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On Edge Dominating Number of Tensor Product of Cycle and Path

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***Abstract*—*A subset S' of E(G) is called an edge dominating set ofG if every edge not in S' is adjacent to some edge in S'. The edge dominatingnumber of G, denoted by γ'(G), of G is the minimum cardinality takenover all edge dominating sets of G. Let G_1 (V_1, E_1) and G_2(V_2,E_2) betwo connected graph. The tensor product of G_1 and G_2, denoted by$G_1 \otimes G_2$ is a graph with the cardinality of vertex $|V| = |V_1| \times |V_2|$and two vertices (u_1,u_2) and (v_1,v_2) in V are adjacent in $G_1 \otimes G_2$ifu$_1$ $v_1 \in E_1$ and $u_2,v_2 \in E_2$. In this paper we study an edge dominatingnumber in the tensor product of path and cycle. The results show that$\gamma'(C_n \otimes P_2) = \left\lceil \frac{2n}{3} \right\rceil$ for n is odd, $\gamma'(C_n \otimes P_3) = n$ for n is odd, and theedge dominating number is undefined if n is even. For n ∈even number,we investigated the edge dominating number of its component on tensorproduct of cycle C_n and path. The results are $\gamma'_c(C_n \otimes P_2) = \left\lceil \frac{n}{3} \right\rceil$ and$\gamma'_c(C_n \otimes P_3) = \left\lceil \frac{n}{2} \right\rceil$ which C_n ,P_2 and P_3, respectively, is Cycle order n,Path order 2 and Path order 3.*

Keywords*—*edge dominating number, tensor product, path, cycle.

I. INTRODUCTION

One of the interesting topics in graph theory is dominating of graph.In recent years, there are some kinds of dominating in graph have been investigated. Most of those belong to the vertex dominating of graph. A fewresults have been obtained about the edge dominating of graph. In this paper,we mainly discuss about the edge dominating on some product operations ofgraph.

A subset *H* of *E*(*G*) is called an edge dominating set of *G* if every edge notin *H* is adjacent to some edge in *H*. The edge dominating number γ'(*G*) of*G* is the minimum cardinality taken over all edge dominating sets of *G*. Theconcept of edge dominating number are introduced by Mitchell and Hedetniemi[4] and was studied by several researches, such as V.R. Kulli [6,7,8,9,10], S.Arumugam and S. Velamal [1], Araya Chemchan [2], etc.

In graph theory, we have some graph operations, one of them is tensorproduct of graph. Let G_1 (V_1, E_1) and G_2 (V_2, E_2) be two connected graph.The tensor product of $G_1$and G_2, denoted by $G = G_1 \otimes G_2$ is a graph withthe cardinality of vertex $|V| = |V_1| \times |V_2|$ and two vertices (u_1,u_2) and (v_1, v_2)in *V* are adjacent in $G_1 \otimes G_2$ if $u_1v_1 \in E1$ and $u_2v_2 \in E_2$. All graph consideredhere are finite, nontrivial, undirected, connected, without loops and multipleedges. We will verify the tensor product operation between cycle and path.

Cycle is a single vertex with a self-loop or simple graph *C* with $|V_c| = |E_c|$that can be drawn so that all of its vertices and edges lie on a single circle. An*n−vertex* cycle graph is denoted by C_n[3]. A path is a simple graph with$|V_p| = |E_p| + 1$ that can be drawn so that all of its vertices and edge lie on asingle straight line. A path graph with m vertices and *m−1 edges* is denotedby P_m [3]. From the previous research, S. Arumugam and S velamal show thatthe edge dominating number of cycle graph is $\gamma'(C_n) = n$ for $n \geq 3$ [1] and based on the paper "The Neighborhood Total Edge dominating Number of A Graph" by V.R. Kulli, we know that the edge dominating number of the pathgraph $\gamma'(P_n) = n$ for $n \geq 2$, $n = 0 \bmod 3$ [10]. In this paper, we are focus onfinding the edge dominating number on tensor product of graph,respectivelytensor product of cycle C_n and path which haven't been investigated before.

II. EDGE DOMINATING NUMBER

An edge in graph *G* dominates itself and its adjacent. A set of edges *S'* ingraph *G* is an edge dominating set, if each edge of *G* is dominated by some edgesin *S'* . The dominating number γ'(*G*) of *G* is the minimum cardinality of edgedominating set of *G*. The concept of edge dominating was firstly introducedby Mitchel and Hedetniemi [4]. Let we take an example in cycle C_9 (see figure2), we can verify some edge dominating sets such as $X_1 = \{e_1, e_3, e_4, e_6, e_8\}$,$X_2 = \{e_1, e_4, e_6, e_8\}$ and $X_3 = \{e_1, e_4, e_7\}$. The cardinality of X_1, X_2 and $X_3$are respectively 5, 4, and 3. Based on the definition, the dominating numberof C_9 is the minimum cardinality of edge dominating set of C_9 . So, we get$\gamma'(C_9) = 3$.

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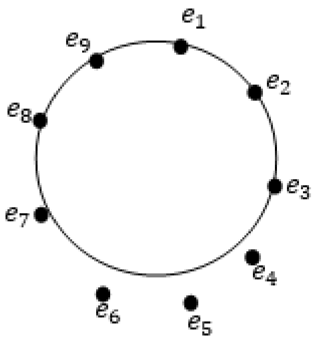

Fig.1: cycle graph order 9

Consider the graph C_9 shown in figure 1, we can see that the edge dominating set of $C_9$with cardinality 3 is not only X_3= {e_1, e_4, e_7}. There also existanother edge dominating sets with cardinality 3, such that X_4 = {e_2, e_5, e_8}and X_5 = {e_3, e_6, e_9}. Our concern in this paper is not about the element ofedge dominating set, but the minimum cardinality taking on edge dominatingset. As long as the cardinality of edge dominating set is minimum, it can beconcluded as edge dominating number of graph G. In order to verify the edgedominating number on tensor product of path and cycle in this paper, we needthe following term. The degree of an edge $e = uv$ of G is defined by $deg\ (e) = deg\ (u) + deg\ (v - 2)$ [1]. For a real number x, the greatest integer less thanor equal to x is denoted by $\lceil x \rceil$ and the smallest integer greater than or equalto x is denoted by$\lfloor x \rfloor$.

III. MAIN RESULTS

The following theorems show the properties of the edge dominating number on tensor product of path and cycle. Path order 2, path order 3, and cyclegraph order n, respectively, are denoted by P_2, P_3, and C_n. Here are someresults of edge dominating number on tensor product of cycle and path.

Theorem 3.1 Let C_n and P_2 be, respectively cycle order n and path order2. For $n \geq 3$ and n is odd,

$$\gamma'(C_n \otimes P_2) = \left\lceil \frac{2n}{3} \right\rceil$$

Proof. The graph $C_n \otimes P_2$ is a regular graph 2. By definition, for any regulargraph with degree 2 is a cycle. If we observe graph $C_n \otimes P_2$, this graph isisomorphic with cycle graph with the cardinality of edges is $2n$. Thus, weshould consider the edge dominating number of cycle graph. Because everyedge in $C_n \otimes P_2$ has degree 2, So, for every $E \in S'$, E can dominated maximum3 sides in $Cn \otimes P_2$, $|S'| \geq \frac{2n}{3}$. Let says that the minimum cardinality of edgedominating number is $\gamma'(C_n \otimes P_2) = \left\lceil \frac{2n}{3} \right\rceil$ for $n \geq 3$. In order to prove that$\left\lceil \frac{2n}{3} \right\rceil$is the minimum cardinality of the dominating set S' in $C_n \otimes P_2$, weuse contradiction. Let S is a minimum edge dominating set with cardinality$\left\lceil \frac{2n}{3} \right\rceil - 1$. The maximum edge can be dominated is 3($\left\lceil \frac{2n}{3} \right\rceil - 1) \leq 3($ $\left(\frac{2n+2}{3}\right) - 1$ $=2n - 1$. We know that $\|C_n \otimes P_2\| = 2n$. If the cardinality of minimum edgedominating set S' is $\left\lceil \frac{2n}{3} \right\rceil - 1$, there are one edge which cannot be dominatedby S' . So, $|S'| = \gamma'(C_n \otimes P_2) = 2n$ is the minimum cardinality taken all of the dominating set. It can be concluded $\gamma'(C_n \otimes P_2)$ $= \left\lceil \frac{2n}{3} \right\rceil$.

For $n \geq 3$ and even, there is no domination number since ($C_n \otimes P_2$) isdisconnected graph. We can find the edge dominating number only on connected graph. For $n \in$ even number, the tensor product $C_n \otimes P_2$ consist oftwo component of graph which is isomorphic one and other. If we separatethe two component, we can see that each graph is isomorphic to cycle graphC_n. Then, we can verify a new theorem about edge dominating number oncomponent of tensor product $C_n \otimes P_2$ (denoted by γ'_s), for $n \geq 3$ and even.

Theorem 3.2 Let C_n and P_2 be, respectively, cycle order n and path order2. For any component of $C_n \otimes P_2$, for $n \geq 3$ and even,

$$\gamma'_c (C_n \otimes P_2) = \left\lceil \frac{n}{3} \right\rceil$$

Proof. We have noticed that the component of $C_n \otimes P_2$for $n \in$ even numberis isomorphic to cycle graph order n. Thus, we should consider the edge dominating number of cycle graph. Because every edges in cycle graph has degree2, So, for every $E \in S'$, E can dominated maximum 3 sides in C_n, $|S'| \geq \frac{n}{3}$ .Because $n \in Z^+$, which $Z^+$is denoted an integer more than zero, so $S' = \left\lceil \frac{n}{3} \right\rceil$.

Now, we proof $\gamma'(C_n) \geq n$. By contradiction, let $|S'| = \gamma'(C_n) \leq \left\lceil \frac{n}{3} \right\rceil - 1$.Without loose of generality, we can assume that S' is a minimum edge dominating set of cardinality $\left\lceil \frac{n}{3} \right\rceil - 1$. The maximum edge can be dominated is$3(\left\lceil \frac{n}{3} \right\rceil - 1) \leq 3(\frac{n+2}{3}) - 1 = n - 1$. Not all edges can be dominated by S'. So, $|S'| = \gamma'(C_n) \leq \left\lceil \frac{n}{3} \right\rceil$ is the minimum cardinality taken all of the dominatingset. We can conclude $\gamma'_c (C_n \otimes P_2) = \left\lceil \frac{n}{3} \right\rceil$, for any component of $C_n \otimes P_2$ and $n \in even\ number$.

Theorem 3.3 Let $C_n$and P_3 be, respectively, cycle order n and path order3. For $n \geq 3$ and $n\ is\ odd$,

$$\gamma'(C_n \otimes P_3) = n$$

Proof. The tensor product of $C_n \otimes P_3$ consist of $2 \times n \times$

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$(3-1)=4n$ edges.The maximum edge degree in $C_n \otimes P_3$ is $deg\ (u_1v_1) = deg\ (u_1) + deg\ (v_1) - 2 = 4$.So, every edge with maximum degree can dominate 5 other edges includeditself. Suppose that, the edge with maximum degree is one of the element ofedge dominating set in $C_n \otimes P_3$. There are $\left\lfloor\frac{2n}{3}\right\rfloor$ edges with maximum degreewhich can dominate 5 edges included it's self. In order to determine the edgedominating number of $C_n \otimes P_3$, Let we observe the following condition ofdominating set if the element of edge dominating set is the edge with maximumdegree.

1. If $n$is multiple of three, there are$\frac{2n}{3}$edges which can dominate 2edges and $\left\lfloor\frac{2n}{3}\right\rfloor$ edges with maximum degree that can dominated 5 edges. Thetotal edges can be dominated by those edges are $5.\frac{2n}{3} + 2.\frac{2n}{3} = 4n$. So, we needminimal $|S'| = \frac{2}{3} + \frac{2n}{3} = n$ edges so that all edges in $C_n \otimes P_3$can be dominated.If the cardinality of S' is $n-1$, there will be minimal 2 edges which can't bedominated byS'.
2. If $n-1$ is multiple of three, there are $2\left(\frac{n-1}{3}\right)$ edges which can dominate 2 edges, $\left\lfloor\frac{2n}{3}\right\rfloor$ edges with maximum degree, and one edge that can dominate4 other edges include it's self. So, the cardinality of edge dominating set is$|S'| \geq \frac{2n-2}{3} + \frac{n-1}{3} + 1 = n$. The total edges can be dominated by $|S'| = n$ edges are $5.\left\lfloor\frac{2n}{3}\right\rfloor + 2.\left(\frac{n-1}{3}\right) + 4 = 4n$. If the cardinality of S' is $n-1$, there will beminimal 2 edges which can't be dominated by S' .
3. If $n+1$ is multiple of three, there are $2\left\lfloor\frac{n}{3}\right\rfloor$edges which can dominate 2 edges, $\left\lfloor\frac{2n}{3}\right\rfloor$ edges with maximum degree, and one edge that can dominate3 other edges include it's self. So, the cardinality of edge dominating set is$|S'| \geq \frac{2n-1}{3} + \frac{n-2}{3} + 1 = n$. The total edges can be dominated by $|S'| = n$edges are $5.\left\lfloor\frac{2n}{3}\right\rfloor + 2.\left(\frac{n-1}{3}\right) + 4 = 4n$. If the cardinality of S' is $n-1$, there willbe minimal 2 edges which can't be dominated by S'.

From those three conditions, we can see that the minimum cardinality ofedge dominating set in each condition is n. So, it can be concluded $\gamma'\ (C_n \otimes P_3) = n$, for $n \in even\ number$.

For $n \geq 3$ and even, there is no domination number since $C_n \otimes P_3$ isdisconnected graph. Because tensor product $C_n \otimes P_3$for $n \in even\ number$consist of disconnected graph, so the edge dominating number is undefined.Although we can not determine the edge dominating number of $C_n \otimes P_3$, butwe can consider the component of $C_n \otimes P_3$to be analyzed. The tensor product $C_n \otimes P_2$consist of two component which is isomorphic. From the componentof $C_n \otimes P_3$, we can determine the edge dominating number of each componentin $C_n \otimes P_3$. Here is the theorem of edge dominating number on component oftensor product $C_n \otimes P_2$(denoted by γ'_c), for $n \geq 3$ and even.

Theorem 3.4 Let $Cn$and $P3$be, respectively, cycle order n and path order3. For any component of $C_n \otimes P_3$, for $n \geq 3$ and even,

$$\gamma'_s(C_n \otimes P_3) = \frac{n}{2}$$

Proof. The component of $C_n \otimes P_3$consist of $2n$ edges. Suppose that,the edge with maximum degree is one of the element of edge dominating setin subgraph $C_n \otimes P_3$. The maximum edge degree in subgraph of $C_n \otimes P_3$is $deg\ (u_1v_1) = deg\ (u_1) + deg\ (v_1) - 2 = 4$. So, every edge with maximumdegree can dominated 5 other edges included itself. If n is multiple of 3, therewill be $\frac{n}{3}$ edges with maximum degree that can dominate 5 edges and $\frac{n}{6}$ whichcan dominate 2 edges. So, the total edge can be dominated by those edgesare $5.\frac{n}{3} + 2.\frac{n}{6} = 2n$. We can say that $\frac{n}{3} + \frac{n}{6} = \frac{n}{2}$ edges can dominate alledges in $C_n \otimes P_3$. If $n-1$ is multiple of 3, there will be $\left\lfloor\frac{n}{3}\right\rfloor$ edges withmaximum degree, $\frac{n-4}{6}$ which can dominate 2 edges, and one edge which candominated 3 edges. So, the total edge can be dominated by those edges are$5.\left\lfloor\frac{n}{3}\right\rfloor + 2.\frac{n-4}{6} + 1.3 = 2n$. There will be $|S'| \geq \frac{n-1}{3} + \frac{n-4}{6} + 1 = \frac{n}{2}$edges whichcan dominate all edges in $C_n \otimes P_3$. If $n-2$ is multiple of 3, there will be$\left\lfloor\frac{n}{3}\right\rfloor$edges which can dominate 5 edges, $\frac{n-2}{6}$which can dominate 2 edges, andone edge which can dominate 4 edges. So, the total edge can be dominated by those edges are $5.\left\lfloor\frac{n}{3}\right\rfloor + 2.\frac{n-2}{6} + 1.4 = 2n$. It means that $|S'| \geq \frac{n-2}{3} + \frac{n-2}{6} + 1 = \frac{n}{2}$edges can dominate all edges in $C_n \otimes P_3$. From those explanation, we can seethat we need minimal $\frac{n}{2}$ edges to dominate all edges in $C_n \otimes P_3$.

In order to check that $|S'| = \frac{n}{2}$ is minimum cardinality of edge dominatingset, we use a contradiction. Assume that $|S'| = \frac{n}{2} - 1$ is minimum cardinalityof edge dominating set. If $|S'| = \frac{n}{2} - 1$, there will be minimal two edges whichcan not be dominated. So, it can be concluded that $\gamma'_s(C_n \otimes P_3) = \frac{n}{2}$, for $n \in even\ number$ and $n \geq 3$.

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IV. CONCLUSION

In this paper, we have already investigated the edge dominating numberon tensor product of cycle and path. The results are as follow:

- $\gamma' (C_n \otimes P_2) = \left\lceil \frac{2n}{3} \right\rceil$, for $n \geq 3$ and n is odd.
- $\gamma'_c (C_n \otimes P_2) = \left\lceil \frac{n}{3} \right\rceil$, for any subgraph of $C_n \otimes P_2$ and $n \in even\ number$.
- $\gamma'(C_n \otimes P_3) = 3$, for n ≥ 3 and n is odd.
- $\gamma'_c (C_n \otimes P_3) = \frac{n}{2}$, for any subgraph of Cn ⊗ P3 and n ∈ even number

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