

# Cost-Benefit Analysis of a Redundant System with Server having Refreshment Facility Subject to Inspection

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**Abstract**— In this paper two units cold standby system has been discussed with the facility that server inspect the failed unit before repair/replacement of the unit and server may allow to take refreshment whenever needed. The operative unit may fail directly from normal mode and the cold standby unit may be failed owing to remain unused for a longer period of time. There is single server who serves the dual purpose of inspection and repair immediately whenever required. Also, after having refreshment the server may eventually perform the better service efficiently. The time to take refreshment and repair activity follows negative exponential distribution whereas the distributions of unit failure and server failure are taken as arbitrary with different probability density functions. The expressions of various stochastic measures are analyzed in steady state using semi-Markov process and regenerative point technique. The graphs are sketched for arbitrary values of the parameters to delineate the behavior of some important performance measures to check the efficacy of the system model under such situations.

**Keywords**— Profit Analysis, Cold-standby, regenerative point, steady state and Semi-Markov process, inspection and refreshment.

## I. INTRODUCTION

Redundancy is the provision of alternate means or parallel paths in a system for performing the given assignment to the system. Application of redundancy in the system design is found in almost all types of system due to its numerous advantages to improve reliability and availability of a system. Various forms of redundancy-active (hot) redundancy, standby (cold) redundancy, warm redundancy, component redundancy, system redundancy etc can be installed in a system, depending upon their feasibility. The use of a particular approach depends upon many factors such as the operating characteristics of components or systems, weight, size and initial cost. In literature, the stochastic behavior of cold

standby system has been widely discussed by many researchers including Osaki and Nakagawa [1971] discussed a two-unit standby redundant system with standby failure. Nakagawa and Osaki [1975] analyzed stochastic behavior of a two-unit priority standby redundant system with repair. Subramanian et al [1976] explored reliability of a repairable system with standby failure. Gopalan and Naidu [1982] analyzed cost-benefit of a one-server system subject to inspection. Gopalan and Nagarwalla [1985] evaluated cost benefit of a one server two unit cold standby system with repair and age replacement. Singh and Srinivas [1987] investigated stochastic analysis of a two unit cold standby system with preparation time for repair. Singh [1989] evaluated profit of a two-unit cold standby system with random appearance and disappearance time of the service facility. Dhillon [1992] evaluated reliability and availability analysis of a system with standby and common cause failures. Lam [1997] developed a maintenance model for two-unit redundant system. Kumar [2005] analyzed of reliability models with different types of failure and repair policies. Malik and Barak [2007] analyzed a single-server system operating under different weather conditions. Malik [2009] discussed reliability modeling and cost-benefit analysis of a system— a case study. Bhardwaj et al [2014] have described semi-Markov approach for asymptotic performance analysis of a standby system with server failure. Malik et al [2015] analyzed performance of a stochastic system with standby failure and maintenance. Some of them have generally imagined the server to be always in good condition and it never fails while working. But this imagination seems to be quite impractical when a server has to work in varying environmental conditions. We may observe many cases where the server fails during his performance. Recently, Barak and Dhiraj [2016] investigated stochastic analysis of a cold standby system with server failure.

In a cold standby redundant system whenever the operating unit fails, the standby unit takes its existence

and the failed unit goes under repair. But it may be possible that the standby unit is already damaged owing to remain unused for a longer period of time or erosion etc. So keeping the view of above research work in mind, we developed a stochastic model of redundant standby system with server failure. The model consists of two identical units; one unit is in operative mode and other in cold standby. The cold standby unit becomes operative after failure of the operative unit. The failure of the server during any service activity can produce undesirable results in terms of safety as well as economic losses and server may go for refreshment to increase his efficiency whenever required. The server works afresh after taking refreshment with full efficiency. The time to take refreshment and repair activity follows negative exponential distribution whereas the distributions of unit and server failure are taken as arbitrary with different probability density functions. The expressions for various reliability measures such as transition probabilities, mean sojourn times, mean time to system failure, steady state availability are deduced by using semi-Markov process and regenerative point technique. The graphical behaviors of some important performance measures to check the efficacy of the system model under such situations are delineated for arbitrary values of the parameters.

**System Assumptions and Notations:**

- E : Set of regenerative states  $\{S_0, S_1, S_2, S_3\}$ .
- O/Cs : The unit is operative /cold standby
- $FUi / FUI$  :The failed unit is under inspection/under inspection continuously from previous state.
- $FWi / FWI$  : The failed unit is waiting for inspection/waiting for inspection continuously from previous state.
- $Fur / FUR$  : The failed unit is under repair/ under repair continuously from previous state.
- SUt/ SUT : The server is busy to getting refreshment due to his requirement / continuously busy to getting refreshment from previous state.
- $Fwr / FWR$  : The failed unit is waiting for repair / waiting for repair continuously from previous state due to server is continuously busy to taking refreshment from previous state.

- $\lambda / \mu / \gamma$  : Constant failure rate of unit / rate by which server feel requirement of refreshment / the rate by which unit goes for repair after inspection.
- a/b :Probability that cold standby unit is ready for use / not ready for use.
- $f(t)/F(t)$  : pdf / cdf of refreshment rate by which the server to recover his freshness.
- $g(t)/G(t)$  : pdf / cdf of repair rate of the failed unit.
- $h(t)/H(t)$  : pdf / cdf of replacement rate of the failed unit.
- $q_{ij}(t)/Q_{ij}(t)$  : pdf/cdf of direct transition time from a regenerative state  $S_i$  to a regenerative state  $S_j$  without visiting any other regenerative state.
- $q_{i,j;k}(t)/Q_{i,j;k}(t)$  : pdf / cdf of first passage time from a regenerative state  $S_i$  to a regenerative state  $S_j$  or to a failed state  $S_j$  visiting state  $S_k$  once in  $(0,t]$ .
- $q_{i,j;k(r,s)}(t) / Q_{i,j;k(r,s)}(t)$  : pdf / cdf of first passage time from a regenerative state  $S_i$  to a regenerative state  $S_j$  or to a failed state  $S_j$  visiting state  $S_k, S_r$  and  $S_s$  once or more than one times in  $(0,t]$ .
- $M_i(t)$  : Probability that the system is up initially in state  $S_i \in E$  is up at time t without visiting to any other regenerative state or before returning to the same state via one or more non-regenerative states.
- $W_i(t)$  : Probability that the server is busy in state  $S_i$  up to time t without making any transition to any other regenerative state or before returning to the same state via one or more non-regenerative states.
- $m_{ij}$  : Contribution to mean sojourn time  $\mu_i$  in state  $S_i$  when system transit directly to state  $S_j$  so that
 
$$\mu_i = \sum_j m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^* (0)$$
- $\otimes / \odot$  : Symbol for Stieltjes convolution / laplace convolution
- L.T/L.S.T : Laplace transformation/Laplace stieltjes transformation.

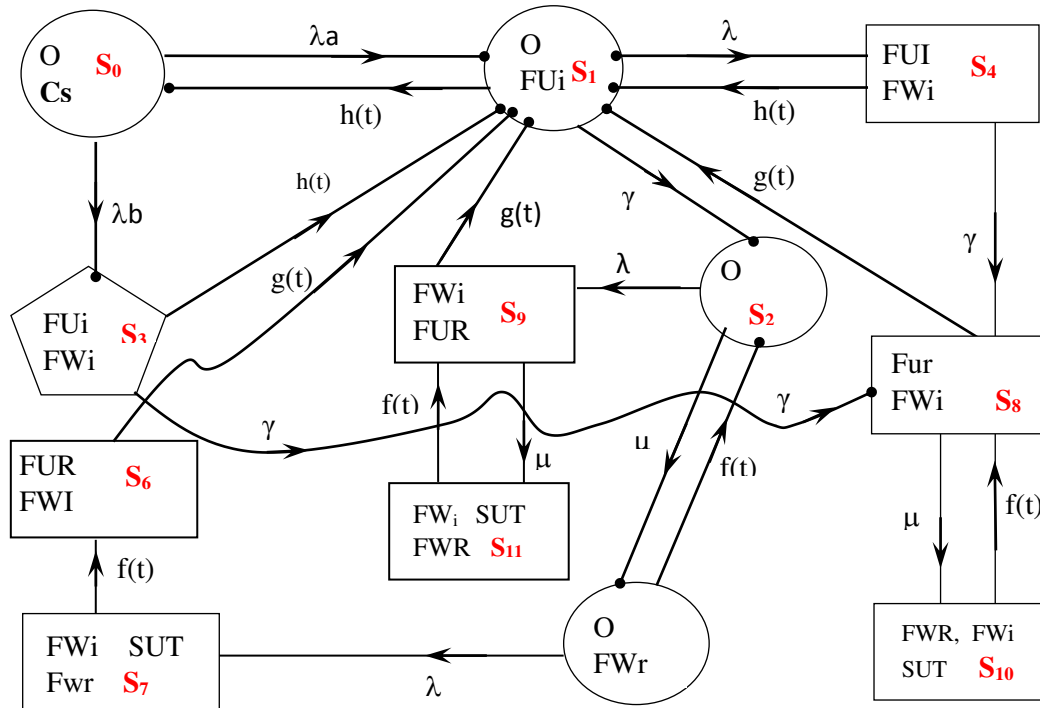


Fig.1: State Transition Diagram

**Transition Probabilities;**

Simple probabilities considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t) dt \tag{1}$$

$$p_{10} = a, p_{03} = b, p_{10} = \frac{\psi}{\psi + \lambda + \gamma}, p_{12} = \frac{\gamma}{\psi + \lambda + \gamma}, p_{14} = \frac{\phi}{\psi + \lambda + \gamma}, p_{25} = \frac{\mu}{\lambda + \mu}$$

$$p_{29} = \frac{\lambda}{\lambda + \mu}, p_{31} = p_{41} = \frac{\psi}{\psi + \gamma}, p_{38} = p_{48} = \frac{\gamma}{\gamma + \psi}, p_{52} = \frac{\theta}{\theta + \lambda}, p_{57} = \frac{\lambda}{\theta + \lambda}$$

$$p_{61} = p_{76} = p_{10,8} = p_{11,9} = 1, a + b = 1, p_{81} = p_{91} = \frac{\phi}{\phi + \mu}, p_{8,10} = p_{9,11} = \frac{\mu}{\mu + \phi}$$

$$p_{11,4(8,10)} = \frac{\lambda\gamma}{(\psi + \lambda + \gamma)(\psi + \gamma)}, p_{21,(9,11)} = \frac{\lambda\mu}{(\lambda + \mu)(\phi + \mu)}, p_{31,(8,10)} = \frac{\gamma\mu}{(\psi + \gamma)(\phi + \mu)} \tag{2}$$

for these transition probabilities, it can be verified that:

$$p_{01} + p_{03} = p_{10} + p_{12} + p_{14} = p_{25} + p_{29} = p_{31} + p_{38} = p_{41} + p_{48} = p_{52} + p_{57} = 1 \tag{3}$$

$$p_{31} + p_{31,8} + p_{31,(8,10)} = 1 = p_{10} + p_{12} + p_{11,4} + p_{11,4(8,10)} = p_{25} + p_{21,9} + p_{21,(9,11)} = p_{52} + p_{51(76)} = 1$$

**Mean Sojourn Time**

Let T denotes the time to system failure then the mean sojourn times (μ<sub>i</sub>) in the state S<sub>i</sub> are given by

$$\mu_i = E(t) = \int_0^{\infty} P(T > t) dt$$

Hence,  $\mu_0 = \frac{1}{\lambda}, \mu_1 = \frac{1}{\psi + \lambda + \gamma}, \mu_2 = \frac{1}{\lambda + \mu}, \mu_3 = \frac{1}{\psi + \gamma}, \mu_5 = \frac{1}{\theta + \lambda}$

$$\mu'_1 = \frac{[\{\gamma(\psi + \gamma) + \lambda\psi\}\theta\psi + \lambda\{(\lambda + 2\gamma)\theta\phi + \gamma(\phi + \mu)(\lambda + \gamma)\}]}{\theta\phi(\lambda + \gamma)(\psi + \gamma)(\psi + \lambda + \gamma)}, \mu'_2 = \frac{\theta\phi + \theta\lambda + \mu\lambda + \mu^2}{\theta\phi(\lambda + \mu)}$$

$$\mu'_3 = \frac{\theta\phi + \gamma(\theta + \mu)}{\theta\phi(\psi + \gamma)} \quad \text{and} \quad \mu'_5 = \frac{\theta\phi + \lambda(\theta + \phi)}{\theta\phi(\theta + \lambda)} \quad (4)$$

**Mean Time to System Failure**

Let  $\phi_i(t)$  be the c.d.f. of the passage time from regenerative state  $S_i$  to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for  $\phi_i(t)$

$$\begin{aligned} \phi_0(t) &= Q_{01}(t) \otimes \phi_1(t) + Q_{03}(t) \\ \phi_1(t) &= Q_{10}(t) \otimes \phi_0(t) + Q_{12}(t) \otimes \phi_2(t) + Q_{14}(t) \\ \phi_2(t) &= Q_{21}(t) \otimes \phi_5(t) + Q_{29}(t) \\ \phi_5(t) &= Q_{52}(t) \otimes \phi_2(t) + Q_{57}(t) \end{aligned} \quad (5)$$

Taking L.S.T. of relation (5) and solving for  $\phi_0^*(s)$

$$\text{MTSF} = \lim_{s \rightarrow 0} R^*(s) = \lim_{s \rightarrow 0} \frac{1 - \phi_0^*}{s} = \frac{(1 - p_{25}p_{52})(\mu_0 + p_{01}\mu_1) + p_{01}p_{12}(\mu_2 + p_{25}\mu_5)}{(1 - p_{25}p_{52})(1 - p_{01}p_{10})} \quad (6)$$

**Steady State Availability**

Let  $A_i(t)$  be the probability that the system is in up-state at instant t, given that the system entered the regenerative state  $S_i$  at t = 0. The recursive relations for  $A_i(t)$  are as follows:

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \otimes A_1(t) + q_{03}(t) \otimes A_3(t) \\ A_1(t) &= M_1(t) + q_{10}(t) \otimes A_0(t) + q_{12}(t) \otimes A_2(t) + q_{1,4}(t) \otimes A_1(t) + q_{1,4,8}(t) \otimes A_1(t) + q_{1,4,(8,10)} \otimes A_1(t) \\ A_2(t) &= M_2(t) + q_{25}(t) \otimes A_5(t) + q_{2,9}(t) \otimes A_1(t) + q_{2,9,(1,1)}(t) \otimes A_1(t) \\ A_3(t) &= q_{31}(t) \otimes A_1(t) + q_{3,8}(t) \otimes A_1(t) + q_{3,8,(10)}(t) \otimes A_1(t) \\ A_5(t) &= M_5(t) + q_{52}(t) \otimes A_2(t) + q_{5,76}(t) \otimes A_1(t) \end{aligned} \quad (7)$$

$M_i(t)$  is the probability that the system is up initially in state  $S_i \in E$  at time t without visiting to any other regenerative state where

$$M_0(t) = e^{-\lambda t}, \quad M_1(t) = e^{-(\lambda+\gamma)t} \overline{H(t)}, \quad M_2(t) = e^{-(\lambda+\mu)t} \quad \text{and} \quad M_5(t) = e^{-\lambda t} \overline{F(t)} \quad (8)$$

Taking Laplace transform of relation (7 & 8) and solving for  $A_0^*(s)$ , the steady state availability is given by

$$A_0 = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{(1 - p_{25}p_{52})(\mu_0 p_{10} + \mu_1) + \mu_2 p_{12} + \mu_5 p_{12} p_{25}}{(1 - p_{25}p_{52})(p_{10}\mu_0 + \mu'_1 - \mu'_3 p_{03} p_{10}) + \mu'_2 p_{12} + \mu'_5 p_{12} p_{25}} \quad (9)$$

**Busy period of the server due to inspection of the failed unit**

Let  $B_i^I(t)$  be the probability that the server is busy due to inspection of the failed unit at instant 't' given that the system entered the regenerative state  $S_i$  at t = 0. The recursive relations for  $B_i^I(t)$  are as follows :

$$\begin{aligned} B_0^I(t) &= q_{01}(t) \otimes B_1^I(t) + q_{03}(t) \otimes B_3^I(t) \\ B_1^I(t) &= W_1(t) + q_{10}(t) \otimes B_0^I(t) + q_{12}(t) \otimes B_2^I(t) + q_{1,4,8}(t) \otimes B_1^I(t) + q_{1,4,(8,10)}(t) \otimes B_1^I(t) \\ B_2^I(t) &= q_{25}(t) \otimes B_5^I(t) + q_{2,9}(t) \otimes B_1^I(t) + q_{2,9,(1,1)}(t) \otimes B_1^I(t) \\ B_3^I(t) &= W_3(t) + q_{31}(t) \otimes B_1^I(t) + q_{3,8}(t) \otimes B_1^I(t) + q_{3,8,(10)}(t) \otimes B_1^I(t) \\ B_5^I(t) &= q_{52}(t) \otimes B_2^I(t) + q_{5,76}(t) \otimes B_1^I(t) \end{aligned} \quad (10)$$

where  $W_i(t)$  is the probability that the server is busy in state  $S_i$  due to repairing of unit up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states so

$$W_1(t) = e^{-(\lambda+\gamma)t} \overline{H(t)} \otimes \lambda e^{-(\lambda+\gamma)t} \overline{H(t)} \otimes e^{-\lambda t} \overline{H(t)}, \quad W_3(t) = e^{-\lambda t} \overline{H(t)} \quad (11)$$

Taking L.T. of relation (10&11) and solving for  $B_0^{I*}(s)$  the time for which server is busy is given as

$$B_0^I = \lim_{s \rightarrow 0} sB_0^{I*}(s) = \frac{(W_1^*(0) + W_3^*(0)p_{03}p_{10})(1 - p_{25}p_{52})}{(1 - p_{25}p_{52})(p_{10}\mu_0 + \mu'_1 - \mu'_3 p_{03}p_{10}) + \mu'_2 p_{12} + \mu'_5 p_{12}p_{25}} \quad (12)$$

**Busy period of the server due to repair of the failed unit**

Let  $B_0^R(t)$  be the probability that the server is busy due to repair of the failed unit at instant 't' given that the system entered the regenerative state  $S_i$  at  $t = 0$ . The recursive relations for  $B_i(t)$  are as follows :

$$\begin{aligned} B_0^R(t) &= q_{01}(t) \odot B_1^R(t) + q_{03}(t) \odot B_3^R(t) \\ B_1^R(t) &= q_{10}(t) \odot B_0^R(t) + q_{12}(t) \odot B_2^R(t) + q_{1,4,8}(t) \odot B_1^R(t) + q_{1,4,(8,10)}(t) \odot B_1^R(t) \\ B_2^R(t) &= W_2(t) + q_{25}(t) \odot B_5^R(t) + q_{21,9}(t) \odot B_1^R(t) + q_{21,(9,11)}(t) \odot B_1^R(t) \\ B_3^R(t) &= q_{31}(t) \odot B_1^R(t) + q_{31,8}(t) \odot B_1^R(t) + q_{31,(8,10)}(t) \odot B_1^R(t) \\ B_5^R(t) &= W_5(t) + q_{52}(t) \odot B_2^R(t) + q_{51,76}(t) \odot B_1^R(t) \end{aligned} \quad (13)$$

where  $W_i(t)$  is the probability that the server is busy in state  $S_i$  due to repairing of unit up to time 't' without making any transition to any other regenerative state or before returning to the same via one or more non-regenerative states so

$$\begin{aligned} W_2(t) &= e^{-(\lambda+\mu)t} + \lambda e^{-(\lambda+\mu)t} \odot e^{-\mu t} \overline{G(t)} + \lambda e^{-(\lambda+\mu)t} \odot \mu e^{-\mu t} \overline{G(t)} \odot f(t) \odot e^{-\mu t} \overline{G(t)} + \lambda e^{-(\lambda+\mu)t} \odot \mu e^{-\mu t} \overline{G(t)} \odot f(t) \odot \\ &\mu e^{-\mu t} \overline{G(t)} \odot f(t) \odot e^{-\mu t} \overline{G(t)} + \dots \\ W_5(t) &= \lambda e^{-\lambda t} \overline{F(t)} \odot f(t) \odot \overline{G(t)} \end{aligned} \quad (14)$$

Taking L.T. of relation (13 & 14) and solving for  $B_0^*(s)$ , the time for which server is busy is given as

$$B_0^R = \lim_{s \rightarrow 0} sB_0^{R*}(s) = \frac{W_2^*(0)p_{12} + W_5^*(0)p_{25}p_{12}}{(1 - p_{25}p_{52})(p_{10}\mu_0 + \mu'_1 - \mu'_3 p_{03}p_{10}) + \mu'_2 p_{12} + \mu'_5 p_{12}p_{25}} \quad (15)$$

**Expected Number of visits by the server due to repair of the unit**

Let  $R_i(t)$  be the expected number of visits by the server in  $(0,t]$ , given that the system entered the regenerative state  $S_i$  at  $t=0$ . The recursive relations for  $R_i(t)$  are as follows:

$$\begin{aligned} R_0(t) &= Q_{01}(t) \odot (R_1(t) + 1) + Q_{03}(t) \odot (R_1(t) + 1) \\ R_1(t) &= Q_{10}(t) \odot R_0(t) + Q_{12}(t) \odot R_2(t) + Q_{1,4}(t) \odot R_1(t) + Q_{1,4,8}(t) \odot R_1(t) + Q_{1,4,(8,10)}(t) \odot R_1(t) \\ R_2(t) &= Q_{25}(t) \odot R_5(t) + Q_{21,9}(t) \odot R_1(t) + Q_{21,(9,11)}(t) \odot R_1(t) \\ R_3(t) &= Q_{31}(t) \odot R_1(t) + Q_{31,8}(t) \odot R_1(t) + Q_{31,(8,10)}(t) \odot R_1(t) \\ R_5(t) &= Q_{52}(t) \odot R_2(t) + Q_{51,76}(t) \odot R_1(t) \end{aligned} \quad (16)$$

Taking Laplace Stieltjes transform of the above relation and solving for  $R_0^*(s)$ . The expected numbers of visits by the server are given by

$$R_0 = \lim_{s \rightarrow 0} sR_0^*(s) = \frac{p_{10}(1 - p_{25}p_{52})}{(1 - p_{25}p_{52})(p_{10}\mu_0 + \mu'_1 - \mu'_3 p_{03}p_{10}) + \mu'_2 p_{12} + \mu'_5 p_{12}p_{25}} \quad (17)$$

**Expected Number of refreshments given to server**

Let  $T_i(t)$  be the expected number of treatments given to server in  $(0,t]$  such that the system entered the regenerative state at  $t = 0$ . The recursive relations for  $T_i(t)$  are as follows:

$$\begin{aligned} T_0(t) &= Q_{01}(t) \odot T_1(t) + Q_{03}(t) \odot T_3(t) \\ T_1(t) &= Q_{10}(t) \odot T_0(t) + Q_{12}(t) \odot T_2(t) + Q_{1,4}(t) \odot T_1(t) + Q_{1,4,8}(t) \odot T_1(t) + Q_{1,4,(8,10)}(t) [1 + T_1(t)] \\ T_2(t) &= Q_{25}(t) \odot [T_5(t) + 1] + Q_{21,9}(t) \odot T_1(t) + Q_{21,(9,11)}(t) \odot [T_1(t) + 1] \\ T_3(t) &= Q_{31}(t) \odot T_1(t) + Q_{31,8}(t) \odot T_1(t) + Q_{31,(8,10)}(t) \odot [T_1(t) + 1] \\ T_5(t) &= Q_{52}(t) \odot T_2(t) + Q_{51,76}(t) \odot T_1(t) \end{aligned} \quad (18)$$

Taking laplace stieljels transform of the above relation and solving for  $T_0^*(s)$ , we get

$$T_0 = \lim_{s \rightarrow 0} T_0^*(s) = \frac{P_{12}(1 - P_{21,9}) + (1 - P_{25}P_{52})(P_{11,4}(8,10) + P_{03}P_{10}P_{31,8}(10))}{(1 - P_{25}P_{52})(P_{10}\mu_0 + \mu'_1 - \mu'_3P_{03}P_{10}) + \mu'_2P_{12} + \mu'_5P_{12}P_{25}} \quad (19)$$

**Particular Case:**

Let us take  $g(t) = \theta e^{-\theta t}$ ,  $f(t) = \phi e^{-\phi t}$ ,  $h(t) = \psi e^{-\psi t}$

$$MTSF = \frac{\{\psi + (\lambda + \gamma)(1 + a)\}}{\lambda(\lambda + \gamma + b\psi)} \quad (20)$$

$$Availability (A_0) = \frac{(\theta + \lambda + \mu)(\psi + \lambda + \gamma)}{[(\theta + \lambda + \mu)\{\psi(1 - \lambda b\mu'_3) + \mu'_1\lambda(\psi + \lambda + \gamma)\} + \gamma(\theta + \lambda)\{\mu'_2(\mu + \lambda) + \mu\mu'_5\}]} \quad (21)$$

Busy period of the server due to inspection

$$B_0^I = \frac{\lambda(\theta + \lambda + \mu)\{(1 + b)\psi + \lambda + \gamma\}}{(\gamma + \psi)[(\theta + \lambda + \mu)\{\psi(1 - \lambda b\mu'_3) + \mu'_1\lambda(\psi + \lambda + \gamma)\} + \gamma(\theta + \lambda)\{\mu'_2(\mu + \lambda) + \mu\mu'_5\}] \quad (22)$$

Busy period of the server due to Repair

$$B_0^R = \frac{\gamma\{(\lambda + \theta)(\lambda + \phi) + \lambda\mu\}}{\phi[(\theta + \lambda + \mu)\{\psi(1 - \lambda b\mu'_3) + \mu'_1\lambda(\psi + \lambda + \gamma)\} + \gamma(\theta + \lambda)\{\mu'_2(\mu + \lambda) + \mu\mu'_5\}] \quad (23)$$

Expected Number of visits due to repair

$$R_0 = \frac{\lambda\psi(\lambda + \theta + \mu)}{[(\theta + \lambda + \mu)\{\psi(1 - \lambda b\mu'_3) + \mu'_1\lambda(\psi + \lambda + \gamma)\} + \gamma(\theta + \lambda)\{\mu'_2(\mu + \lambda) + \mu\mu'_5\}] \quad (24)$$

Expected Number of treatments given to Server

$$T_0 = \frac{\gamma[\lambda(\mu + \theta + \lambda)(\lambda + b\psi)(\phi + \mu) + \mu(\lambda + \phi + \mu)(\theta + \lambda)(\psi + \gamma)]}{(\phi + \mu)[(\theta + \lambda + \mu)\{\psi(1 - \lambda b\mu'_3) + \mu'_1\lambda(\psi + \lambda + \gamma)\} + \gamma(\theta + \lambda)\{\mu'_2(\mu + \lambda) + \mu\mu'_5\}] \quad (25)$$

**Cost -Benefit Analysis:**

The profit occurred in the system model in steady state can be calculated as

$$P_0 = K_0A_0 - K_1B_0^I - K_2B_0^R - K_3R_0 - K_4T_0 \quad (26)$$

where  $K_0=(5000)$  Revenue per unit up- time of the system.

$K_1=(600)$  Cost per unit time for which server is busy due to inspection

$K_2=(650)$  Cost per unit time for which server is busy due to repair

$K_3=(450)$  Cost per unit visits by the server

$K_4=(300)$  Cost per unit time treatment given to server.

**II. DISCUSSION**

In this study the effect of various parameters on performance measure of system model is envisioned. Table-1 reflects that the availability of the system increase when server failure rate  $\theta$  decrease. So we can improve the system availability by checking failure of the server. The third column of the table clearly shows that availability of the system again increase after making

increment in the replacement rate ' $\phi$ ' after inspection. Effect of increasing the rate ' $\phi$ ' (by which the unit goes for repair after inspection) clearly express that the availability of the system is also in increasing manner but slightly less than the previous values. At last, the availability of the system is increasing when  $\phi$  increasing from .1 to 1.0 with the possible change of the other parameters.

Table.1: (Availability Vs Repair rate)

$\theta$ ↓	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.35, \gamma=.40$	$\lambda=.55, \mu=.35, \phi=.65$ $\Psi=.35, \gamma=.40$	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.45, \gamma=.40$	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.35, \gamma=.50$
.1	0.184084	0.191171	0.208286	0.170436
.2	0.28995	0.295273	0.323445	0.272288
.3	0.358861	0.360882	0.396686	0.340171
.4	0.407366	0.406079	0.447456	0.388728
.5	0.443403	0.43914	0.484769	0.42523
.6	0.471256	0.464393	0.513376	0.453697
.7	0.493444	0.484324	0.536022	0.476535

.8	0.511546	0.500461	0.554406	0.495275
.9	0.526602	0.513799	0.569635	0.510936
1.0	0.539325	0.52501	0.58246	0.524223

Table.2: (Profit Vs Repair Rate)

$\theta$ ↓	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.35, \gamma=.40$	$\lambda=.55, \mu=.35, \phi=.65$ $\Psi=.35, \gamma=.40$	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.45, \gamma=.40$	$\lambda=.55, \mu=.48, \phi=.65$ $\Psi=.35, \gamma=.50$
.1	757.6293	773.9268	870.8326	703.215
.2	1191.149	1191.81	1348.865	1121.036
.3	1471.934	1453.244	1650.942	1397.946
.4	1668.634	1632.165	1859.108	1594.972
.5	1814.114	1762.277	2011.264	1742.34
.6	1926.083	1861.134	2127.333	1856.733
.7	2014.928	1938.778	2218.792	1948.111
.8	2087.146	2001.368	2292.717	2022.788
.9	2147.007	2052.89	2353.71	2084.963
1.0	2197.434	2096.038	2404.891	2137.534

Table-2 also reflects that profit is also increasing with increasing of repair rate  $\theta$  from .1 to 1.0. By comparing column one and column two of the table-2 it is found that profit of the function increasing whenever the server failure rate  $\mu$  declined from 0.48 to 0.35. It is observe that form column third the replacement rate  $\Psi$  after inspection increased from 0.35 to 0.45 then the profit of the system also rapidly increased. In the forth column clearly shows that when the rate ' $\gamma$ ' (by which unit goes for repair after inspection) increased from .40 to .50 the profit of the system less than the other cases but still in the trend of increasing.

### III. CONCLUSION

The idea of provide refreshment to the server which improves the efficiency of the server is more beneficial and economical for smooth functioning of the system.

### REFERENCES

- [1] Osaki, S. and Nakagawa T. [1971]; on a two-unit standby redundant system with standby failure, **Operation Research**, 19 (2) pp.510-523.
- [2] Nakagawa T. and Osaki S.[1975]; stochastic behavior of a two-unit priority standby redundant system with repair, **Microelectronics Reliability**, 14(3) pp.309-313.
- [3] Subramanian R. Venkatakrisnan K. S. and Kistner K.P. [1976]; reliability of a repairable system with standby failure, **Operation Research**, 24(1) pp. 169-176.
- [4] Gopalan M.N. and Naidu R.S.[1982]; cost-benefit analysis of a one-server system subject to inspection, **Microelectronics Reliability**, 22 pp. 699-705.
- [5] Gopalan M.N. and Nagarwalla H.E.[1985]; cost-benefit analysis of a one server two unit cold standby system with repair and age replacement, **Microelectronics Reliability**, 25 pp.977-990.
- [6] Singh S.K. and Srinivas B. [1987]; stochastic analysis of a two unit cold standby system with preparation time for repair, **Microelectronics Reliability**, 27(1) pp. 55-60.
- [7] Singh S.K.[1989]; profit evaluation of a two-unit cold standby system with random appearance and disappearance time of the service facility, **Microelectronics Reliability**, 29 pp. 705-709.
- [8] Dhillon B.S.[1992]; reliability and availability analysis of a System with standby and common cause failures, **Microelectronics Reliability**, 33(9) pp. 1343-1349.
- [9] Lam Y.F.[1997]; a maintenance model for two-unit redundant system, **Microelectronics Reliability**, 37(3) pp. 497-504.
- [10] Kumar, Ramesh [2005]; analysis of reliability models with different types of failure and repair Policies, Ph.D. Thesis, M.D. University Rohtak.
- [11] Malik S.C. and Barak M.S.[2007]; probabilistic analysis of a single-server system operating under different weather conditions, **Journal of Mathematical Analysis and Approximation Theory**, 2(2) pp. 173-182.
- [12] Malik S.C.[2009]; reliability modeling and cost-benefit analysis of a system – A case study, **Journal of Mathematics and System Sciences**, 5(2) pp. 26-38.
- [13] Bhardwaj R.K. and Singh R. [2014]; semi-Markov approach for asymptotic performance analysis of a

standby system with server failure, **International journal of computer Applications**, 98(3) pp.9-14.

[14] Malik,S.C., Bhardwaj R.K. and Komaldeep Kaur [2015]; performance analysis of a stochastic system with standby failure and maintenance, **International journal of computer Applications**, 125(1) pp. 6-11.

[15] Barak M.S. and Dhiraj [2016]; stochastic analysis of a cold standby system with server failure. **International Journal of Mathematics and Statistics Invention(IJMSI)** ([www.ijmsi.org](http://www.ijmsi.org) ISSN 2321-4767) 4, Issue 6 ||August 2016|| pp. 18-22.