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The Iteratively Regularized Newton Method for Reconstructing Image of Microwave Tomography

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Abstract— *The iteratively regularized Newton method has been investigated to obtain stable solutions of Microwave Tomography inverse problems. The problem is presented in term of minimization of the difference between measured scattered field and the guessed field which is calculated from initial guess of the exact solution. The iterative sequence is defined based on the Newton method, in which, decreasing sequence of positive number regulates the ill-posed operator of the nonlinear problem. The method proposed is tested using numerical example and compared with non-iterative regularization method.*

Keywords*— *iteratively regularized, inverse problem microwave tomography.

I. INTRODUCTION

The image of Microwave Tomography (MT) can be produced by illuminating a dielectric body with a microwave field in different directions, and measuring the scattered field around the object at each illumination. The data of the measured scattered fields are processed using image reconstruction algorithms to produce the electric property distribution of the object. The reconstructions are completed in two steps: forward and inverse problems.

Numerically the forward problem which is also well known as direct scattering problem can directly be solved accurately. On the other hand, the inverse problem is a highly nonlinear system and suffers from ill posed, thereto, regulated inverse method is needed to solve the problem. Currently, the Newton based methods are popular techniques to solve the MT inverse problem for speed reason. The Newton Kantorovich method [1], Inexact Newton Method [2], Quasi-Newton method [3] [4] and Gauss-Newton inversion [5] had been intensively applied to the MT inverse problem. The results show that the regulator is essential to stabilize the reconstruction process.

Thikonov regularization is probably the most recognized regulator in MT which is applied in [1] [3] [4], beside the filter method in the form of Truncated SVD. Multiplicative and Additive regulator has also been applied in MT inverse problem [2]. Nevertheless, the last regulator need second derivative of the cost function which increases the computational burden. Furthermore, the iteratively regularized Gauss-Newton method has been proved that it can be used to provide stable solution of non-linear ill-posed problem [6]. In this paper the method is applied to solve the MT inverse problem and tested using synthetic data.

II. FORWARD PROBLEM

Direct scattering problem is presented in electric field integral equations (EFIE) under TM polarization. The incident field inside the domain object is stated as domain equation as follows

$$E_z^i(x,y) = E_z^t(x,y) + \frac{k_0\eta_0}{4}\iint J_z(x',y')H_0^{(2)}(k_0 r)dx'dy' \quad (1)$$

Where total electric field may be replaced by the current source $J = j\omega\varepsilon_0(\varepsilon_r - 1)E$. The scattered fields are computed using data equation, which is stated as

$$E_z^s = -\frac{k_0\eta_0}{4}\iint J_z(x',y')H_0^{(2)}(k_0 r)dx'dy' \quad (2)$$

The equations are solved using the method of Moment. The object is divided into N number of cells with equal in size, then the $J(x,y)$ can be approximated by superposition of Pulse Basis function. The integral in (1), (2) are evaluated analytically with the assumption that all the cell are circles [7] and written in matrix equations (4) and (5). To avoid singularity at background medium the contrast of dielectric is stated as :

$$\chi_\varepsilon = (\varepsilon_r - 1)/\varepsilon_r \quad (3)$$

and the integral equations are written as:

$$\boldsymbol{\chi}\mathbf{E}^i = \left[\boldsymbol{\chi}\mathbf{Z}_N - \frac{\eta_0}{k_0}\mathbf{I}\right]\mathbf{J} \quad \text{(domain equation)} \quad (4)$$

$$\mathbf{E}^s = -\mathbf{Z}_M\mathbf{J} \quad \text{(data equation)} \quad (5)$$

$\mathbf{Z}_N, \mathbf{Z}_M$ is $N \times N$ and $N \times N$ matrix sequentially, where

$$Z_{mn} = \begin{cases} m \neq n, & \frac{\eta_0\pi a_n}{2} J_1(k_0 a)H_0^2(k_0\rho) \\ m = n, & \frac{\eta_0\pi a_n}{2} H_1^2(k_0 a_n) \end{cases}$$

Once the $\boldsymbol{\chi}$ is given in (4), the $\mathbf{J}$ is easily computed using domain equation. Furthermore, the electric fields can be determined using the data equations (5), in which the cells size is determinant to the level of accuracy.

III. INVERSE PROBLEM

The MT inverse problem can be stated as the operator equation which is continous dan differentiable at $D(\mathcal{F})$. If $\boldsymbol{E}_p^i$ are illuminated to the dielectric object from P number of projections, then the operator equation for MT inverse problem is written as

$$\mathcal{F}(\chi) = -\sum_{p=1}^{P}\left[\mathbf{Z}_M\,[\boldsymbol{\chi}\mathbf{Z}_N - I]^{-1}\,\boldsymbol{\chi}\boldsymbol{E}_p^i\right] \quad (6)$$

The target solution, which is measured data is stated as:

$$y = \sum_{p=1}^{P}\left[\boldsymbol{E}_{mea,p}^s\right] \quad (7)$$

The inverse problem determines the unknow physical quantity $\boldsymbol{\chi}$ from set of data measurement. In practical situation only approximated data (y^δ) are available. However, to simplify the experiment, in this paper the data used are synthetic which are free from noise. Thus the problem in term of operator equation is written as

$$\mathcal{F}(\chi) = y \quad (8)$$

The well known classical regularized solution (χ_α) of (8) which is non-linear and ill-posed are gained by minimizing the problem using Tikhonov regularization as follows:

$$\chi_\alpha^\delta \in \text{argmin}[\|\mathcal{F}(\chi) - y\|^2 + \alpha\|\chi - \chi_0\|^2] \quad (9)$$

where α is a positive real number and χ_0 is the initial guess of the solution of (8). In practice, solving (9) using Newton method will face two main problems, which are determining the value of Tikhonov regulator α and determining direction of iteration. Both variables, which are hard to be found, directly influence the stability of the inverse problem. Therefore, stable solution of (8) sometimes difficult to obtain.

To overcome the instability of the solution of (8), the iteratively regularized Newton method is proposed. It solves

$$\chi_{\alpha_k}^\delta \in \text{argmin}[\|\mathcal{F}(\chi) - y\|^2 + \alpha_k\|\chi - \chi_0\|^2] \quad (10)$$

where α_k is dynamic over the iteration.

The iterative sequence of solution $(\chi_{\alpha_k}^\delta)$ is stated as

$$\chi_{\alpha_k}^\delta = \chi_{\alpha_{k-1}}^\delta + d\Delta\chi_{\alpha k}^\delta \quad (11)$$

where $d < 1$, and $\Delta\chi_{\alpha k}^\delta$ is the step of iteration.

The step of iteration (11), in which the iteratively regularized Newton method is applied, is calculated using Gauss-Newton algorithm as follows:

$$\Delta\chi_{\alpha k}^\delta = -\left[\mathcal{F}'(\chi_{\alpha k}^\delta)^*\mathcal{F}'(\chi_{\alpha k}^\delta) + \alpha_k I\right]^{-1}\left(\mathcal{F}'(\chi_{\alpha k}^\delta)^*\left(\mathcal{F}(\chi_{\alpha k}^\delta) - y\right) + \alpha_k(\chi_{\alpha k}^\delta - \chi_0)\right) \quad (12)$$

where $\mathcal{F}'(\chi_{\alpha k}^\delta)$ is Frechet differential of (6) at $\chi_{\alpha k}^\delta$ which is defined as

$$\mathcal{F}'(\chi) = \sum_{p=1}^{P}\left[\mathbf{Z}_M\,[\boldsymbol{\chi}\mathbf{Z}_N - I]^{-1}\left[\boldsymbol{E}_p^i + \mathbf{Z}_N\mathbf{J}_p\right]\right] \quad (13)$$

Some authors apply the algorithm by considering iteratively regularization with the number of iterations $n_\delta = N$ being fixed [6]. In this paper, α_k plays the role of the regularization parameter. The α_k is a positive real number that sequentially updated along the iteration. Therefore, the sequence of the α_k and the initial value of $\alpha_{k=0}$ are considered to build stable solution.

The stable approximated solution with initial guess $\chi_0 \in D(\mathcal{F})$, is obtained as $\chi_{\alpha_k}^\delta \to \chi$ and $\|\mathcal{F}(\chi_{\alpha k}^\delta) - y\|$ decreases to 0 for $k \to \infty$. Therefore, the proposed iteratively regulator is a sequence that follows

$$\frac{\alpha_k}{\alpha_{k+1}} = r \quad (14)$$

where r is a real positive number bigger than 1, which is determined in the experiment.

Furthermore, the starting value of the iteratively regulator is iteratively determined. Initially, the value is set as a small positive real number, which is $\alpha_0 = 0.0001$. Then it is iteratively increase to keep the solution inside the domain $\chi_{\alpha k}^\delta \in D(\mathcal{F})$. The sequence of α_0 follows

$$\alpha_0 = \alpha_0 + \tfrac{1}{2}\alpha_0 \quad (15)$$

and ends as the following condition satisfied

$$\|\mathcal{F}(\chi_{\alpha 0}^\delta) - \mathcal{F}(\chi_{\alpha 1}^\delta)\| < tolerance \quad (16)$$

The tolerance is a positive real number less than 1, which is defined at the experiment.

Once α_0 and r are determined, the algorithm (12) can be executed. The algorithm is controlled by stopping index of iteration n_δ, which is selected according to

$$\|\mathcal{F}(\chi_{n_\delta}^\delta) - y^\delta\| \le c\delta < \|\mathcal{F}(\chi_{\alpha k}^\delta) - y^\delta\| \quad (17)$$

where $c > 1$ and $0 \le k < n_\delta$

IV. NUMERICAL EXPERIMENT

The research design is numerical experiment. The measured data are synthetic which are obtained from forward problem (4)(5). The data are reconstructed by the iteratively regularized Newton algorithm (12). Comparative study is used to analyze the stability of the solutions. The images resulted by iteratively regularization Newton method are compared to the images resulted from classical Tikhonov regularization (9). The degree of reconstruction difficulty is determined by the complexity of the object.

Two difference object are used to test the algorithm. Two dielectric circular cylinders and four similar dielectric circular cylinders are placed inside the object domain. The object domain is illuminated using TM signal at 4 GHz from 16 antennas at data domain sequentially. At each illumination, the scattering field data are gained at 16 antennas around the object domain. Then, all 16 projection data are reconstructed using method proposed. The results of reconstruction are shown in fig 1.

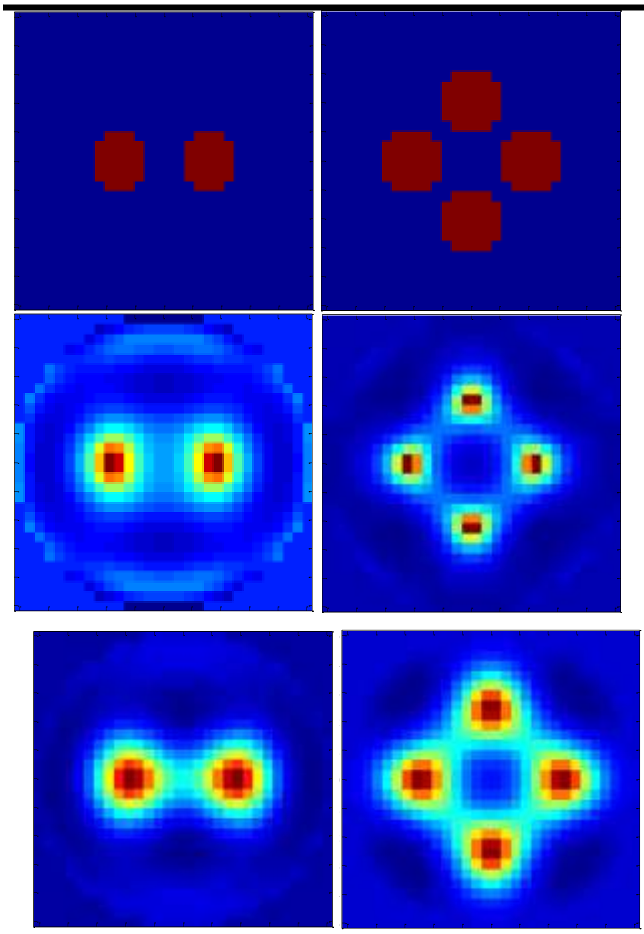

Fig.1: The image of original object (top), reconstructed using Tikhonov regularization (center) and reconstructed using iteratively regularized Newton method (bottom)

It can be seen that iteratively regularized Newton method produces a stable solution. The number and the shape of the dielectric object can be clearly distinguished, while the other regulator produces less quality image. The proposed method capable in eliminating the background material. Thus the error of the solution is lower than the error of solution of classical Tikhonov regularization. The sequence of the α_k and its initial guess selection stabilize the solution.

The geometric series of α_k with ratio in between 1.5 to 3 is selected as the sequence of the regulator. It brings established solution of (10) for both cases tested as seen in the fig 1. The $r = 2$ seems to be acceptable value to set for this type of problem. Furthermore, the decreasing of the regulator is different from [6], which is cq^k with $q < 1$. Thus further studied need to be done to see the effect of regulator series to the speed and stability of solution of MT inverse problem.

The procedure for defining the initial value of the regulator α_k ensures the solution stay at object domain. This eliminates the difficulty in selecting regulator and direction at classical Tikhonov regularization. Experimentally the tolerance is in between 0.2 to 0.5.

V. CONCLUSION

The results of the numerical experiment reveal that the iteratively regularized Newton method combined with the applying geometric series of regulator and setting initial value of regulator produce stable the solution for MT image inverse problem. The optimal solution are gain when the regulator is sequentially decreased of optimal order for $r = 2$ and the initial value of regulator equal is set that produce the norm of sequential operator equation $\mathcal{F}(\chi)$ equal to 0.4.

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