A Unique Approach for Planar Parallel Robotic Arms

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Abstract— *In the design of mechanism, a decision must first be taken regarding the type of mechanism to be employed. The number of links and connections required to give the desired degree of freedom must then be determined. Finally, the required dimensions needed to bring about a particular motion must be deduced. In the present study the main focus is to select a mechanism for parallel robotic arms. However there are a number of mechanisms available which can be used as robot hands. In the selection of mechanism for robotic hands, rigidity and grasping power are the main important considerations. In the present paper, a unique numerical method is used to measure the parallelism between the object and the ground link. This can be used to compare the robotic hands for rigidity and grasp.*

Keywords— Grasping capacity, Mechanism, Parallelism, Rigidity, Robotic arms

I. INTRODUCTION

When greater rigidity is required then closed kinematic chains with multi degree of freedom are mostly used as grasping power and rigidity are the most important parameters that should be considered for planar parallel robot hands [1-8]. Number of fingers and parallelism decides grasp capacity. Parallelism between ground and object also affects grasping capacity. Greater parallelism means greater grasp and rigidity.

A systematic study of robots and manipulators [1], now concentrating on "in-parallel" actuator-arrangements, reveals much geometry applicable either to entire robot-arms or to parts of otherwise series-actuated arms.

The study [2] presents solutions to the forward position and velocity problems of a planar eight-bar, three degree-of-freedom, and closed-loop linkage. The linkage is proposed as a programmable platform-type robot which can both position and orient the platform. A sixth-order polynomial equation in the angular displacement of the platform is derived which indicates that six configurations, for a given set of input angular displacements, are possible. The polynomial equation is important in the study of the limit positions of the linkage. The forward velocity problem is solved using first-order partial derivatives of the four output angular displacements with respect to the three independent input

displacements. The partial derivatives provide geometric insight into the kinematic analysis of the linkage. A graphical method, which utilizes the instantaneous centers of zero velocity, is introduced as a check of the velocity analysis. The method is solely a function of the configuration of the linkage and is, therefore, a practical alternative to other methods.

The forward displacement analysis (FDA) in closed form of two classes of new parallel mechanisms derived from the Stewart Platform Mechanism (SPM) is presented in the study [3]. These mechanisms, when a set of actuator displacements is given, become multi-loop structures of type PRR-3S and PPR-3S, with P, R and S for prismatic, revolute and spherical pairs, whereas the SPM has the structure RRR-3S. Solving the FDA in closed form means finding all the possible positions and orientations of the output controlled link when a set of actuator displacements is given, or equivalently, finding all possible closures of the corresponding structure. The closed form analysis of the PRR-3S and PPR-3S structures here presented results in algebraic equations in one unknown of degree 16 and 12, respectively. Hence 16 and 12 closures of the corresponding structures can be obtained.

A comparative study of chains and mechanisms at the conceptual stage of design is expected to help the designer in selecting the best possible chain or mechanism for the specified task. To accomplish this designer should be able to read the characteristics of the kinematic chains based on their topology. It is only necessary to associate logically certain characteristics, weakness and strength of a chain to perform a task, with the structure of the chain and then generalize. Based on this belief work has been initiated to assess the ability of a chain to reveal some of the characteristics like structural error performance, dynamic behavior etc. in a comparative sense. In this study [4] criteria and measurements to compare kinematic chains and inversions for other characteristics like static behavior (mechanical advantage), compactness, stiffness and suitability as platform type robots, which are gaining importance, are presented.

The number synthesis of kinematic chains is applied in this study [5] in several different ways in order to

synthesize chains suitable for application as robot hands; several examples of the structures so found are presented. So as to identify those kinematic chains that are more promising than others, the new concept of minimal sets of kinematic chains is defined. Another new concept, the variety of a kinematic chain, is defined and used to make generalizations about relative connectivity within kinematic chains, which has application in the selection of actuated pairs.

Based on the topology of chains, quantitative methods are presented [6] in order to compare all the distinct chains, with the specified number of links and degree-of-freedom (d.o.f.), (i) for workspace and rigidity, (ii) to select the joint of the input link to introduce motion, and (iii) to test isomorphism, simply and uniquely.

Parallelism can be associated with every closed kinematic chain or its representative graph [8]. Parallelism throws light on work space, rigidity, speed ratios (mechanical advantage), etc., and is of great help in selecting multi degree-of-freedom (dof) chains for robotic applications. Numerous distinct chains with the same number of links and dof exist. The extent of parallelism differs from chain to chain and hence a numerical measure is necessary to quantify the same so that the designer gains insight simply based on the structure without having to actually design all the distinct chains before selecting the best chain for the specified task.
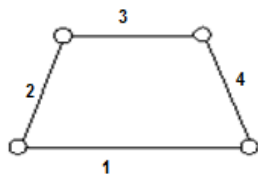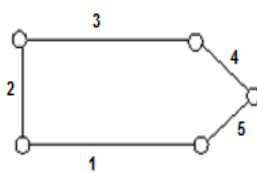

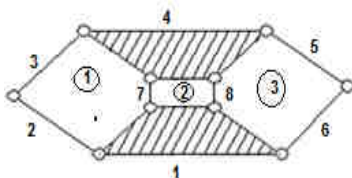*Fig. 1: Four bar chain*

Fig. 2: Five bar chain

Fig. 3: Eight bar chain

II.   CALCULATION OF PARALLELISM FOR SINGLE LOOP CHAIN

Single loop means the motion can be transferred from one link to other link by means of two paths only (path contains links and joints), as shown in Fig. 1, Fig. 2, etc.

For example, if we consider Fig. 1 then motion can be transferred from link 1 to link 3 by two paths i.e. path 1 – 2 – 3 or path 1 – 4 – 3. Path 1 – 2 – 3 and path 1 – 4 – 3 forms a single loop, called loop 1 – 2 – 3 – 4. Therefore we can say that loop is formed when the joints of a link are connected to the corresponding joints of other link by means of different paths. For example, link 1 of Fig. 1 contains two joints and link 3 also contains two joints. If we connect these joints then a loop is formed. Number of joints in a path depends upon number of links.

If J_i, J_j, etc. be the number of joints along the path i, j, etc, and if J be the total number of joints in a loop then

$$J = J_i + J_j + \text{--------} \qquad (1)$$

The farthest link will have least parallelism and this is the link for which product of factorial of J_i, J_j, etc, is minimum. Factorial is taken in order to increase the interconnectivity between the links. Keeping this in mind, following relation can be used to calculate parallelism P between two links k and l

$$P_{kl} = (\llcorner J_i \times \llcorner J_j) / L \qquad (2)$$

Where L is the loop size i.e. sum of number of links or number of joints.

Consider three bar chain, as shown in Fig. 4, parallelism between links 1 and 2 is:

$$P_{12} = (\llcorner 1 \times \llcorner 2) / 3 = 0.666$$

Similarly, parallelisms between links 1, 3 and 2, 3 are 0.666 and 0.666 respectively. When this happens then it forms a structure and no motion can be transferred.

Consider four bar chain, as shown in Fig. 1, parallelism between links 1 and 3 is:

$$P_{13} = (\llcorner 2 \times \llcorner 2) / 4 = 1$$

Similarly, parallelisms between links 1, 2 and 1, 4 are 1.5 and 1.5 respectively.

Consider five bar chain, as shown in Fig. 2, parallelism between links 1 and 3 is:

$$P_{13} = (\llcorner 2 \times \llcorner 3) / 5 = 2.4$$

Similarly, parallelisms between links 1, 2 and 1, 4 and 1, 5 are 4.8, 2.4 and 4.8 respectively.

III.   MULTI LOOP CHAIN

Multi loop means the motion can be transferred from one link to other link by means of more than two paths, as shown in Fig. 3 and Fig. 5. For example, if we consider Fig. 5, then motion can be transferred from link 1 to link 3 by three paths i.e. 1 – 2 – 3, 1 – 4 – 3 and 1 – 5 – 3. Path 1 – 2 – 3 and path 1 – 4 – 3 forms first loop. Path 1 – 4 – 3 and path 1 – 5 – 3 forms second loop. Two loops are formed because one path is common in two loops. Similarly, if we consider Fig. 3, then motion can be transferred from link 1 to link 4 by four paths i.e. path 1 – 2 – 3 – 4, path 1 – 7 – 4, path 1 – 8 – 4 and path 1 – 6 – 5

– 4. Path 1 – 2 – 3 – 4 and path 1 – 7 – 4 forms first loop. Path 1 – 7 – 4 and path 1 – 8 – 4 forms second loop and path 1 – 8 – 4 and path 1 – 6 – 5 – 4 forms the third loop. Three loops are formed because two paths are common in every two adjacent loops.

IV. CALCULATION OF PARALLELISM FOR TWO SYMMETRICAL LOOPS

Parallelism P between two links k and l in a multi loop chain (containing two symmetrical loops), as shown in Fig. 5, can be calculated by using the following steps:

1. If the links k and l participate in both loops, then calculate the parallelism between the links by the method as mentioned in Section II, for each loop. Take either of the two values for parallelism between the links under consideration.

2. If the links k and l participate in one loop only, then calculate the parallelism between the links by the method as mentioned in Section II.

Consider the parallelism between the links 1 and 3 of five bar chain, as shown in Fig.5. Link 1 and link 3, both participates in loop 1 and loop 2. In other words we can say that link 1 and link 3 are connected by three paths 1 – 2 – 3, 1 – 4 – 3, 1 – 5 – 3 and two separate loops are formed. This is because one path 1– 4 – 3 is common in both loops. Therefore parallelism between links 1 and 3 will be due to both loops. Therefore, parallelism between links 1 and 3 considering loop 1 only:

$P_{13} = (\llcorner 2 \times \llcorner 2) / 4 = 1$

Parallelism between links 1 and 3 considering loop 2 only:

$P_{13} = (\llcorner 2 \times \llcorner 2) / 4 = 1$

In this case, value of parallelism is same in both loops. Therefore, parallelism between the links is 1.

Consider the parallelism between the links 1 and 2. Here, link 1 participates in loop1 and loop 2, but link 2 participates in loop 1 only. In other words we can say that link 1 and link 2 are connected by two paths 1 – 2 and 1 – 4 – 3 – 2, therefore only one loop is formed. Therefore parallelism between link 1 and link 2 will be due to one loop only. Therefore, parallelism between the links 1 and 2 is:

$P_{12} = (\llcorner 1 \times \llcorner 3) / 4 = 1.5$

Consider the parallelism between the links 1 and 4. Link 1 and link 4, both participate in loop 1 and loop 2. In other words we can say that link 1 and link 4 are connected by three paths 1 – 4, 1 – 2 – 4 and 1 – 5 – 3 – 4. Path 1 – 4 is common, therefore two loops are formed. Therefore parallelism between link 1 and link 4 will be due to both loops. Therefore, parallelism between the links 1 and 4 considering loop 1, is:

$P_{14} = (\llcorner 1 \times \llcorner 3) / 4 = 1.5$

Parallelism between the links 1 and 4 considering loop 2, is:

$P_{14} = (\llcorner 1 \times \llcorner 3) / 4 = 1.5$

In this case, value of parallelism is same in both loops. Therefore, parallelism between the links is 1.5. Similarly, the parallelism between links 1 and 5 is also 1.5.

V. CALCULATION OF PARALLELISM FOR TWO UNSYMMETRICAL LOOPS

Parallelism P between two links k and l in a multi loop chain (containing two unsymmetrical loops), as shown in Fig. 3, can be calculated by using the following steps:

1. If the links k and l participate in both loops, then calculate the parallelism between the links by the method as mentioned in Section II, for each loop. Take minimum of all the values for the farthest link and maximum of all the values for the nearest link. This is because the farthest link and nearest links will have least and greatest parallelism respectively.

2. If the links k and l participate in one loop only, then calculate the parallelism between the links by the method as mentioned in Section II.

Consider the parallelism between the links 1 and 4 of eight bar chain, as shown in Fig. 3. Link 1 and link 4, both participate in loop 1, loop 2 and loop 3. In other words, we can say that links 1 and 4 are connected by the paths 1 – 2 – 3 – 4, 1 – 7 – 4, 1 – 8 – 4 and 1 – 6 – 5 – 4, forming three different loops. Paths 1 – 7 – 4 and 1 – 8 – 4 are common in every two adjacent loops. Two of them are symmetrical and one is unsymmetrical. Therefore parallelism between link 1 and link 4 will be due to all the three loops. Parallelism between the links 1 and 4 considering loop 1, is:

$P_{14} = (\llcorner 2 \times \llcorner 3) / 5 = 2.4$

Parallelism between the links 1 and 4 considering loop 2, is:

$P_{14} = (\llcorner 2 \times \llcorner 2) / 4 = 1$

Parallelism between the links 1 and 4 considering loop 3, is:

$P_{14} = (\llcorner 2 \times \llcorner 3) / 5 = 2.4$

Therefore, parallelism between link 1 and 4 is minimum of all the values i.e. 1. This is because link 4 is the farthest link from link 1 and, therefore, link 4 will have least parallelism.

Consider the parallelism between the links 1 and 2. Here, link 1 participates in loop1, loop 2 and loop3, but link 2 participates in loop 1 only. In other words, we can say that link 1 and link 2 are connected by two paths 1 – 2 and 1 – 7 – 4 – 3 – 2. Therefore, only one loop is formed and parallelism between link 1 and link 2 will be due to one loop only. Therefore, parallelism between the links 1 and 2 is:

$P_{12} = (\llcorner 1 \times \llcorner 4) / 5 = 4.8$

Consider the parallelism between the links 1 and 3. Here, link 1 participates in loop1, loop 2 and loop3, but link 3 participates in loop 1 only. In other words, we can say that link 1 and link 3 are connected by two paths $1 - 2 - 3$ and $1 - 7 - 4 - 3$. Therefore, only one loop is formed and parallelism between link 1 and link 3 will be due to one loop only. Therefore, parallelism between the links 1 and 3 is:

$P_{13} = (\llcorner 2 \times \llcorner 3) / 5 = 2.4$

Consider the parallelism between the links 1 and 7. Here, link 1 participates in loop1, loop 2 and loop3, but link 7 participates in loop 1 and loop 2 only. In other words, we can say that link 1 and link 7 are connected by two paths $1 - 2 - 3 - 4 - 7$, $1 - 7$ of loop 1 and $1 - 7$, $1 - 8 - 4 - 7$ of loop 2. Therefore, two loops are formed and parallelism between link 1 and link 7 will be due to two loops. Therefore, parallelism between the links 1 and 7, due to loop 1, is:

$P_{17} = (\llcorner 1 \times \llcorner 4) / 5 = 4.8$

Parallelism between the links 1 and 7, due to loop 2, is:

$P_{17} = (\llcorner 1 \times \llcorner 3) / 4 = 1.5$

Therefore, parallelism between link 1 and 7 is maximum of all the values i.e. 4.8. This is because link 7 is the nearest link from link 1 and, therefore link 7 will have greatest parallelism. Similarly the parallelism between the links $1 - 6$, $1 - 5$ and $1 - 8$ are 4.8, 2.4 and 4.8 respectively.

VI. EXPLANATION OF PARALLELISM, RIGIDITY AND GRASPING POWER IN CASE OF ROBOT HANDS

Farthest link has least parallelism and nearer link has greater parallelism. Greater parallelism means more rigidity i.e. more stiffness and ultimately the more grasping power, a property useful to the designer when he considers kinematic chains for application as robotic hands.

Though two links are considered serially connected, say links 1 and 3, as shown in Fig. 6, the second path necessary for parallelism to exist between those links 1 and 3 is considered to consist of very large number of links and joints, as shown by dotted lines. In other words, parallelism between two links can also be considered as links in series separated by joints equal to the inverse of the parallelism.

Consider the robot hand 1 of figure 7, developed by Tichler et. al [5]. This hand has two fingers "a" and "b" and three finger tips. This figure is redrawn as Fig. 8 for simplicity. Links are numbered 1, 2, etc and p, q and r are the finger tips which make point contact with the object. Frictional point contact between the object and the finger tips can be considered equivalent to spherical joints

between the object and the finger tips [7]. In planar hands, these joints will be reduced to revolute joints.

Consider the robot hand 1, which contains two fingers "a" and "b". Finger "a" has two tips "p" and "q". Finger "b" has one tip "r". Since finger "a" have two tips "p" and "q", therefore, first of all it should be converted into a single tip having single path above the ternary link. For that parallelism between the object and the ternary link 6 is calculated first by using Equation 2 and then combined with the rest of the link assembly.

Therefore, considering object, link 6 and link 7 as three bar loop, parallelism between link 6 and object is:

$P_{60} = (\llcorner 1 \times \llcorner 2) / 3 = 2 / 3$

Inverse of this is $3 / 2$. It means that whole assembly above ternary link can be replaced by a binary link having a joint value of 1 and 0.5. This is shown in Fig. 9. Finally the parallelism between ground link and object is obtained by dividing the total joint value of the equivalent robot hand by the product of sum of joint values of each path. Consider robot hand 1, as shown in Figure 8. Its equivalent robot hand is shown in Fig 9. One path contains 4 joints and therefore has total joint value of 4. Other path contains 7 joints and therefore has total joint value of 6.5. Now the total joint value of the equivalent robot hand is 10.5. According to our theory, the parallelism between ground link and object is obtained by $10.5 / (4 \times 6.5) = 0.404$. Similarly, the parallelism between ground link and object of other robot hands can also be calculated and the result is shown in the Table 1. For robot hand 9 of Fig. 7, calculate the parallelism between ground link and object for the two individual symmetrical loops and then take either of the two values, as discussed in Section 3.1.
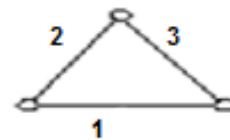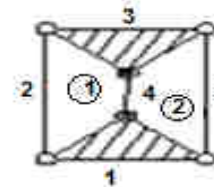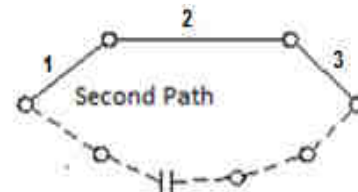

Fig. 4: Three bar chain
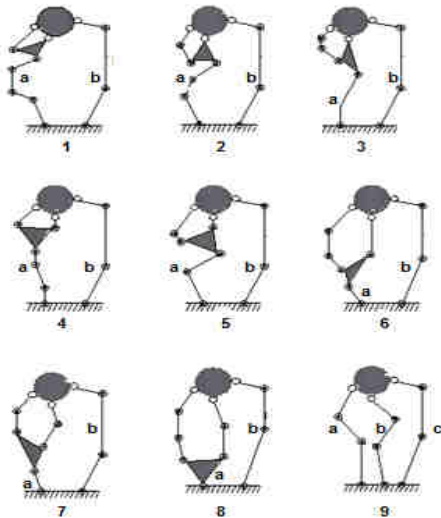

Fig. 5: Five bar chain
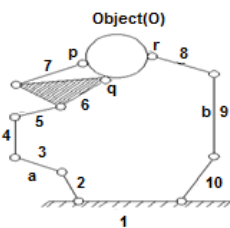

Fig. 6: Equivalent graph

Fig. 7: Robot hands

Fig. 8: Robot hand (1) of Fig.7

Fig. 9: Equivalent Robot hand of Fig.8

considers kinematic chains for application as robotic hands. From the Table 1 it is clear that robot hand 9 has greater parallelism and therefore greater rigidity. Unit parallelism between two links indicates that they are likely to have unit velocity ratio while greater parallelism indicates the possibility of high speed ratio.

Table.1: Parallelism of robot hands of Fig. 7 (in descending order)

Robot hands	Parallelism
9	1.500
8	1.203
6	0.720
7	0.711
3	0.639
5	0.542
2	0.464
4	0.450
1	0.404

Table.2: Parallelism of robot hands of Fig. 7 (in descending order)

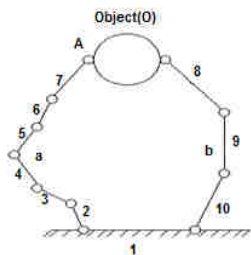| Robot hands	Parallelism
9	0.75
8	0.66
6	0.55
7	0.53
3	0.51
5	0.48
2	0.46
4	0.45
1	0.42

VII. RESULT

Result can be summarized into the following points:

1. Factorial is taken in order to increase the inter-connectivity between the links.

2. A different method [8] when applied to the robot hands, Fig. 7, gave the results which are shown in Table 2.

3. Though results obtained by the present method (shown in Table 1) and the result obtained by the method [8] are different, the pattern of the results are same.

4. Unit parallelism between two links indicates that they are likely to have unit velocity ratio while greater parallelism indicates the possibility of high speed ratio.

VIII. CONCLUSION

Farthest link has least parallelism and nearer link has greater parallelism. Greater parallelism means more rigidity i.e. more stiffness and ultimately the more grasping power, a property useful to the designer when he

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