MATHEMATICS PROBLEM SOLVING SKILL ACQUISITION: LEARNING BY PROBLEM POSING OR BY PROBLEM SOLVING

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Abstract: Problem posing is an instructional method where students are asked to create problems based on the given information, then solve them. While in an instructional method of problem solving, students learn by solving given problems. The aim of this study was to test: (1) the differences of efficacy between learning by problem posing and the problemsolving method of individual and small group instruction strategies; (2) the interaction effect of learning methods and grouping strategies.With regard to the independent variables, problemsolving skill or cognitive load, a quasi experiment with post-test-only-non-equivalent control group designwas used. Year 7 contextual mathematics problems were tested in this experiment, and one hundreds students, who had sufficient prior knowledge, participated. A 2 by 2 anova was employed for data analysis. The results showed that: (1) problem posing method was significantly more effective than problem-solving method; (2) there was no significant difference in efficacy between individualized instruction and small group instruction strategies; (3) the interaction between learning methods and grouping strategies, where it is more likely that learning problem posing was better than problem solving for individual instruction.

Keywords: cognitive load, individual, mathematics, problem posing, problem solving, small group

PENGUASAAN KETERAMPILAN PEMECAHAN MASALAH MATEMATIKA: BELAJAR MELALUI PROBLEM POSING ATAU PROBLEM SOLVING

Abstrak: *Problem posing* adalah suatu metode pembelajaran dimana siswa diminta untuk menciptakan masalah-masalah berdasarkan informasi yang diberikan, kemudian siswa diminta menyelesaikan masalah tersebut. Sedangkan dalam metode pembelajaran *problem solving*, siswa belajar melalui penyelesaian masalah yang telah ditentukan. Tujuan penelitian ini adalah untuk menguji: (1) perbedaan efektivitas metode pembelajaran problem posing dan problem soving secara individual atau kelompok; (2) Efek interaksi antara metode pembelajaran dan strategi pengelompokan belajar. Dengan meninjau pada variabel terikat, keterampilan pemecahan masalah dan muatan kognitif, kuasi eksperimen dirancang dengan desain *post-test-only-non-equivalent control groups*. Materi pembelajaran dalam eksperimen adalah masalah matematika kontekstual untuk kelas 7, dengan sampel sejumlah 100 siswa yang telah mempunyai pengetahuan awal yang memadai. Anova dua jalur digunakan untuk analisis data. Hasil penelitian menunjukkan bahwa: (1) ada perbedaan yang signifikan dari kedua metode pembelajaran, dimana *problem posing* lebih efektif daripada *problem solving*; (2) tidak ada perbedaan yang signifikan antara strategi belajar individu atau kelompok; (3) ada efek interaksi antara metode pembelajaran dengan strategi pengelompokan, dimana dalam strategi belajar individu, menggunakan *problem posing* lebih baik daripada menggunakan *problem solving*, tetapi ada kecenderungan sebaliknya untuk strategi belajar kelompok.

Kata kunci: muatan kognitif, individual, matematika, *problem posing, problem solving*, kelompok kecil

INTRODUCTION

Education is fundamental to the development of human life, and so mathematics education. For this reason perhaps mathematics has been a compulsory subject at all level of study. Specifically, the common objectives of mathematics teaching at secondary level are that students are able to solve and understand conceptual knowledge of contextual problems, design mathematical models, solve models, and interpret thesolutions. The advancement of problem-solving skill is a main instructional goal, as well as the ability to employ reasoning. To acquire this skill, students need to do problem solving in order to both learn mathematics and to learn the problem solving strategy.

Unfortunately, mathematics problem solving always seems difficult (Dowker, Sarkar, & Looi, 2016) or be misunderstood that mathematics could only be learned by gifted students (Arikan & Unal, 2015). Besides mathematics contain abstract concepts, mathematics problem solving requires sufficient knowledge base in order to solve and learn. One of the mathematical learning materials that use problem solving is related to geometric shape. Part of the material related to geometric shape is the circumference and area of the quadrilateral. The rectangular area and square area consisting of the circumference and the area of the square, rectangle, split, parallelogram, trapezoid, and kite are interesting problem-solving materials for students to learn because they are useful indaily life.

A cognitive load theory(Sweller, Ayres, & Kalyuga, 2011) provides an insightful framework for examining the relationship between the mastery of learning outcomes and the challenges faced during internalizing knowledge. The theorists explain that effective learning methods are those that minimize cognitive load. They describe cognitive loadas the total mental effort in working memory when students conciously think, solve problems or learn from problems. Consideration of a students' cognitive load is essential in optimizing their ability to understand and construct studied knowledge.

Moreover,cognitive load theory suggests that the problem-based instruction with cognitive loadprovides positive effects for students who have sufficient initial knowledge (P. A. Kirschner, Sweller, & Clark, 2006; Sweller et al., 2011). Problem-based instruction involves students learning from presented problems and employing their knowledge to solve the subsequent problems in their own way. The findings of cognitive load theory show that if problem solving instruction is adequate, then studentsare more active during the learning process.

On the other hand, students may study problem solving skill by posing problems (Leung, 2013; Silver, 1994, 2013; Silver & Cai, 1996). Silver states that the problem posing method has been used as a means to enourage students to analyze the problem holistically, thus improving their problem-solving skills. Furthermore, Silver explains that theproblem posing gives instruction to studentsto formulate problems/questions from the information or situation available. For instance, given an information: a square park is surrounded by pine trees whose distance from each other is four meters. From this information, it is expected that students can come up with questions, such as 'if the length of the park is 65 m, then how many pine trees are in the park?'

The problem posing method directs the students to access their schematic knowledge from their problem formulation and to use in theproblem solution. There are three types of problem posing methods according to Silver and Cai (1996). These are: 1) pre-solution posing, i.e problem-solving based on situation or information provided;2) within-solution posing, i.e making or formulating the questions being solved; 3) post-solution posing, i.e the student modifies or revises the objectives or problem conditions that have been resolved to produce new, more challenging questions.Most junior high school students are more than twelve years old, and assumed already have acquired some abstract concepts or precodures (NCTM, 2000), however, they may have different levels of knowledge base on particular topic (Donovan & Bransford, 2005; Silver & Cai, 1996). Therefore, pre-solution posing was hypothesied the most conducive to students at beginning level, as within-solution posing and post-solution posing require students to work effectively and innovatively at advanced level.

The pre-solution posing method guides students to effectively use presented information to acquire further knowledge (Silver, 1994). The main stages of the problem posing of pre-solution posing type may include: (a) delivery of materials, (b) students undertaking practice tests in line with teaching material, (c) students are given chance to compile questions from given information, (d) students solve the problems they havemade, and (e) students discuss the results. The teaching strategies in this problem posing method target student analysis of the statement presented, ability to understand the given command, to identify the relevant information (relevant to the material or conceptual knowledge already known by the student), to compile questions, and evaluate the conclusions drawnfrom solving these questions. From these stages, students are trained to improve their problem-solving skills by simulating self-made problems. Particularly, creating problems from an information triggers creative use of previously learned knowledge (Arikan & Unal, 2015).

Unlike the problem posing method, the problem solving method leads the students to solve a given problem. The problem solving method is a teaching guideline that theoretically or conceptually trains students to solve mathematical problems using various strategies and existing problem solving steps (NCTM, 2000). According to Polya (1981), in general there are four general steps in solving mathematical problems, among others: 1) understanding the problem, 2) creating a problem-solving plan, 3) implementing a problem-solving plan, and 4) re-examining the answer. These four steps can be used as a guide to solve problems, but students need to have sufficient knowledge of relevant mathematical theorems/algorithms to be able to rreach the problem solution efficiently. The application of the problem solving method significantly improved students' mathematical problem solving abilities, especially in understanding a problem and planning its solution(Arterberry, Cain, & Chopko, 2007; Hmelo-Silver, 2004; Youssef, Ayres, & Sweller, 2012). Comprehension ofa problem and explicit solution planning are fundamental of the problem solving process, and are key to measure the success of learning goals.

Both the problem posing method and the problem-solving method are categorised student-centered learning. The problem posing teaching method encourages students to actively inquire, or work on a problem using supplied information, while the problem-solving method promotes active analysis to find the solution of the given problem. A study comparing the two methods may have not yet been undertaken. However, an effective collation could allow teachers better understand when to suitabily apply which particular method, whether they are applicable to all students, to any mathematics learning topics or only specific situations. Therefore, knowing cognitive load imposed by learning process is beneficial to apply suitableteaching practices in mathematics instruction.

Turning into two grouping strategies, individualized and small groups are commonly employed instructions in reguler mathematics classrooms. Many studies have been conducted to compare the two strategies, however, the results are varied (Avouris, Dimitracopoulou, & Komis, 2003; F. Kirschner, Paas, & Kirschner, 2011; Retnowati & Aqiila, 2017; Retnowati, Ayres, & Sweller, 2010, 2016). Kirschner, Paas, and Kirschner (2011) compared instruction with individual to with group on problem solving methods (jigsaw). During the instruction phase,students were given problem-solving tasks that were either solved individually or in groups of three students (jigsaw group). In the jigsaw group, students were designed with peer guided instruction, as each group member had only one-third of all information, so the students had to exchange information to solve the problem. Forindividual instruction, each student was presented with all necessary information to solve the problem alone. The results showed that the group instruction was more effective than the individual. However, when conventional groups of three to four students was used, the contrasting result was shown (Retnowati et al., 2010), as well as when using dyads (Retnowati & Aqiila, 2017), where notably the subjects were novice students. It is assumed that the presentation of information from others during solving a challenging problem caused high extraneous cognitive load. As the theory of expertise reversal describe that the problem solving method is efficient for students with high prior knowledge; students with low prior knowledge will experience high cognitive load resulting in low learning outcomes (Sweller et al., 2011). Consequently, teachers must ensure the level of knowledge base before applying a problem solving method in the classroom, whether students learn alone or in small groups. It should be noted that a conventional small group consists of three to five students who meet face-to-face to solve the same problem through discussion or knowledge sharing. Such instructional strategy sounds beneficial because students could help in other during learning (Arterberry et al., 2007; Hmelo-Silver, 2004). Such comparison will be useful to generate conclusion what instruction is best applied for learning mathematics individually or in small groups when students have sufficient prior knowledge. Therefore, this study was specifically aimed to test three hypotheses:

1) When students have sufficient prior knowledge, learning from problem posing strategy is more effective than learning from problem solving strategy.
2) When students have sufficient prior knowledge, learning in small groups is more effective than learning in individually.
3) When students have sufficient prior knowledge, learning from problem posing is more effective in small groups rather than individually.

RESEARCH METHOD

This research was a quasi-experiment, with the post-test-only-non-equivalent control design. The population sampled had moderate math ability based on mathematical competency (based on the district rank) and had not studied the complex problems related to the learning material used in the experiment: word problems of circle and rectangular areas. Nevertheless, all students have mastered the pre-requisite knowledge on this material. The sample that was established using the convenience sampling techniqueconsisted of 100 year-7 students (in four classes) from a public junior high school. These reguler classrooms are randomly assigned to (1) problem solving – individual study; (2) problem solving – small group study; (3) problem posing – individual study; and (4) problem posing – small group study.

After each group is assigned,the researcher implemented the teaching steps:

1) The teacher reminds students about previously related material, where students were asked to answer some questionsrelated

to the pre-requisite material: the type and nature of quadrilateral and units, and rectangle images, and also previously learned area or perimeter concepts (20 minutes). The teacher clarified the correctness of their answers.

2) The teacher conducts the main instructions. For the problem solving groups: students learned by solving problems; either individually or in small groups. For the problem posing groups: students learned by posing problems; either individually or in small groups. Every student was given worksheet to complete in 40 minutes. The problem posing worksheet contained 6 information (statement). The instruction was "create as many problem as possible based on the information" and "select one of the problems you want to solve". Example of the information given for the problem posing group: "The side length of a square room is 4 m. Create as many problem as possible based on the information." The problem solving worksheet contained 6 word problems to solve by students. The instruction was "solve the problem". The problems were written in Indonesian language. Example of the word problem for the problem solving group: "The side length of a square room is 4 m. The tile is a square shape as large as 400 cm^2. If one box tile consists of 20 tiles and costs Rp42.000,00, then how much is the total cost of the tile?". Afterwards, students presented their work in a classroom discussion led by the teacher and then, the teacher guided students to the conclusion of their learning (20 minutes).

3) Students were administered a post-test to assess the problem solving skill and cognitive load straight after the learning phase collapsed. The research instrument was 10 problem-solving test (Cronbach's alpha = 0,86) to solve in 80 minutes (Table 1) and a 9-point rating scale for cognitive load developed by(Paas & van Merriënboer, 1994). The cognitive load question given was "how difficult is it to solve the problem?".

Table 1. The problem solving skill test grid

Question Number	Materials / Competencies
1.	Resolving contextual issues (theme: gardens) related to the area and circumference of a square and rectangle.
2.	Resolving the contextual issues (theme: field) related to the area and circumference of a rectangle.
3.	Resolving contextual issues (theme: tiles) related to the area and circumference of a rectangle.
4.	Resolving contextual issues (theme: chicken coop) related to the area and circumference of a square.
5.	Resolving the contextual issues (theme: park) related to the area and circumference of a parallelogram.
6.	Resolving contextual issues (theme: tiles) related to the area and circumference of a parallelogram.
7.	Resolving contextual issues (theme: kite) related to the area and circumference of a kite.
8.	Resolving contextual issues (theme: paper) related to the area and circumference of a trapezoidal.
9.	Resolving contextual issues (theme: wall hangings) related to the area and circumference of a rhombus.
10.	Resolving the contextual issues (theme: garden) related to the area and circumference of a trapezoidal.

RESULTS AND ANALYSIS

Result

Three hypothesis were tested in this study involving two learning strategies: problem posing and problem solving; and two groups: individual and small group learning. Two dependent variables were measured: problem solving skill and cognitive load. Table 2 shows the average score of problem solving skill (maximum score: 100, minimum score: 0). The higher the score indicates the better the skill.

Table 2. Interaction effect of problem solving skill

Instruction strategy	Learning methods	
	Problem Posing	Problem Solving
Individual	$\bar{x} = 84.31$	$\bar{x} = 58.00$
	$SD = 11.89$	$SD = 19.65$
	$n = 26$	$n = 21$
Small Group	$\bar{x} = 66.85$	$\bar{x} = 75.81$
	$SD = 13.65$	$SD = 12.52$
	$n = 27$	$n = 26$

Table 3 shows the cognitive load rating (maximum rating: 9, minimum rating: 1). The higher the cognitive load describes the less efficient their thinking process is.

Table 3. Interaction effect of cognitive load

Instruction strategy	Learning Methods	
	Problem Posing	Problem Solving
Individual	$\bar{x} = 3.242$	$\bar{x} = 5.657$
	$SD = 1.12$	$SD = 1.77$
	$n = 26$	$n = 21$
Group	$\bar{x} = 5.356$	$\bar{x} = 4.415$
	$SD = 1.71$	$SD = 2.02$
	$n = 27$	$n = 26$

Based on the results of the analysis of variance (ANOVA) test for the effectiveness of learning strategy (problem posing vs. problem solving) in terms of students' problem solving skill at the significance level (α) of 0.05, the *F* obtained was 8.934, $p = 0.004 < 0.05$, $\eta_p^2 = 0.085$, which means that there was a significant difference of effectiveness between the problem posing and the problem-solving method in terms of problem solving skill (hypothesis 1 confirmed). Based on$\eta_p^2 = 0.085$ or $\eta_p^2 = 8.5\%$ learning method contributes 8.5% to student problem solving skill, wheret the problem posing ($\bar{x} = 75.42$ and $SD = 15.454$) was more effective than the problem-solving ($\bar{x} = 67.85$ and $SD = 18.252$) in terms of student problem-solving skill.

Hypothesis one is also confirmed referring to the results of the analysis of variance (ANOVA) test for the effectiveness of learning strategy in terms of students' cognitive load at the significance level (α) of 0.05, the F obtained was 4.747, $p = 0.032 < 0.05$, $\eta_p^2 = 0.047$, which means that there was a significant difference of effectiveness between the problem posing and the problem-solving in terms of students' cognitive load, and it was indicated that the problem posing ($\bar{x} = 4.32$ and $SD = 1.7891$) is more effective than the problem-solving ($\bar{x} = 4.97$ and $SD = 1.9933$) in terms of student cognitive load.

However, the second hypothesis was rejected. Based on the results of the analysis of variance (ANOVA) test for the effectiveness of the instruction strategy in terms of problem solving skill at the same significance level (α) of 0.05, obtained value of $F = 0.004$, $p = 0.952 > 0.05$, $\eta_p^2 = 0.000$, which means that there was no significant difference in effectiveness between individualized instruction and small group instruction strategies in terms of problem solving skill. As well, for the students' cognitive load the *F* value obtained was 1.658, $p = 0.201 > 0.05$, $\eta_p^2 = 0.017$, which means that there was no significant difference in effectiveness between individualized instruction and small group instruction.

Moreover, hypothesis three was not proved,that there was an interaction effect between learning methods and grouping strategies in terms of problemsolving skill, the F value obtained was 36.899, $p = 0.000 < 0.05$, $\eta_p^2 = 0.278$ (see Figure 1). Simple effect tests indicated that for the individualized instruction, problem posing was significantly better than problem solving based learning, t(45) = 5.82, p < 0.0001. On the other hand, for the small group instruction, problem solving is significantly better than problem posing, t(51) = 2.49, p = 0.016 (see Table 2 for the means and SDs).

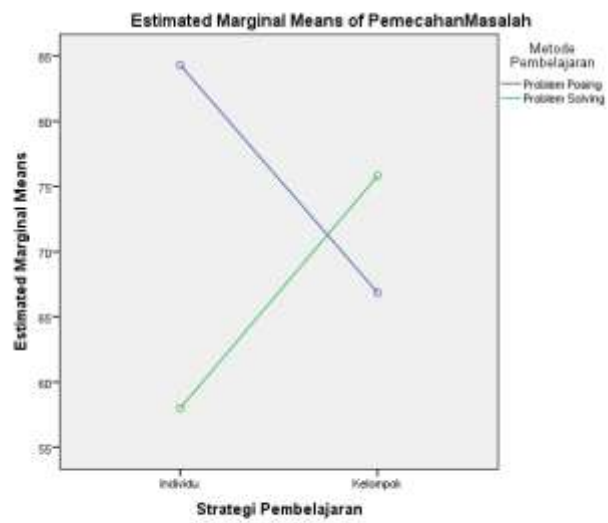

Figure 1. Interaction effect in terms of problem solving skill

Furthermore, there was also significant interaction effects between learning methods and grouping strategies in terms of student cognitive load the *F* value obtained was 24.573, $p = 0.000 < 0.05$, $\eta_p^2 = 0.204$ (see Figure 2). Simple effect tests performed that for the individualized instruction, as the cognitive load rating was lower (see Table 3 for the means and SDs), problem posing was significantly better than problem solving based learning, t(45) = 5.82, p < 0.0001. However, for the small group instruction, no significant difference between problem posing and problem solving, t(51) = 1.83, p = 0.073.

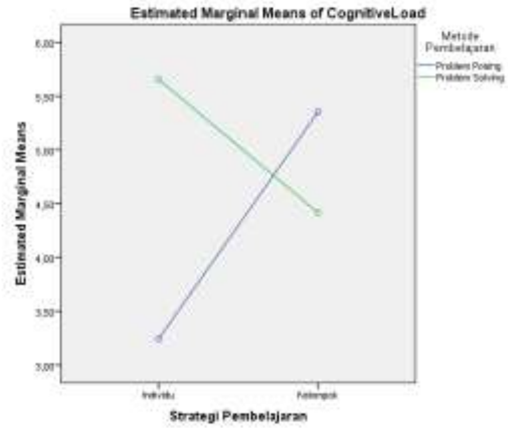

Figure 2. Interaction effect of cognitive load

Discussion

The study showed empirical evidence that problem posing instruction is better than problem solving instruction for students who had sufficient knowledge base. However, the instruction of small group was not confirmed to be better than the individualized instruction. Specifically, it also seems that learning by problem posing was more effective than by problem solving in individualized instruction.

Instructional strategy of the problem posing required students to devise problems from the information presented, which encouraged students to utilise and extend their creative thinking skills, as asserted by Silver and Cai (1996), students with mastery of knowledge in the domain performed better problem posing. The following is an example of students' work on the worksheet of the problem posing teaching method of the pre-solution posing type. Figure 4 is an example of student's work on problem number 2. The information is: "Pak Subur has a rectangular yard and the area is 2 hectars. From this information, create a problem as many as possible." The student posed two problems: "What is the width of the yard?" and "If the price of the land is Rp1.500.000 per m^2, how much does his land cost?" Before creating the problem, they converted 2 hectars into m^2. To devise a problem based on the information provided, the students had first to understand the information. The students' work showed comprehension of the given information, and an ability to manipulate this data to devise their own problems. Converting of area unit from hectare to m^2 before they created their own problems shows a good understanding of the concept in the information. The other students created different problems based on the same information, like shown in Figure 5.

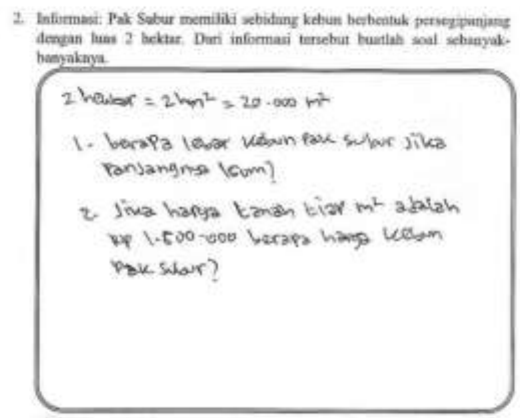

Figure 4. Examples of student work1

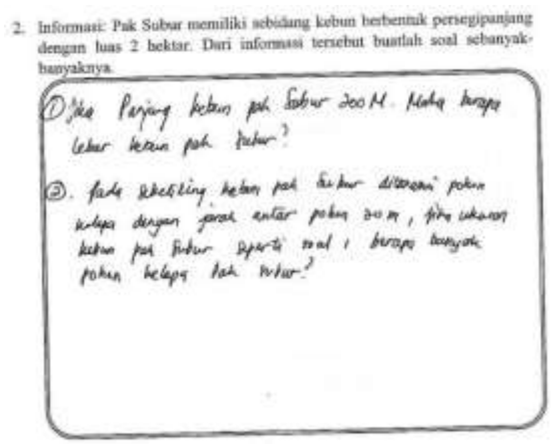

Figure 5. Examples of student work 2

Some students could quickly create a problem based on the information provided. The example above shows the different questions of two students. This difference might be stemmed from their creative thinking processes based on their knowledge base. The student results support previous research, which shows that the problem posing method in mathematics teaching improves students' theoretical and creative thinking, and the understanding of mathematical concepts(Silver & Cai, 1996). Choosing appropriate approaches or problem-solving strategies, students need to be creative and show understanding of the problem, and this supports the idea that the problem posing method is good for improving student problem solving skills.Lin's research also supports the theory that the process of formulating a problem could improve students' problem-solving skills and their attitudes toward mathematics (Lin, 2004). At the time of the making or formulating a problem based on the information provided, the students must understand the problem well, and this is the first step in solving the problem. By problem posing based learning, problems created by students must be solved by the students themselves.During the instruction, students will certainly try to solve the problem after understanding their devised problem.

It is suspected that many students in the problem solving instruction failed in finishing the instruction in the worksheet during the study phase, or they did not do metacognitive strategy to learn problem solving deeply, such as by checking the problem solution. Reviewing answers or evaluating solutions when problemsolving is very important as part of the completion of each step of the problem solution (Destan & Roebers, 2015; Donovan & Bransford, 2005).It is interesting to recall though the students in this research had appropriate prior knowledge of the problem solving. The cognitive load analysis indicated that the problem solving instruction caused higher load than the problem posing. The problem posing trains students to express their ideas through formulating questions based on the information given and working out the answers. Students should find answering questions that they devised easier, because they should understand their own questions. Indeed, learning materials or teaching presentation that does not complicate understanding will decrease extraneous cognitive load (Sweller et al., 2011).

The problemsolving method encourages students to work on problems by understanding existing problems and devising strategies to find the solutions to them(Hmelo-Silver, 2004). The students were instructed to work on the given questions, which they might have the knowledge base. Notably, there were given varied contextual themes they might have not attempted before, therefore experienced an increased cognitive load(P. A. Kirschner et al., 2006; Paas & van Gog, 2006).An effective teaching method is the one that minimizes cognitive load. Thatlearning byproblem posing was found to be more effective in terms of student cognitive load in this study shows that solving self-created questions lower cognitive load. In other words, to some extent, learning by problem solving might still be challenging for students with sufficient knowledge base.

Supposedly, in the small group instruction, students had the opportunity to discuss the learning with their friends, and this can help the students in collecting the required information. However, the current study found there was no significant difference of effectiveness between individual instruction and small group instruction strategies in terms of students' problem solving skill as well as cognitive load. That grouping strategy has no impact on learning was also indicated by previous research (Retnowati & Aqiila, 2017; Retnowati et al., 2010, 2016).Nevertheless, F. Kirschner et al. (2011)argue that during jigsaw small group instruction, the cognitive load experienced by each student is shared among group members, thus maximising learning outcomes. It should be noted that in this study, conventional small group was used, which means each group member received the same learning material. The presentation of other students seems to be extraneous factors because the students already possesed adequate prior knowledge which enable them to acomplish the task alone.

Regarding the third hypothesis, there was an interaction between learning strategy and teaching strategies in terms of student problem solving skill. The simple effect test revealed that the individual students those in small groups. It could be said that problem posing might be easier to complete individually. That the problem posing was beneficial if done individually was due to that solving or creating problems based on the information given individually could make the students' reasoning ability grow and let their creative thinking develops (Arikan & Unal, 2015), regardless of their friends' opinion. The increase of intensity in accomplishing the problem posing taskscould result in increased problem-solving skills, especially in terms of developing problem-solving strategies that require creativity.

There was anextra difference of interactional effectbetweenlearning strategyand instruction strategies in terms of cognitive load of the students. The simple effect test showed that the class taught using the problem posing hada lower cognitive load overall. The problem posing method in individual trains the students to build their own knowledge. Through the procedural steps of problem posing, students find a way to create problems from the simple to compex ones based on the information given, and so that the constructed knowledge more schematic. Consequently, learning through the problem posing, the students could easily understand the lesson because they learned in their own way. Arguably, this method might decrease the extraneous cognitive load,as the presence of others might add complication of understanding conceptual knowledge or take the focus away from the learning aims.

The current study may conclude that if the problem posing instruction is suggested to be implemented for learners who have sufficient knowledge base. Individual study of posing own problems based on a contectual mathematics information is more promising than in small groups since the group members might hinder learning instead. Nevertheless, questions remain on what kind of small group instruction could be beneficial for learners, or how to overcome challenges when designing problem posing instruction. As suggested by Silver (2013) how students think mathematically while problem posing or problem solving in many mathematics areas also is also need further research.

CONCLUSION

Based on the results of the data analysis and discussion above, the conclusions that can be taken in this study are as follows.1) There were significant differences between the problem posing and the problemsolving instruction, in terms of problem solving skill and cognitive load, where the former was more effective than the latter. 2) There was no significant difference of effectiveness between individual instruction and small group instruction strategies in terms of the students' problem-solving skill and cognitive load. 3) There was an interaction between the teaching method and instruction strategy in terms of the students' problem-solving skill or cognitive load. The problem posing instruction for students with sufficient knowledge base is better than the problem solving instruction when the individual instruction strategy was used.

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