

# Simulation model of the flow in the drainage system of Ambon city with explicit finite difference method

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**Abstract**— Analysis of flood modeling scenarios conducted on the simulation of urban drainage systems in Ambon city based on mathematical equations finite difference method which allows analyzing the water depth and flowing rate as a function of space and time. In order to study the propagation of flooding in the drainage channel with the 1D model of Saint Venant hydrodynamic equations are approximated by explicit MacCormack numerical method which is used as a simulation model. Numerical solution with MacCormack discretization technique on the parameters that influence the system can provide stable and final results that can be trusted. The purpose of this model is that it can reduce the volume of puddles on the local channel of drainage system area.

**Keywords**— Drainage System, Saint Venant, Mac Cormack, Ambon City.

## I. INTRODUCTION

Flow in open channels can be classified in a steady flow and unsteady flow. The steady stream flow is classified in the absence of changes in water level over time. While the flow unsteady flow is characterized by a change in elevation and velocity with respect to time and changes in discharge or velocity against distance. Flow that occurs in nature almost cannot be classified in an unsteady flow.

Unsteady flow is classified into two types, they are unsteady flow that gradually varied flow and the rapidly varied flow. Unsteady flow phenomena can be formulated into mathematical equations in one, two, or three dimensions. The mathematical equation resulting is quite complicated and difficult to be solved analytically. So the common solution is to use numerical methods, one of which is the finite difference. One of the finite difference method is a method that has been known as Mac Cormack explicit.

In an open channel flow river / canal there are some common model to use in order to describe the flow of the channel as Saint Venant models [1,2,3,4,8,9,10] patterns of flood flow in open channel which can be approximated by differential equations partial and derived from of Saint Venat equation [13]. Hydraulics model is based on two forms of the equation is the equation of conservation of mass and momentum conservation equations [11] (Chow, 1988). The purpose of this study is to simulate case of one-dimensional flow in the drainage network system in Ambon city.

## II. METHODOLOGY

### 2.1 The Study Area

Ambon city is located on the coast and has a flat topography with elevation of 0 meters - 10 meters from the sea level and has a catchment area covering an area of 1.43 km<sup>2</sup>. Drainage system in the city of Ambon has 5 parts water catchment area with 3 small rivers which can directly flow from the drainage to dispose of the body directly into the river or sea. In accordance with the characteristics of the system area the climatic conditions in the city of Ambon has daily rainfall maximum of 455 mm which occurred on 28 August 1988 for data analysis rainfall 29 years and temperature maximum temperature that occurred was in January of 2004 at 30 ° C and The minimum in February 2001 of 21.6 ° C. During the period of the last six years (2007 - 2013) Ambon City area experienced flooding in the area and eventually the system impact of material and human losses. Causes of flooding caused changes in the function of land use, poor drainage system and will be less conscious of the impact of flood hazards.

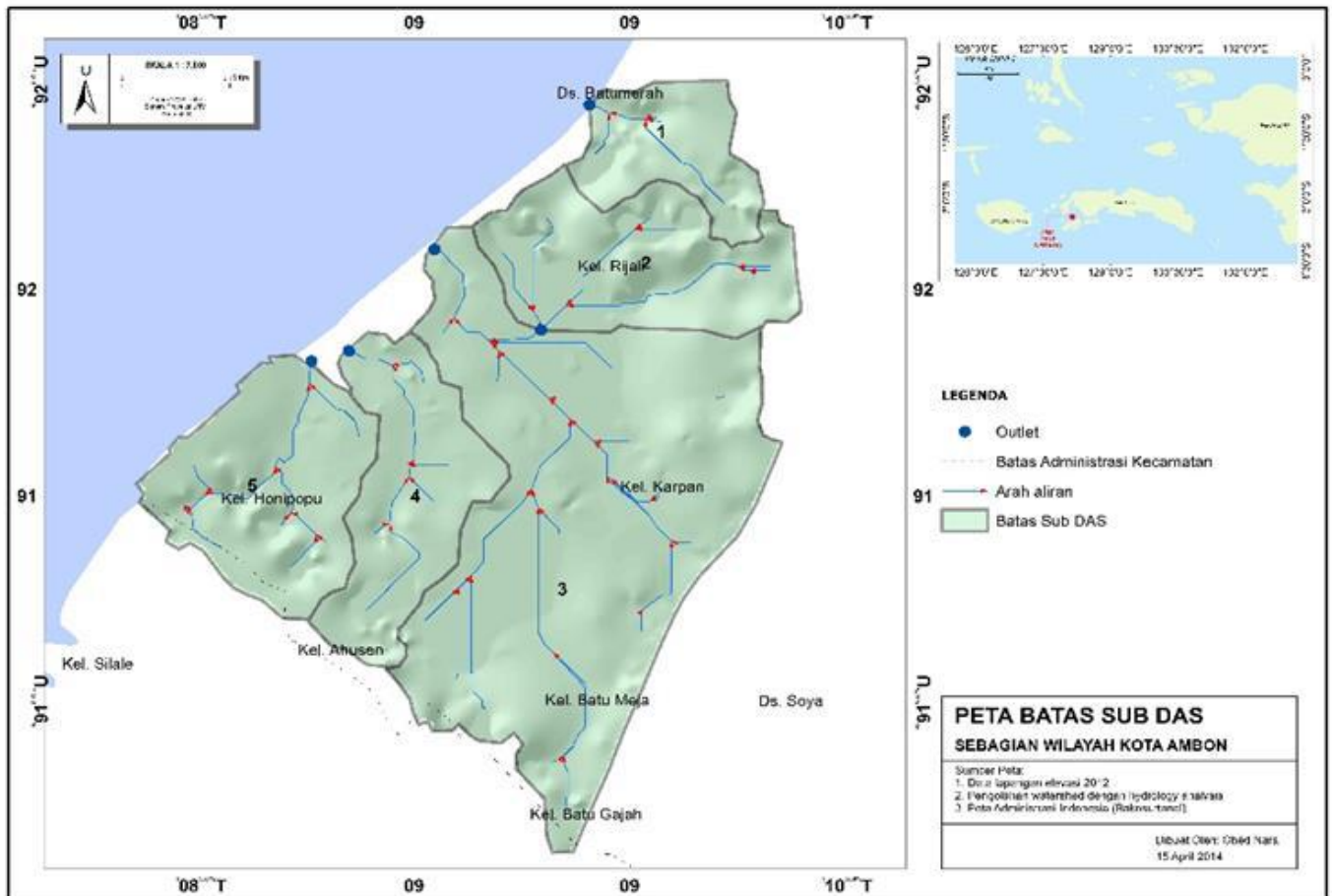


Fig.1: Catchments area in Ambon city

## 2.2 Numerical Model

The concept of a mathematical model used for the simulation of Ambon city drainage network system is a collection of mathematical description of the components hydrology and hydraulics. The structure of the model and is determined by the objectives of development planning

models. To facilitate the solution of mathematical model, then the model reduces to the numerical model. The purpose of the simulation model to learn a numeric behavior that is an event that occurs which is influenced by variables modifiers and other components as described in the concept of a model like image 2 below.

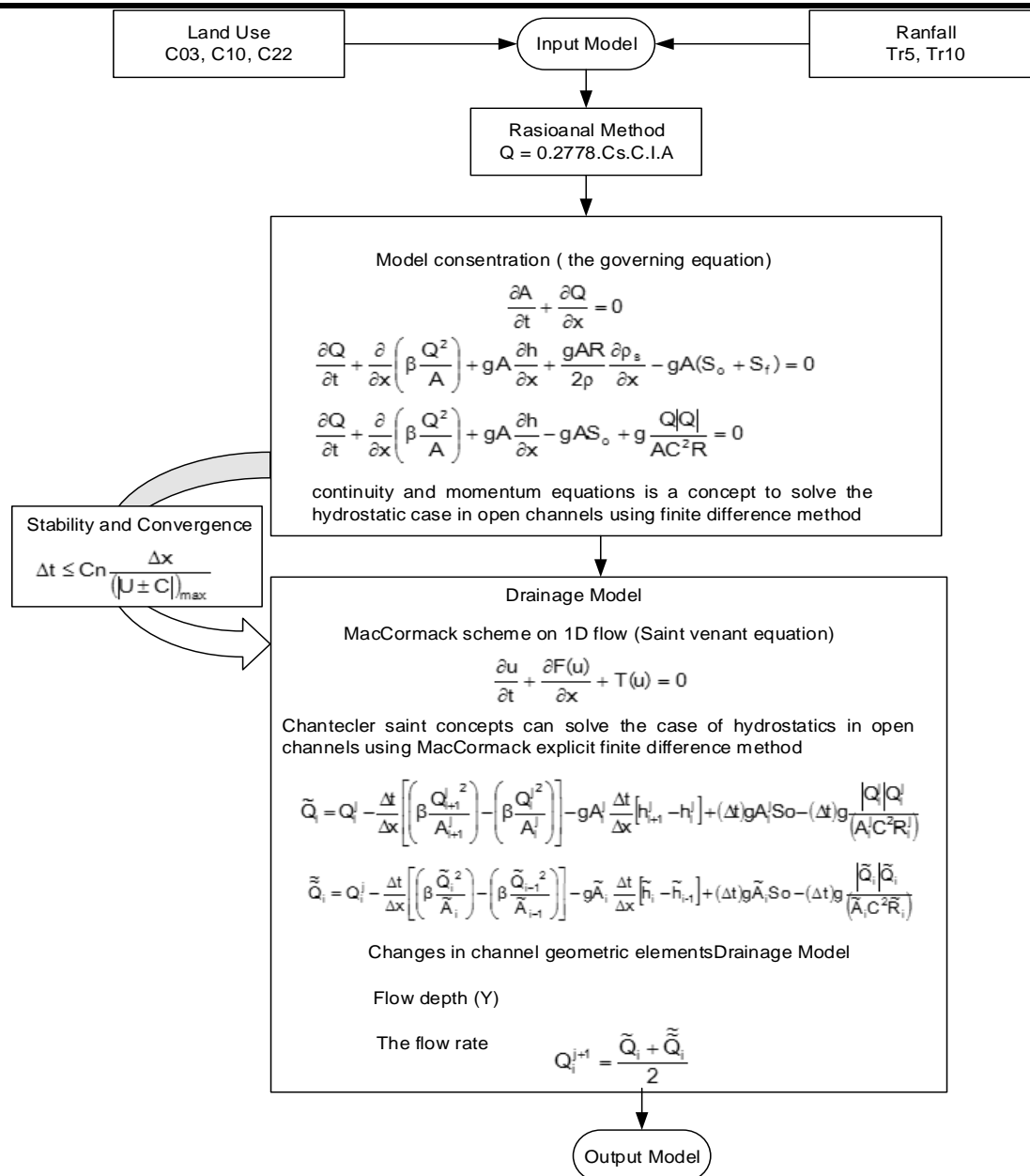


Fig.2: Framework numerical of modeling scenarios

### 2.2.1 The governing equationsThe continuity equation

The continuity equation [1,3,11] for unsteady flow there can be structured based on the mass conservation of a teller space, that is the mass rate of water entering the space look - mass flow of water coming out the room look the same as the rate of increase in volume / mass in the eye chamber , for the assumption of the continuity equation as follows:

- For the entire cross section of the channel

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q$$

(1)

- If  $q = 0$ , then ;

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

(2)

### Momentum Equation

Momentum equation [7] derived from the concept of conservation of momentum, where the net rate of momentum entering the room look + Number of the forces acting on the space look = The rate of accumulation of momentum in the eye chamber.

The momentum equation taking into account the net rate of momentum, hydrostatic force, changes in fluid density and friction force, with the simplifying assumptions stated earlier, is obtained as follows:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} + \frac{gAR}{2\rho} \frac{\partial p_s}{\partial x} - gA(S_o + S_f) = 0 \quad (3)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} + \frac{gAR}{2\rho} \frac{\partial p_s}{\partial x} - gAS_o + g \frac{Q|Q|}{AC^2R} = 0 \quad (4)$$

if the fluid density is assumed constant, then we obtain :

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q^2}{A} \right) + gA \frac{\partial h}{\partial x} - gAS_o + g \frac{Q|Q|}{AC^2R} = 0 \quad (5)$$

### 2.2.2 Model discretization and MacCormack Schemes

One of the schemes that are commonly used for analysis of variable explicitly on the channel is the difference until MacCormack method [5]. After using the shape derivative approach to explicit difference, that enter the differential equation, the price of a time function on the grid point interval =  $t + \Delta t$  waktu along the x-axis and y-axis can be calculated directly by using the values of the function at the point near him at the time interval  $t = \Delta t$  known. Development explicitly have a weakness, because it is necessary to obtain stability calculation restrictions interval  $\Delta t$ .

The concept models of the flow in the drainage system are;

- The time interval ( $\Delta t$ ): By using the MacCormack explicit finite difference equation where the price function of a lattice point at time interval =  $t + \Delta t$  along the x-axis and y-axis can be calculated directly by using the values of the function at a nearby point on the hose time  $t = \Delta t$  known. Development explicitly have a weakness, because it is necessary to obtain stability calculation restrictions interval  $\Delta t$ .

- The interval distance ( $\Delta x$ ): Changes in the distance added to the terms of the initial conditions along the drainage of free water level.
- Volume Control: The volume control is used to control the continuity and momentum equations in fluid density changes in hydrostatic force and the friction force on the liquid. In the picture above is the  $Q_{in}$  discharge  $Q_{out}$  incoming volume control which is a discharge coming out of the control volume where  $\Delta x$  is the length of the control volume  $\partial Q / \partial x$  which is the change in the value of  $Q$  along  $\Delta x$ .
- Stabilization flow: In order to assert a condition where the flow conditions are considered stable, then  $C_n$  (Courant number) is a necessary condition.
- Elements of geometric channel

Channel width ( $b$ ), High water in the channel ( $h$ ), channel length ( $L$ ) is a parameter used in modeling the flow.

In general, explicit MacCormack numerical scheme is divided into two phases: phase predictor and corrector. For more details can be seen In Figure 3 below;

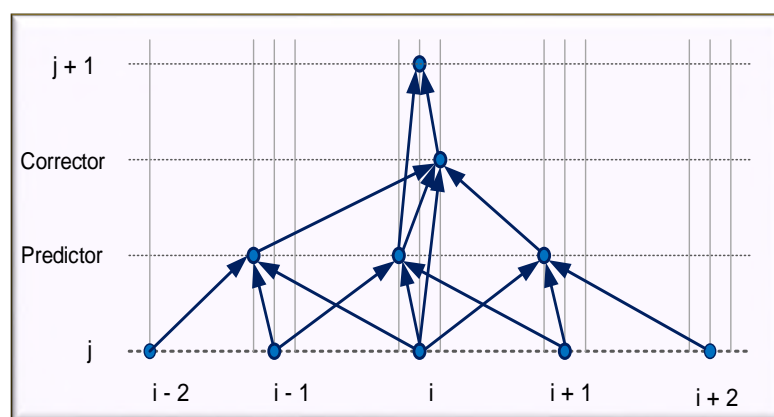


Fig.3: MacCormack scheme

Application of the MacCormack scheme Flow Equation 1 Dimensions (Saint Venant Equation) will be as follows:

$$\frac{\partial u}{\partial t} + \frac{\partial (\tau u)}{\partial x} = 0 \quad (6)$$

fittings flow equation above with Mac Cormack gives [1,2,5]

Predictor:

$$\tilde{U}_i = U_i^j \frac{\Delta t}{\Delta x} [\tau u)_{i+1}^j - \tau u)_{i-1}^j] - \Delta t \tau u)_{i-1}^j \quad (7)$$

Corrector:

$$\bar{U} = U_i \frac{\Delta t}{\Delta x} [\tau u)_{i-1}^j - \tau u)_{i+1}^j] - \Delta t \tau u)_{i-1}^j \quad (8)$$

$$U_i^{j+1} = 0.5 * (U_{\text{predictor}} + U_{\text{corrector}}) \quad (9)$$

The continuity equation for a wide cross-section per unit

$$U_i^{n+1} = \frac{\tilde{U}_i + \bar{U}_i}{2} \quad (10)$$

The continuity equation for a wide cross-section per unit

$$\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} = 0 \quad (11)$$

$$\frac{\partial h}{\partial t} + h \frac{\partial u}{\partial x} + u \frac{\partial h}{\partial x} = 0 \quad (12)$$

Pengespingan equation (12) with Mac Cormack:  $\tilde{h}_i = h_i^j - h_i^j \frac{\Delta t}{\Delta x} [u_{i+1}^j - u_i^j] - u_i^j \frac{\Delta t}{\Delta x} [h_{i+1}^j - h_i^j]$

(13)

$$\begin{aligned} \tilde{h}_i &= h_i^j - \tilde{h}_i \frac{\Delta t}{\Delta x} [\tilde{u}_i - \tilde{u}_{i-1}] - \tilde{u}_i \frac{\Delta t}{\Delta x} [\tilde{h}_i - \tilde{h}_{i-1}] \\ h_i^{j+1} &= \frac{\tilde{h}_i + \tilde{\tilde{h}}_i}{2} \\ h_i^{j+1} &= \frac{\tilde{h}_i + \tilde{\tilde{h}}_i}{2} \\ &= \frac{1}{2} \left\{ h_i^j - h_i^j \frac{\Delta t}{\Delta x} [u_{i+1}^j - u_i^j] - u_i^j \frac{\Delta t}{\Delta x} [h_{i+1}^j - h_i^j] + h_i^j - \tilde{h}_i \frac{\Delta t}{\Delta x} [\tilde{u}_i - \tilde{u}_{i-1}] - \tilde{u}_i \frac{\Delta t}{\Delta x} [\tilde{h}_i - \tilde{h}_{i-1}] \right\} \\ &= h_i^j - \frac{\Delta t}{2\Delta x} \{ h_i^j [u_{i+1}^j - u_i^j] + u_i^j [h_{i+1}^j - h_i^j] + \tilde{h}_i [\tilde{u}_i - \tilde{u}_{i-1}] + \tilde{u}_i [\tilde{h}_i - \tilde{h}_{i-1}] \} \\ h_i^{j+1} &= h_j - \frac{\Delta t}{2\Delta x} \{ h_i^j [u_{i+1}^j - u_i^j] + u_i^j [h_{i+1}^j - h_i^j] + \tilde{h}_i [\tilde{u}_i - \tilde{u}_{i-1}] + \tilde{u}_i [\tilde{h}_i - \tilde{h}_{i-1}] \} \end{aligned} \quad (14)$$

- In state n, t, dx = 0 then all the parameters on the node (i, j) are the values that are used as initial conditions in terms of this model.

- In state n, t, dx > 0; high value of water in the channel (h) is proportional to the width of the channel cross section (b) in (i, j).

Momentum equation cross-section per unit width

$$\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} (\beta u^2 h) + gh \frac{\partial h}{\partial x} + g \frac{hR}{\rho} \frac{\partial p_s}{\partial x} - g(h S_o - S_f) = 0$$

The above equation is the momentum equation in the absence of adverse runoff. If the hydrostatic force due to

density differences are ignored then the equation (11) becomes

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} (\beta u^2 h) + g h \frac{\partial h}{\partial x} - g h (S_0 - S_f) = 0 \quad (16)$$

$$u \frac{\partial h}{\partial t} + h \frac{\partial u}{\partial t} + \frac{\partial}{\partial x} (\beta u^2 h) + g h \frac{\partial h}{\partial x} - g h (S_0 - S_f) = 0 \quad (17)$$

By using equations (2) and (4) to pengespangan to equation (1) the importance of the equation (18) and (19) below;

$$\tilde{Q}_i = Q_i^j - \frac{\Delta t}{\Delta x} \left[ \left( \beta \frac{Q_{i+1}^j{}^2}{A_{i+1}^j} \right) - \left( \beta \frac{Q_i^j{}^2}{A_i^j} \right) \right] - g A_i \frac{\Delta t}{\Delta x} [h_{i+1}^j - h_i^j] + (\Delta t) g A_i^j S_0 - (\Delta t) g \frac{|Q_i^j| Q_i^j}{(A_i^j C^2 R_i^j)} \quad (18)$$

$$\tilde{\tilde{Q}}_i = Q_i^j - \frac{\Delta t}{\Delta x} \left[ \left( \beta \frac{\tilde{Q}_i^2}{\tilde{A}_i} \right) - \left( \beta \frac{\tilde{Q}_{i-1}^2}{\tilde{A}_{i-1}} \right) \right] - g \tilde{A}_i \frac{\Delta t}{\Delta x} [\tilde{h}_i - \tilde{h}_{i-1}] + (\Delta t) g \tilde{A}_i S_0 - (\Delta t) g \frac{|\tilde{Q}_i| \tilde{Q}_i}{(\tilde{A}_i C^2 \tilde{R}_i)} \quad (19)$$

Equation (18) and (19) become equation (20)

$$Q_i^{j+1} = \frac{\tilde{Q}_i + \tilde{\tilde{Q}}_i}{2} \quad (20)$$

The above equation is the equation of the predictor and corrector MacCormack explicit numerical methods for simulation of 1D flow in the channel [5].

Value Debit (Q) on the initial conditions at each node (i, j) is the same as using the Manning equation whereas the discharge value (n + 1) has the same value as the initial condition and the condition (i, j + 1) value obtained discharge initial conditions of discharge for discharge predictions in total the correction divided by two or discharge (Qp + Qc) / 2.

### 2.2.3 Initial Conditions and Boundary Conditions

Mac Cormack method is the explicit calculation of the non-staggered **pembagasan** scheme. To resolve the above discrete equation with this method required some values that are known / defined. These values are called the boundary condition (boundary condition) and the initial conditions (initial condition) [5,6,]

Initial conditions (initial condition) is the determination of the initial values, good water level (h) and discharge (Q), at any point in time the calculation was first started. Making this initial value arranged for not too much with the actual conditions. Intake values are much different from the true value will make the calculations more iterations, so the calculation time is longer (time consuming).

Boundary condition (boundary condition) that is required consists of the downstream boundary condition (down stream boundary) and upstream boundary conditions (up stream boundary) to a fluctuating water level  $h = f(t)$  or can also discharge data (water inflow) at any time,  $Q = f(t)$ .

### 2.2.4 Stability Criteria

Figures from the Courant-Friedrichs-Lewy (CFL) condition express a necessary condition for the convergence of the explicit finite difference scheme dependence domain. Where the issue involves discrete domain differential equations in finite difference boundary is close to zero. Conditions Courant-Friedrichs-Lewy (CFL condition) is a necessary condition for stability while in order to solve certain partial differential equations (PDE usually hyperbolic) numerically with a finite difference method. This figure appears in an explicit numerical analysis of time-based scheme, when it is used for numerical solution. For MacCormack scheme on time step as other explicit scheme, must meet the Courant stability criterion Friedrich-Lewy (CFL) [6,11] as the following equation:

$$\Delta t \leq C n \frac{\Delta x}{(|U \pm C|)_{\max}} \quad (21)$$

## III. APPLICATION SIMULATION MODEL

In order to determine the effect of water level and discharge in the channel as shown in figure 5 below the channel that is used as a model for in simulate with the red line. The parameters in the model are defined as the channel length (L) = 112 m, channels with a rectangular cross-section of the channel width (b) = 2 m; the slope of the line (S0) = 0.00071; cross-section correction factor ( $\beta$ ) = 1; channel wall roughness (n) = 0.025. Simulations performed with the number of nodes on the line segment by 21 grid so that the channel is divided into several sections with  $\Delta x = 5.6$  m and the interval  $\Delta t = 1$  sec. With the Courant number (Cn) = 0.583. In this case the initial value of high water in saluran1, 163 m obtained by using the Manning equation [12]. Boundary condition on the



steady flow case is to set the flow rate at the upstream ( $x = 0$ ) at  $Q = 1.062 \text{ m}^3/\text{sec}$ . The results of the simulation

model can be seen in the table Q and Q predictor corrector below.

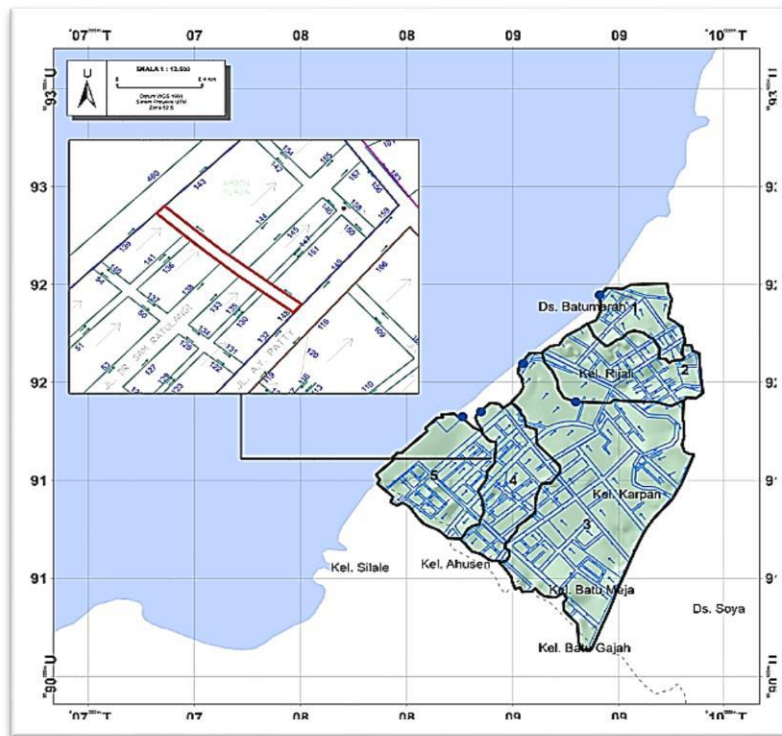


Fig.4: Layout of drainage network distribution system

Table.1: Initial conditions stedy flow

t=0	Node																					
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		21
h	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758	0,758
Q	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062	1,062

Table.2: Conditions steady simulation models

Node	n	t	dx	h	A	P	R	C2	Q	Ap	Qp	hp	Pp	Rp	(C2)p	Ac	Qc
1	0	0	0	0,758	1,516	3,516	0,43	1208,8	1,0624	1,516	1,06	0,758					1,0624
5				0,758	1,516	3,516	0,43	1208,8	1,0624	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0627
10				0,758	1,516	3,516	0,43	1208,8	1,0624	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0627
15				0,758	1,516	3,516	0,43	1208,8	1,0624	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0627
21				0,758	1,516	3,516	0,43	1208,8	1,0624								1,0624

Table.3: Analysis of numerical models

t	dx	h	A	P	R	C2	Q	Ap	Qp	hp	Pp	Rp	(C2)p	Ac	Qc
5	28	0,759	1,516	3,516	0,43	1208,8	1,062	1,516	1,06	0,758					1,062
		0,759	1,516	3,516	0,43	1208,8	1,0636	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0639
		0,759	1,516	3,516	0,43	1208,8	1,0636	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0639
		0,759	1,516	3,516	0,43	1208,8	1,0636	1,516	1,06	0,758	3,516	0,43	1209	1,516	1,0639
		0,759	1,516	3,516	0,43	1208,8	1,0636								1,062
t	dx	h	A	P	R	C2	Q	Ap	Qp	hp	Pp	Rp	(C2)p	Ac	Qc
10	56	0,767	1,516	3,516	0,431	1208,8	1,062	1,516	1,06	0,758					1,062
		0,767	1,516	3,516	0,431	1208,8	1,0652	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0656
		0,767	1,516	3,516	0,431	1208,8	1,0652	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0655
		0,767	1,516	3,516	0,431	1208,8	1,0652	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0655
		0,767	1,516	3,516	0,431	1208,8	1,0652								1,062
t	dx	h	A	P	R	C2	Q	Ap	Qp	hp	Pp	Rp	(C2)p	Ac	Qc
15	84	0,779	1,516	3,516	0,431	1208,8	1,062	1,516	1,06	0,758					1,062
		0,779	1,5164	3,5164	0,431	1208,8	1,0672	1,516	1,07	0,758	3,516	0,43	1209	1,517	1,068
		0,779	1,516	3,516	0,431	1208,8	1,0668	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0671
		0,779	1,516	3,516	0,431	1208,8	1,0668	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0671
		0,779	1,516	3,516	0,431	1208,8	1,0684								1,062
t	dx	h	A	P	R	C2	Q	Ap	Qp	hp	Pp	Rp	(C2)p	Ac	Qc
20	112	0,800	1,516	3,516	0,431	1208,8	1,062	1,517	1,06	0,759					1,062
		0,800	1,5179	3,5179	0,431	1208,8	1,0713	1,518	1,07	0,759	3,518	0,43	1208	1,518	1,0736
		0,800	1,516	3,516	0,431	1208,8	1,0684	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0687
		0,800	1,516	3,516	0,431	1208,8	1,0684	1,516	1,07	0,758	3,516	0,43	1209	1,516	1,0687
		0,800	1,5555	3,5555	0,437	1208,8	1,1736								1,062

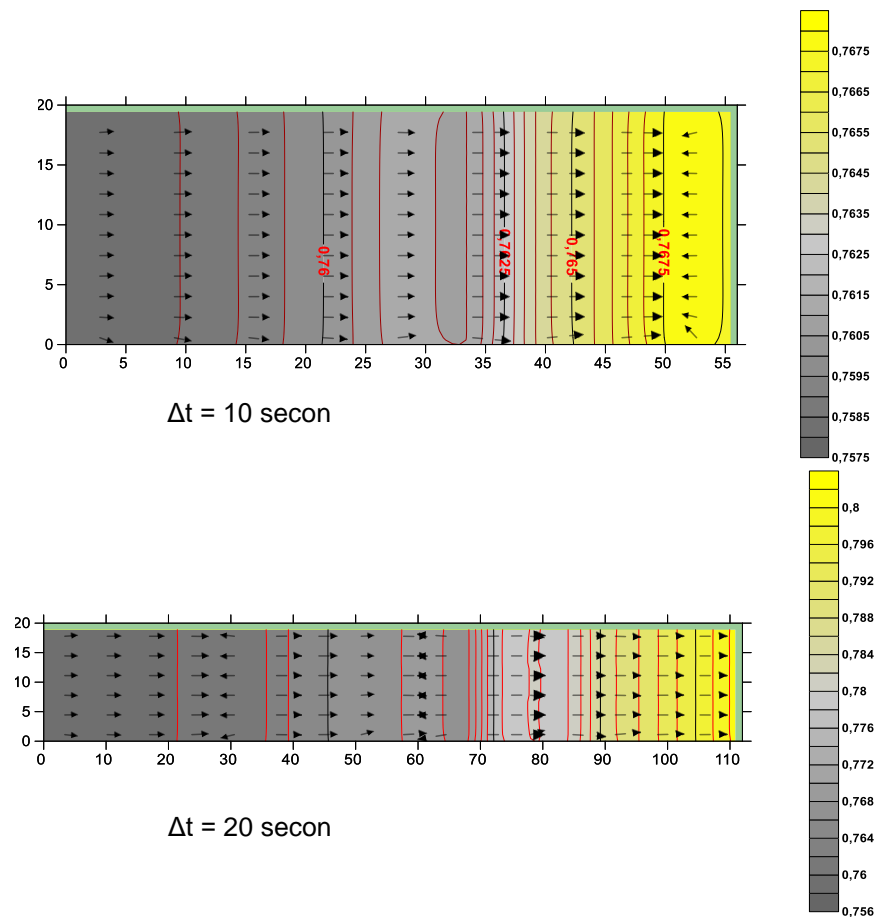


Fig.5: Contours of flow and change the time on the channel at node 148



From the results of numerical experiments running on the channel at node 148 with the boundary condition is the condition of high water in the channel on condition of  $t = 0$ ;  $h = 0.78$  meters and the inflow at node 1 =  $Q$  at the beginning and the end node 21 =  $Q$ . Channel propagation process that occurs at node 148 provides an overview to the fluctuation of water level changes resulting from a change in the distance and the change of time increase. The results of numerical calculations for changes in overtime 10 seconds and 20 seconds at a distance equal to the channel at node 148 which gives an overview of the direction of flow has distributed nearly perpendicular to the flow pattern in a cross section of the channel and the water surface contour changes in longitudinal cross section gives an overview of where each channel presence increment  $\Delta x$  and  $\Delta t$ , the closer the contour lines which led to increased fluctuations in discharge and water level in the channel will be higher than normal.

#### IV. Conclusion

The use of explicit MacCormack method provides an easy to model analysis but has a weakness in the interval ( $\Delta t$ ) in the simulation.

- Determination of boundary conditions and initial conditions must be adapted to the geometry of the channel element because in this case the channel cross section is relatively small.
- The more grids used in the simulation indicates a condition will be stable flow conditions

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