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On *r*-Dynamic Chromatic Number of the Corronation of Path and Several Graphs

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Abstract—*This study is a natural extension of k-proper coloring of any simple and connected graph G. By an r-dynamic coloring of a graph G, we mean a proper k-coloring of graph G such that the neighbors of any vertex v receive at least min{r, d(v)} different colors. The r-dynamic chromatic number, written as $\chi_r(G)$, is the minimum k such that graph G has an r-dynamic k-coloring. In this paper we will study the r-dynamic chromatic number of the coronation of path and several graph. We denote the corona product of G and H by $G \odot H$. We will obtain the r-dynamic chromatic number of $\chi_r(P_n \odot P_m)$, $\chi_r(P_n \odot C_m)$and $\chi_r(P_n \odot W_m)$ for m, n≥3.*

Keyword— r-dynamic chromatic number, path, corona product.

I. INTRODUCTION

An *r*-dynamic coloring of a graph *G* is a proper *k*-coloring of graph *G* such that the neighbors of any vertex *v* receive at least min{*r*, *d*(*v*)} different colors. The *r*-dynamic chromatic number, introducedby Montgomery [4] written as $\chi_r(G)$, is the minimum *k* such that graph *G* has an *r*-dynamic *k*-coloring. The *1*-dynamic chromatic number of a graph G is $\chi_1(G) = \chi(G)$, well-known as the ordinary chromatic number of *G*. The *2*-dynamic chromatic number is simply said to be a dynamic chromatic number, denoted by$\chi_2(G)$= $\chi_d(G)$,see Montgomery [4]. The *r*-dynamic chromatic number has been studied by several authors, for instance in[1], [5], [6], [7], [8], [10], [11].

The following observations are useful for our study, proposed by Jahanbekam[11].

Observation 1.[10] Always $\chi(G) = \chi_1(G) \leq \cdots \leq \chi_{\Delta(G)}(G)$. If $r \geq \Delta(G)$, then $\chi_r(G) = \chi_{\Delta(G)}(G)$

Observation 2. Let$\Delta(G)$ be the largest degree of graph *G*. It holds$\chi_r(G) \geq \min\{\Delta(G), r\} + 1$.

Given two simple graphs *G* and *H*, the corona product of *G* and *H*, denoted by $G \odot H$, is a connected graph obtained by taking a number of vertices |*V*(*G*)| copy of *H*, and making the i^{th} of *V(G)*adjacent to every vertex of the i^{th} copy of *V(H)*, Furmanczyk[3]. The following example is $P_3 \odot C_3$.

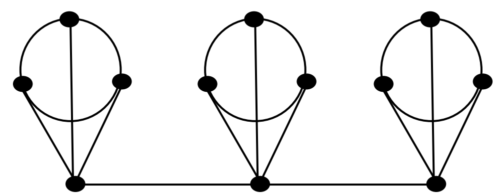

Fig.1: Graph $P_3 \odot C_3$

There have been many results already found, The first one was showed by Akbari et.al [10].They found that for every two natural number *m* and *n*, *m*, *n*≥ 2, the cartesian product of $P_m$and $P_n$is $\chi_2(P_m \square P_n) = 4$ and if 3|*mn*, then$\chi_2(C_m \square C_n) = 3$ and $\chi_2(C_m \square C_n) = 4$. In [2], they then conjectured $\chi_2(G) \leq \chi(G)+2$ when *G* is regular, which remains open. Akbari et.al. [9] alsoproved Montgomery's conjecture for bipartite regular graphs, as well as Lai, et.al. [5] provedthat $\chi_2(G) \leq \Delta(G) + 1$ for $\Delta(G) \geq 4$ when no component is the 5-cycle. By a greedy coloring algorithm, Jahanbekama [11] proved that $\chi_r(G) \leq r\Delta(G) + 1$, and equality holds for $\Delta(G) > 2$ if and only if *G* is *r*-regular with diameter 2 and girth 5. They improved the bound to $\chi_r(G) \leq \Delta(G) + 2r - 2$ when $\delta(G) > 2r \ln n$ and $\chi_r(G) \leq \Delta(G) + r$ when $\delta(G) > r^2 \ln n$.

II. THE RESULTS

We are ready to show our main theorems. There are three theorems found in this study. Those deal with corona product of graph P_n with P_m, C_m, and W_m.

Theorem 1. *Let $G = P_n \odot P_m$ be a corona graph of P_n and P_m. For n, m ≥ 2, the r-dynamic chromatic number is:*

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$$\chi_r(G) = \begin{cases} 3 & , \quad r = 1,2 \\ r+1 & , \quad 3 \le r \le \Delta - 1 \\ m+3 & , \quad r \ge \Delta \end{cases}$$

Proof. The graph $P_n \odot P_m$ is a connected graph with vertex set $V(P_n \odot P_m) = \{y_i, 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n, 1 \le j \le m\}$ and edge set $E(P_n \odot P_m) = \{y_i y_{(i+1)}; 1 \le i \le n-1\} \cup \{y_i x_{ij}; 1 \le i \le n, 1 \le j \le m\} \cup \{x_{ij}, x_{i(j+1)}; 1 \le i \le n, 1 \le j \le m-1\}$. The order of graph $P_n \odot P_m$ is $|V(P_n \odot P_m)| = n(m+1)$ and the size of graph $P_n \odot P_m$ is $|E(P_n \odot P_m)| = 2mn - 1$. Thus, $\Delta(P_n \odot P_m) = m+2$.

By observation 2, $\chi_r(P_n \odot P_m) \ge \min\{r, \Delta(P_n \odot P_m)\} + 1 = \min\{r, m+2\} + 1$. To find the exact value of r-dynamic chromatic number of $P_n \odot P_m$, we define two cases, namely for $\chi_{r=1,2}(P_n \odot P_m)$ and $\chi_r(P_n \odot P_m)$.

Case 1. For $\chi_{r=1,2}(P_n \odot P_m)$, define $c_1 : V(P_n \odot P_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 3$, $m \ge 3$, by the following:

$$c_1(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \le i \le n \\ 2 & , \quad i \text{ even}, 1 \le i \le n \end{cases}$$

$$c_1(x_{ij}) = \begin{cases} 1 & , \quad i \text{ even}, j \text{ odd}, 1 \le i \le n, 1 \le j \le m \\ 2 & , \quad i \text{ odd}, j \text{ odd}, 1 \le i \le n, 1 \le j \le m \\ 3 & , \quad j \text{ even}, 1 \le i \le n, 1 \le j \le m \end{cases}$$

It easy to see that c_1 is map c_1: $V(P_n \odot P_m) \rightarrow \{1, 2, 3\}$, thus it gives $\chi_{r=1,2}(P_n \odot P_m) = 3$.

Case 2.

Subcase 2.1 For $\chi_r(P_n \odot P_m)$, $3 \le r \le \Delta - 1$, define c_2: $V(P_n \odot P_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 3$, $m \ge 3$, by the following:

$$c_2(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \le i \le n \\ 2 & , \quad i \text{ even}, 1 \le i \le n \end{cases}$$

$$c_2(x_{11}, x_{12}, x_{13}) = 2, 3, 4, \quad \text{for } m = 3, r = 3$$

$$c_2(x_{21}, x_{22}, x_{23}) = 1, 3, 4, \quad \text{for } m = 3, r = 3$$

$$c_2(x_{11}, x_{12}, x_{13}) = 3, 4, 5, \quad \text{for } m = 3, r = 4$$

$$c_2(x_{11}, x_{12}, x_{13}, x_{14}) = 2, 3, 4, 5, \quad \text{for } m = 4, r = 4$$

$$c_2(x_{11}, x_{12}, x_{13}, x_{14}) = 3, 4, 5, 6, \quad \text{for } m = 4, r = 5$$

It easy to see that c_2 is a map c_2: $V(P_n \odot P_m) \rightarrow \{1, 2, \ldots, r+1\}$, thus it gives $\chi_r(P_n \odot P_m) = r+1, 3 \le r \le \Delta - 1$

Subcase 2.2 The last for $\chi_r(P_n \odot P_m), r \ge \Delta$, define c_3: $V(P_n \odot P_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 3$, $m \ge 3$, by the following:

$$c_3(y_i) = \begin{cases} 1 & , \quad i = 3t+1, t \ge 0, 1 \le i \le n \\ 2 & , \quad i = 3t+2, t \ge 0, 1 \le i \le n \\ 3 & , \quad i = 3t, t \ge 1, 1 \le i \le n \end{cases}$$

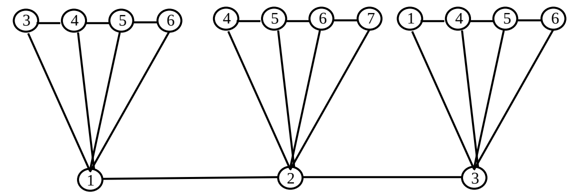

Fig.2: $\chi_6(P_3 \odot P_4) = 7$ *with n= 3, m= 4, r= 6*

$$c_3(x_{11}, x_{12}, x_{13}) = 4, 5, 6, \text{ for } m = 3, r = 5$$

$$c_3(x_{11}, x_{12}, x_{13}, x_{14}) = 3, 4, 5, 6, \quad \text{for } m = 4, r = 6$$

$$c_3(x_{21}, x_{22}, x_{23}, x_{24}) = 4, 5, 6, 7 \quad \text{for } m = 4, r = 6$$

$$c_3(x_{21}, x_{22}, x_{23}, x_{24}, x_{25}) = 4, 5, 6, 7, 8 \quad \text{for } m = 5, r = 7$$

It easy to see that c_3 is a map c_3: $V(P_n \odot P_m) \rightarrow \{1, 2, \ldots, m+3\}$, so it gives $\chi_r(P_n \odot P_m) = m+3, r \ge \Delta$. It concludes the proof

Theorem 2. Let $G = P_n \odot C_m$ be a corona graph of P_n and C_m. For $n \ge 3$, $m \ge 3$, the r-dynamic chromatic number is:

$$\chi_{r=1,2}(G) = \begin{cases} 3 & , \quad m \text{ even or } m = 3k, k \ge 1 \\ 4 & , \quad m \text{ odd or } m = 5 \end{cases}$$

$$\chi_{r=3}(G) = \begin{cases} 4 & , \quad m = 3k, k \ge 1 \\ 6 & , \quad m = 5 \\ 5 & , \quad m \text{ otherwise} \end{cases}$$

$$\chi_r(G) = \begin{cases} r+1 & , \quad 4 \le r \le \Delta - 1 \\ m+3 & , \quad r \ge \Delta \end{cases}$$

Proof. The graph $P_n \odot C_m$ is connected graph with vertex set $V(P_n \odot C_m) = \{y_i; 1 \le i \le n\} \cup \{x_{ij}; 1 \le i \le n, 1 \le j \le m\}$ and edge set $E(P_n \odot C_m) = \{y_i y_{i+1}; 1 \le i \le n-1\} \cup \{x_{ij} x_{i(j+1)}; 1 \le i \le n, 1 \le j \le m-1\} \cup \{x_{i1} x_{im}; 1 \le i \le n\} \cup \{y_i x_{ij}; 1 \le i \le n, 1 \le j \le m\}$. The order of graph $P_n \odot C_m$ is $|V(P_n \odot C_m)| = n(m+1)$ and the size of graph

$P_n \odot C_m$ is $|E(P_n \odot C_m)| = 2mn + n - 1$, thus $\Delta(P_n \odot C_m) = m + 2$. By Observation 2, we have $\chi_r(P_n \odot C_m) \ge \min\{r, \Delta(P_n \odot C_m)\} + 1 = \min\{r, m+2\} + 1$. To find the exact value of r-dynamic chromatic

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number of $P_n \odot C_m$, we define three case, namely for $\chi_{r=1,2}(P_n \odot C_m)$, $\chi_{r=3}(P_n \odot C_m)$ and $\chi_r(P_n \odot C_m)$.

Case 1.

Subcase 1.1 For $\chi_{r=1,2}(P_n \odot C_m)$, define c_4 : $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, m even or $m = 3k, k \geq 1$, by the following:

$$c_4(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_4(x_{ij}) = \begin{cases} 1 & , \ i \text{ even}, j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m-1 \\ 2 & , \ i \text{ odd}, j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m \\ 3 & , \ j \text{ even}, 1 \leq i \leq n, 1 \leq j \leq m \\ 4 & , \ i \text{ even}, 1 \leq i \leq n, j = m \end{cases}$$

It easy to see that c_4 is a map c_4: $V(P_n \odot C_m) \rightarrow \{1, 2, 3\}$, so it gives $\chi_{r=1,2}(P_n \odot C_m) = 3, m$ even or $m = 3k, k \geq 1$

Subcase 1.2 For $\chi_{r=1,2}(P_n \odot C_m)$define c_5: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, , m odd or $m = 5$,, by the following:

$$c_5(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_5(x_{ij}) = \begin{cases} 1 & , \ i \text{ even}, j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m-1 \\ 2 & , \ i \text{ odd}, j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m-1 \\ 3 & , \ j \text{ even}, 1 \leq i \leq n, 1 \leq j \leq m-1 \\ 4 & , \ 1 \leq i \leq n, j = m \end{cases}$$

It easy to see that c_5 is a map c_5: $V(P_n \odot C_m) \rightarrow \{1, 2, 3, 4\}$, so it gives $\chi_{r=1,2}(P_n \odot C_m) = 4, m$ odd or $m = 5$

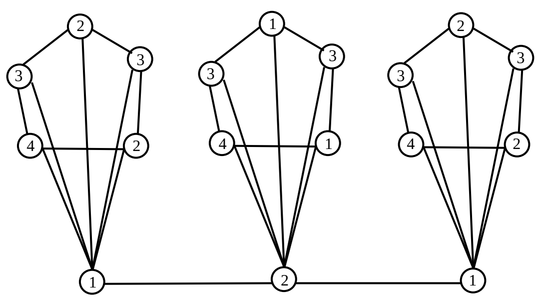

Fig.3: $\chi_2(P_3 \odot C_5) = 4$ *with* $n = 3, m = 5, r = 2$

Case 2.

Subcase 2.1 For $\chi_{r=3}(P_n \odot C_m)$, define c_6: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m = 3k, k \geq 1$, by the following:

$$c_6(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_6(x_{ij}) = \begin{cases} 1 & , \ i \text{ even}, j = 3t+1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 2 & , \ i \text{ odd}, j = 3t+1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 3 & , \ j = 3t+2, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 4 & , \ j = 3t, t \geq 1, 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

It easy to see that c_6 is map c_6: $V(P_n \odot C_m) \rightarrow \{1, 2, 3, 4\}$, so it gives $\chi_{r=3}(P_n \odot C_m) = 4, m = 3k, k \geq 1$.

Subcase 2.2 For $\chi_{r=3}(P_n \odot C_m)$, define c_7: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m = 5$, by the following:

$$c_7(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_7(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = 2, 3, 4, 5, 6$$

$$c_7(x_{21}, x_{22}, x_{23}, x_{24}, x_{25}) = 1, 3, 4, 5, 6$$

It easy to see that c_7 is a map c_7: $V(P_n \odot C_m) \rightarrow \{1, 2, 3, 4, 5, 6\}$.Thus it given $\chi_{r=3}(P_n \odot C_5) = 6$

Subcase 2.3 For $\chi_{r=3}(P_n \odot C_m)$, define c_8: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, m otherwise, by the following:

$$c_8(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_8(x_{ij}) = \begin{cases} 1 & , \ i \text{ even}, j = 4t+1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 2 & , \ i \text{ odd}, j = 4t+1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 3 & , \ j = 4t+2, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 4 & , \ j = 4t+3, t \geq 1, 1 \leq i \leq n, 1 \leq j \leq m \\ 5 & , \ j = 4t, t \geq 1, 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

It easy to see that c_8 is map c_8: $V(P_n \odot C_m) \rightarrow \{1, 2, 3, 4, 5\}$, so it gives $\chi_{r=3}(P_n \odot C_m) = 5$

Case 3.

Subcase 3.1 For $\chi_r(P_n \odot C_m), 4 \leq r \leq \Delta - 1$, define c_9: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m \geq 3$, by the following:

$$c_9(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_9(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = 3, 4, 5, 3, 4, 5, \text{ for } m = 6, r = 4$$

$$c_9(x_{31}, x_{32}, x_{33}, x_{34}, x_{35}, x_{36}) = 3, 4, 5, 3, 4, 5, \text{ for } m = 6, r = 4$$

$$c_9(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = 3, 4, 5, 6, 3, 5, \text{ for } m = 6, r = 5$$

$$c_9(x_{11}, x_{12}, x_{14}, x_{15}, x_{16}) = 3, 4, 5, 6, 7, 3, \text{ for } m = 6, r = 6$$

$$c_9(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = 3, 4, 5, 6, 7, 8, \text{ for } m = 6, r = 7$$

It easy to see that c_9 is a map c_9: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, r+1\}$, so it gives $\chi_r(P_n \odot C_m) = r + 1, 4 \leq r \leq \Delta - 1$

Subcase 3.2 The last for $\chi_r(P_n \odot C_m), r \geq \Delta$, define c_{10}: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m \geq 3$, by the following:

$$c_{10}(y_i) = \begin{cases} 1 & , \ i \text{ odd}, 1 \leq i \leq n \\ 2 & , \ i \text{ even}, 1 \leq i \leq n \end{cases}$$

$$c_{10}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = 4, 5, 6, 7, 8, 9$$

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$$\text{for } m = 6, r = 8$$
$$c_{10}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}) = 4, 5, 6, 7, 8, 9, 10$$
$$\text{for } m = 7, r = 9$$
$$c_{10}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}) = 4, 5, 6, 7, 8, 9, 10, 11$$
$$\text{for } m = 8, r = 10$$

It easy to see that c_{10} is map c_{10}: $V(P_n \odot C_m) \rightarrow \{1, 2, \ldots, m+3\}$, so it given $\chi_r(P_n \odot C_5) = m + 3, r \geq \Delta$.It concludes the proof.

Theorem 3. Let $G = P_n \odot W_m$ be a corona graph of P_n and W_m. For $n \geq 3$, $m \geq 3$, the r-dynamic chromatic number is:

$$\chi_{r=1,2,3}(G) = \begin{cases} 4 & , \quad m \text{ even} \\ 5 & , \quad m \text{ odd} \end{cases}$$

$$\chi_{r=4}(G) = \begin{cases} 5 & , \quad m = 3k, k \geq 1 \\ 7 & , \quad m = 5 \\ 6 & , \quad m \text{ otherwise} \end{cases}$$

$$\chi_r(G) = \begin{cases} r + 1 & , \quad 5 \leq r \leq \Delta - 1 \\ m + 4 & , \quad r \geq \Delta \end{cases}$$

Proof. The graph $P_n \odot W_m$ is a connected graph with vertex set $V(P_n \odot W_m) = \{y_i; 1 \leq i \leq n\} \cup \{x_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{A_i;\ 1 \leq i \leq n\}$ and edge set $E(P_n \odot W_m) = \{y_i y_{i+1}; 1 \leq i \leq n - 1\} \cup \{x_{ij} x_{i(j+1)}; 1 \leq i \leq n, 1 \leq j \leq m - 1\} \cup \{x_{i1} x_{im}; 1 \leq i \leq n\} \cup \{y_i x_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{A_i x_{ij}; 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{A_i y_i; 1 \leq i \leq n\}$.
The order of graph $P_n \odot W_m$ is $|V(P_n \odot W_m)| = mn + 2n)$ and the size of graph $P_n \odot W_m$ is $|E(P_n \odot W_m)| = 3mn + 2n - 1$, thus$\Delta(P_n \odot W_m) = m + 3$.
By observation 2, we have the following $\chi_r(P_n \odot W_m) \geq \min\{r, \Delta(P_n \odot W_m)\} + 1 = \min\{r, m + 3\} + 1$. To find the exact value of r-dynamic chromatic number of $P_n \odot W_m$, we define three case, namely for $\chi_{r=1,2,3}(P_n \odot W_m)$, $\chi_{r=4}(P_n \odot W_m)$ and $\chi_r(P_n \odot W_m)$.

Case 1

Subcase 1.1 For $\chi_{r=1,2,3}(P_n \odot W_m)$, define c_{11} : $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, m even by the following:

$$c_{11}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \leq i \leq n \\ 2 & , \quad i \text{ even}, 1 \leq i \leq n \end{cases}$$
$$c_{11}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \leq i \leq n \\ 2 & , \quad i \text{ odd}, 1 \leq i \leq n \end{cases}$$
$$c_{11}(x_{ij}) = \begin{cases} 3 & , \quad j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m \\ 4 & , \quad j \text{ even}, 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

It easy to see that c_{11} is map c_{11}: $V(P_n \odot W_m) \rightarrow \{1, 2, 3, 4\}$, so it gives$\chi_{r=1,2,3}(P_n \odot W_m) = 4, m$ even .

Subcase 1.2 For $\chi_{r=1,2,3}(P_n \odot W_m)$, define c_{12} : $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, m odd by the following:

$$c_{12}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \leq i \leq n \\ 2 & , \quad i \text{ even}, 1 \leq i \leq n \end{cases}$$
$$c_{12}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \leq i \leq n \\ 2 & , \quad i \text{ odd}, 1 \leq i \leq n \end{cases}$$
$$c_{12}(x_{ij}) = \begin{cases} 3 & , \quad j \text{ odd}, 1 \leq i \leq n, 1 \leq j \leq m - 1 \\ 4 & , \quad j \text{ even}, 1 \leq i \leq n, 1 \leq j \leq m - 1 \\ 5 & , \quad j = m, 1 \leq i \leq n \end{cases}$$

It easy to see that c_{12} is a map c_{12}: $V(P_n \odot W_m) \rightarrow \{1, 2, 3, 4, 5\}$, so it gives$\chi_{r=1,2,3}(P_n \odot W_m) = 5, m$ even.

Case 2

Subcase 2.1 For $\chi_{r=4}(P_n \odot W_m)$, define c_{13}: $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m = 3k, k \geq 1$ by the following:

$$c_{13}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \leq i \leq n \\ 2 & , \quad i \text{ even}, 1 \leq i \leq n \end{cases}$$
$$c_{13}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \leq i \leq n \\ 2 & , \quad i \text{ odd}, 1 \leq i \leq n \end{cases}$$
$$c_{13}(x_{ij}) = \begin{cases} 3 & , \quad j = 3t + 1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 4 & , \quad j = 3t + 2, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m \\ 5 & , \quad j = 3t, t \geq 1, 1 \leq i \leq n, 1 \leq j \leq m \end{cases}$$

It easy to see that c_{13} is a map c_{13}: $V(P_n \odot W_m) \rightarrow \{1, 2, 3, 4, 5\}$, so it given $\chi_{r=4}(P_n \odot W_m) = 5, m = 3k, k \geq 1$.

Subcase 2.2 For $\chi_{r=4}(P_n \odot W_m)$, define c_{14}: $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, $m = 5$ by the following:

$$c_{14}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \leq i \leq n \\ 2 & , \quad i \text{ even}, 1 \leq i \leq n \end{cases}$$
$$c_{14}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \leq i \leq n \\ 2 & , \quad i \text{ odd}, 1 \leq i \leq n \end{cases}$$
$$c_{14}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = 3, 4, 5, 6, 7$$

It easy to see that c_{14} is a map c_{14}: $V(P_n \odot W_m) \rightarrow \{1, 2, 3, 4, 5, 6, 7\}$, so it gives $\chi_{r=4}(P_n \odot W_m) = 7, m = 5$.

Subcase 2.3 For $\chi_{r=4}(P_n \odot W_m)$, define c_{15}: $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \geq 3$, m otherwise by the following:

$$c_{15}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \leq i \leq n \\ 2 & , \quad i \text{ even}, 1 \leq i \leq n \end{cases}$$
$$c_{15}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \leq i \leq n \\ 2 & , \quad i \text{ odd}, 1 \leq i \leq n \end{cases}$$
$$c_{15}(x_{ij}) = \begin{cases} 3 & , \quad j = 3t + 1, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m - 1 \\ 4 & , \quad j = 3t + 2, t \geq 0, 1 \leq i \leq n, 1 \leq j \leq m - 1 \\ 5 & , \quad j = 3t, t \geq 1, 1 \leq i \leq n, 1 \leq j \leq m - 1 \\ 6 & , \quad j = m, 1 \leq i \leq n \end{cases}$$

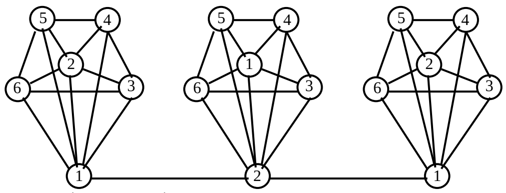

Fig.4: $\chi_4(P_3 \odot W_4) = 6$ with $n = 3$, $m = 4$, $r = 6$

It easy to see that c_{15} is map c_{15}: $V(P_n \odot W_m) \rightarrow \{1, 2, 3, 4, 5, 6\}$, so it gives $\chi_{r=4}(P_n \odot W_m) = 6, m$ otherwise.

Case 3.

Subcase 3.1 For $\chi_r(P_n \odot W_m)$ $5 \le r \le \Delta - 1$, define c_{16} :$V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 3$, $m \ge 3$ by the following:

$$c_{16}(y_i) = \begin{cases} 1 & , \quad i \text{ odd}, 1 \le i \le n \\ 2 & , \quad i \text{ even}, 1 \le i \le n \end{cases}$$

$$c_{16}(A_i) = \begin{cases} 1 & , \quad i \text{ even}, 1 \le i \le n \\ 2 & , \quad i \text{ odd}, 1 \le i \le n \end{cases}$$

$$c_{16}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}) = 3, 4, 5, 3, 4, 5, 6, \text{ for } m = 7, r = 5$$

$$c_{16}(x_{11}, x_{12}, x_{13} x_{14}, x_{15}, x_{16}, x_{17}) = 3, 4, 5, 6, 7, 4, 5, \text{ for } m = 7, r = 6$$

$$c_{16}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}) = 3, 4, 5, 6, 7, 8, 5, \text{ for } m = 7, r = 7$$

$$c_{16}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}) = 3, 4, 5, 6, 7, 8, 9, \text{ for } m = 7, r = 8$$

It easy to see that c_{16} is a map c_{16}: $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, r+1\}$, so it gives $\chi_r(P_n \odot W_m) = r + 1, 5 \le r \le \Delta - 1$.

Subcase 3.2 For $\chi_r(P_n \odot W_m), r \ge \Delta$, define c_{17} :$V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, k\}$ where $n \ge 3$, $m \ge 3$ by the following:

$$c_{17}(y_i) = \begin{cases} 1 & , \quad i = 3t + 1, t \ge 0, 1 \le i \le n \\ 2 & , \quad i = 3t + 2, t \ge 0, 1 \le i \le n \\ 3 & , \quad i = 3t, t \ge 1, 1 \le i \le n \end{cases}$$

$$c_{17}(A_i) = \begin{cases} 1 & , \quad i = 4t + 3, t \ge 0, 1 \le i \le n \\ 2 & , \quad i = 4t, t \ge 1, 1 \le i \le n \\ 3 & , \quad i = 4t + 1, t \ge 0, 1 \le i \le n \\ 4 & , \quad i = 4t + 2, t \ge 0, 1 \le i \le n \end{cases}$$

$$c_{17}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = 4, 5, 6, 7, 8, 9, \text{ for } m = 6, r = 9$$

$$c_{17}(x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}) = 5, 6, 7, 8, 9, 10, \text{ for } m = 6, r = 9$$

$$c_{17}(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}) = 4, 5, 6, 7, 8, \text{ for } m = 5, r = 8$$

$$c_{17}(x_{21}, x_{22}, x_{23}, x_{24}, x_{25}) = 5, 6, 7, 8, 9, \text{ for } m = 5, r = 8$$

$$c_{17}(x_{11}, x_{12}, x_{13}, x_{14}) = 4, 5, 6, 7, \text{ for } m = 4, r = 7$$

$$c_{17}(x_{21}, x_{22}, x_{23}, x_{24}) = 5, 6, 7, 8, \text{ for } m = 4, r = 7$$

It easy to see that c_{17} is map c_{17}: $V(P_n \odot W_m) \rightarrow \{1, 2, \ldots, m+4\}$, so it gives $\chi_r(P_n \odot W_m) = m + 4, r \ge \Delta$.

It concludes the proof.

III. CONCLUSION

We have found some r-dynamic chromatic number of corona product of graphs, namely $\chi_r(P_n \odot P_m) = \chi_r(P_n \odot C_m) = \chi_r(P_n \odot W_m) = r + 1$, for $4 \le r \le \Delta - 1$. and $\chi_r(P_n \odot P_m) = \chi_r(P_n \odot C_m) = m + 3$, for $r \ge \Delta$. All numbers attaina best lower bound. For the characterization of the lower bound of $\chi_r(G \odot H)$ for any connected graphs G and H, we have not found any result yet, thus we propose the following open problem.

Open Problem 1. Given that any connected graphs G and H. Determine the sharp lower bound of $\chi_r(G \odot H)$.

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