceil$ and $l_{Nr^\star}(C_k) \geq \lceil \frac{k+1}{2}\rceil$. On the other hand, consider the set $S = \{v_1, v_3, v_5, \dots, v_{2\lceil \frac{k+1}{2}\rceil -3}\} \bigcup \{v_{k-1}\}$. The set S is an n-set with $|S| = \lceil \frac{k+1}{2}\rceil$ and $|\bar{S}| = \lfloor \frac{k-1}{2}\rfloor < \lceil \frac{k}{2}\rceil$ and hence $\bar{S}$ is not an n-set implies that S is an N-set. Finally, as $k > 4$, we have $|S| > 3$. Hence by Theorem 3.1, S is also an r-set. Thus, $l_{Nr}(C_k) \leq \lceil \frac{k+1}{2}\rceil$. Further when $k = 5$, it is easy to see that $\bar{S}$ contains an adjacent pair of vertices and when $k > 5$, the set $\bar{S}$ has at least 3 vertices. Hence by Remark 3.1 and the 3.1, the set S is also an $r^\star$-set. Hence it also follows that $l_{Nr^\star}(C_k) \leq \lceil \frac{k+1}{2}\rceil$. Lastly, the case $k = 4$ follows easily. □

Remark 3.2. When $k = 3$, it is easy to see that for every nr-set S of C_3, the set $\bar{S}$ is also an n-set and no N-set exists.

Theorem 3.5. *For any integer $k > 4$, $l_{nr^\star}(C_k) = \lceil \frac{k}{2}\rceil$*

Proof. Follows immediately by Theorem 1.4 and Theorem 3.1, as $l_{nr^\star}(C_k) = l_n(C_k) = \lceil \frac{k}{2}\rceil$ for all $k > 4$. □

Remark 3.3. Since $\beta(C_k) = 2$, every r-set of C_k should have at least 2 elements. Therefore, for the existence of an $r^\star$ set of a cycle C_k, k should be at least 5. Further when $k = 3$ or 4, it is easy to see that for every nr-set S of C_k we get $|\bar{S}| = 1$, and hence S is not an $r^\star$-set.

Theorem 3.6. *For any integer $k \geq 4$, $l_{NR}(C_k) = l_{nR}(C_k) = \begin{cases} k-2, & when\ k\ is\ even\ and\ k \neq 4, \\ k-1, & otherwise. \end{cases}$*

Proof. Since $\beta(C_k) = 2$, any two vertices of C_k resolves C_k except the case k is even and the vertices are diagonally opposite. Therefore, for $k > 4$, every R-set S should have minimum of $k-1$ vertices whenever k is odd and $k-2$ if k is even. In either of the cases, the subgraph $\bigcup_{v \in S} N[v] \cong C_k$ for every R-set S and $\bigcup_{v \in \overline{S}} N[v] \neq C_k$ for $k \neq 4$ and hence S is an n-set as well as an N-set. When k=4, every N-set should have at least 3 elements and such a set S with $|S| = 3$ is always an R-set. □

Theorem 3.7. *For every integer $k \geq 3$, $l_{n^\star r^\star}(C_{2k}) = l_{n^\star r}(C_{2k}) = k$.*

Proof. Let S be an $n^\star$-set. Then S and $\overline{S}$ both are edge covering of C_{2k}. Since edge covering number of C_{2k} is $k, |S| = |\bar{S}| = k$. Also, both S and $\bar{S}$ are r-sets (since $k \geq 3$). Finally, every maximal independent set S is an $n^\star r^\star$-set as well as $n^\star r$-set. Hence the result. □

Remark 3.4. For an odd cycle, no $n^\star$-set exists as each n-set contains both end vertices of an edge (so $\bar{S}$ is not an n-set, by Lemma 3.2).

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