# Difference Cordial of Operational Graph Related to Cycle

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***Abstract* —***Let $G$ be a $(p, q)$ graph. A bijective vertex labeling function $f : V(G) \rightarrow \{1, 2, \dots p\}$ is called a difference cordial labeling if for each edge $uv$, assign the label $|f(u) - f(v)|$ then $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper, we prove that cycle with one chord, cycle with twin chords and cycle with triangle admit difference cordial labeling.*

***Keywords -Difference cordial, Cycle with one chord, Cycle with twin chord, cycle with triangle, Swastik graph.***

**AMS Subject classification number: 05C78.**

## I. INTRODUCTION

Here we consider finite, undirected and simple graph. Let $G = (V, E)$ be a $(p, q)$ graph. For graph theoretical terminology and notations we follow Harary[3].
Ponraj et al.[5] introduced difference cordial labeling and proved that path, cycle, complete graph, complete bipartite graph, star, helm are difference cordial graphs. In [6] the same authors have discussed difference cordial labeling behaviour of triangular snake, quadrilateral snake, double triangular snake, double quadrilateral snake and alternate snakes.

## II. MAIN RESULTS

**Definition II.1.** *A chord of a cycle $C_n$ is an edge joining two non-adjacent vertices of cycle $C_n$.*

**Theorem II.1.** *Cycle $C_n$ with one chord is difference cordial graph, where chord forms a triangle with two edges of $C_n$.*

*Proof.* Let $G$ be the cycle $C_n$ with one chord. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v_nv_1\} \cup \{v_2v_n\}$. Here $|V(G)| = n$ and $|E(G)| = n + 1$. To define vertex labeling function $f : V(G) \rightarrow \{1, 2, \dots, n\}$, we consider the following cases.

**Case 1:** $n$ is odd.
$f(v_i) = i; 1 \leq i \leq 2.$

**Subcase 1:** $n \equiv 1(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-1}{4}. \\ (2n+1) - 4i; & \frac{n+3}{4} \leq i \leq \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-1}{4}. \\ 2n - 4i; & \frac{n+3}{4} \leq i \leq \frac{n-3}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 3(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-3}{4}. \\ (2n+1) - 4i; & \frac{n+1}{4} \leq i \leq \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-3}{4}. \\ 2n - 4i; & \frac{n+1}{4} \leq i \leq \frac{n-3}{2}. \end{cases}$$

**Case 2:** $n$ is even.
$f(v_i) = i; 1 \leq i \leq 2.$

**Subcase 1:** $n \equiv 0(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n}{4}. \\ (2n+2) - 4i; & \frac{n+4}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-4}{4}. \\ 2n - 4i; & \frac{n}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 2(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-2}{4}. \\ (2n+1) - 4i; & \frac{n+2}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \le i \le \frac{n-2}{4}. \\ 2n-4i; & \frac{n+2}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

In each case cycle $C_n$ with one chord satisfies condition for difference cordial labeling. □

**Illustration II.1.** *Difference cordial labeling of $C_{11}$ with one chord is shown in Figure 1.*

**Definition II.2.** *Two chords of a cycle $C_n$ are said to be twin chords if they form a triangle with an edge of the cycle $C_n$.*
*For positive integer $n$ and $p$ with $3 \le p \le n-2$, $C_{n,p}$ is the graph consisting of a cycle $C_n$ with a pair of twin chords with which the edges of $C_n$ form cycles $C_p, C_3$ and $C_{n+1-p}$ without chords.*

**Theorem II.2.** *Cycle with twin chords $C_{n,3}$ is difference cordial.*

*Proof.* Let $|V(C_{n,3})| = \{v_1, v_2, \ldots, v_n\}$ and $|E(C_{n,3})| = \{v_iv_{i+1}/1 \le i \le n-1\} \cup \{v_nv_1\} \cup \{v_2v_n\} \cup \{v_3v_n\}$. $|V(C_{n,3})| = n$, $|E(C_{n,3})| = n+2$. To define vertex labeling function $f : V(C_{n,3}) \to \{1, 2, \ldots, n\}$, we consider the following cases.
**Case 1:** $n$ is odd.
**Subcase 1:** $n \equiv 1(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+3}{4}. \\ (2n+5)-4i; & \frac{n+7}{4} \le i \le \frac{n+1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n-1}{4}. \\ (2n+2)-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 3(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+1}{4}. \\ (2n+5)-4i; & \frac{n+5}{4} \le i \le \frac{n+1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n+1}{4}. \\ (2n+2)-4i; & \frac{n+5}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

**Case 2:** $n$ is even.
**Subcase 1:** $n \equiv 0(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n}{4}. \\ (2n+4)-4i; & \frac{n+4}{4} \le i \le \frac{n}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n}{4}. \\ 2n-4i; & \frac{n}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 2(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+2}{4}. \\ (2n+4)-4i; & \frac{n+6}{4} \le i \le \frac{n}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n+2}{4}. \\ (2n+3)-4i; & \frac{n+6}{4} \le i \le \frac{n}{2}. \end{cases}$$

In each case cycle with twin chords $C_{n,3}$ satisfies condition for difference cordial labeling. □

**Illustration II.2.** *Difference cordial labeling of cycle $C_{16}$ with twin chords is shown in Figure 2.*

**Definition II.3.** *A cycle with triangle is a cycle with three chords which by themselves form a triangle.*
*For positive integers $p, q, r$ and $n \ge 6$ with $p+q+r+3 = n$, $C_n(p, q, r)$ denotes a cycle with triangle whose edges form the edges of cycles $C_{p+2}, C_{q+2}, C_{r+2}$ without chords.*

**Theorem II.3.** *Cycle with triangle $C_n(1, 1, n-5)$ is difference cordial.*

*Proof.* Let $|V(C_n(1, 1, n-5))| = \{v_1, v_2, \ldots, v_n\}$ and $|E(C_n(1, 1, n-5))| = \{v_iv_{i+1}/1 \le i \le n-1\} \cup \{v_nv_1\} \cup \{v_1v_3\} \cup \not{v}\{v_3v_{n-1}\} \cup \{v_{n-1}v_1\}$. $|V(C_n(1, 1, n-5))| = n$, $|E(C_n(1, 1, n-5))| = n+3$. To define vertex labeling function $f : V(C_n(1, 1, n-5)) \to \{1, 2, \ldots, n\}$, we consider the following cases.
**Case 1:** $n$ is odd.
$f(v_1) = 1$.
**Subcase 1:** $n \equiv 1(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-1}{4}. \\ 2n-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n-1}{4}. \\ (2n+3)-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 3(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-3}{4}. \\ 2n-4i; & \frac{n+1}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n+1}{4}. \\ (2n+3)-4i; & \frac{n+5}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

**Case 2:** $n$ is even.
$f(v_1) = 1$.
**Subcase 1:** $n \equiv 0(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n}{4}. \\ (2n+1)-4i; & \frac{n+4}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n}{4}. \\ (2n+2)-4i; & \frac{n+4}{4} \le i \le \frac{n}{2}. \end{cases}$$

**Subcase 2:** $n \equiv 2(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-2}{4}. \\ (2n+1)-4i; & \frac{n+2}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n-2}{4}. \\ (2n+2)-4i; & \frac{n+2}{4} \le i \le \frac{n}{2}. \end{cases}$$

In each case the cycle with triangle $C_n(1, 1, n-5)$ satisfies the condition for difference cordial labeling. □

**Illustration II.3.** *Difference cordial labeling of cycle $C_{11}$ with triangle ($C_{11}(1, 1, 6)$) is shown in Figure 3.*

**Definition II.4.** *[4] Swastik graph is a union of four copies of $C_{4n}$ ($n \in \mathbb{N}-1$). Consider $v_{k,i}$ ($1 \le k \le 4, 1 \le i \le 4n$) to be the vertices of $k^{th}$ copy of $C_{4n}$, where $v_{k,4n} = v_{k+1,1}, 1 \le k \le 3$ and $v_{4,4n} = v_{1,1}$. It is denoted as $Sw_n$, $n \in \mathbb{N}-1$.*

**Theorem II.4.** *Swastik $Sw_n$ is difference cordial graph, $n \in \mathbb{N} - 1$.*

*Proof.* Let $v_{k,i}$ $(1 \le k \le 4, 1 \le i \le 4n)$ be the vertices of $k^{th}$ copy of $C_{4n}$ in swastik graph $Sw_n$, where $v_{k,4n} = v_{k+1,1}, 1 \le k \le 3$ and $v_{4,4n} = v_{1,1}$. Here $|V(Sw_n)| = 16n - 4$ and $|E(Sw_n)| = 16n$. We define vertex labeling function $f : V(G) \to \{1, 2, \ldots, |V(Sw_n)|\}$ as follows.

$$f(v_{1,(2i-1)}) = \begin{cases} 4i - 3; & 1 \le i \le n. \\ (8n+3) - 4i; & n+1 \le i \le 2n. \end{cases}$$

$$f(v_{1,(2i)}) = \begin{cases} 4i - 2; & 1 \le i \le n. \\ 8n - 4i; & n+1 \le i \le 2n-1. \end{cases}$$

$$f(v_{2,(2i-1)}) = \begin{cases} 4(n+i-1); & 1 \le i \le n. \\ 2(6n-2j+1); & n+1 \le i \le 2n. \end{cases}$$

$$f(v_{2,(2i)}) = \begin{cases} 4(n+i) - 3; & 1 \le i \le n. \\ 4(3n-i) - 1; & n+1 \le i \le 2n-1. \end{cases}$$

$$f(v_{3,(2i-1)}) = \begin{cases} 4(2n+i) - 5; & 1 \le i \le n. \\ 4(4n-i) + 1; & n+1 \le i \le 2n. \end{cases}$$

$$f(v_{3,(2i)}) = \begin{cases} 4(2n+i-1); & 1 \le i \le n. \\ 2(8n-2j-1); & n+1 \le i \le 2n-1. \end{cases}$$

$$f(v_{4,(2i-1)}) = \begin{cases} 11n + 4i - 2; & 1 \le i \le n. \\ (19n-4) - 4i; & n+1 \le i \le 2n. \end{cases}$$

$$f(v_{4,(2i)}) = \begin{cases} 4(3n+i) - 5; & 1 \le i \le n. \\ (19n+1) - 4i; & n+1 \le i \le 2n-1. \end{cases}$$

□

**Illustration II.4.** *Difference cordial labeling of $Sw_4$ is shown in Figure 4.*

## III. FIGURES

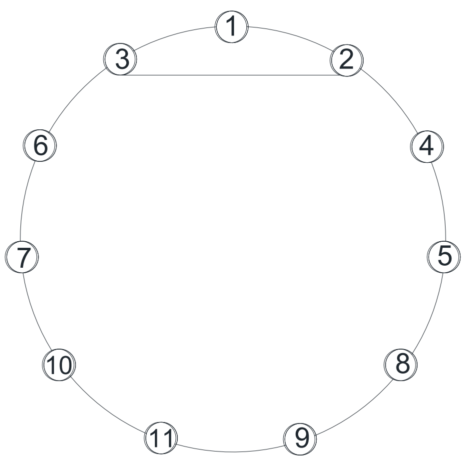


Fig. 1

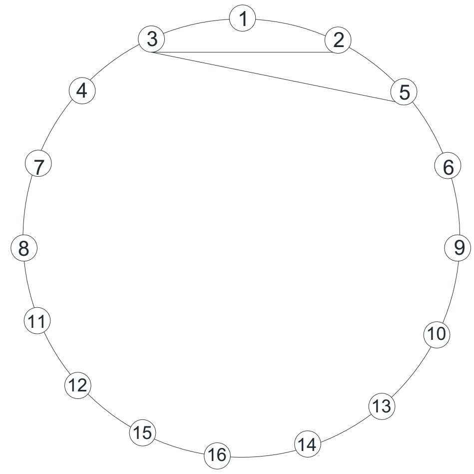


Fig. 2

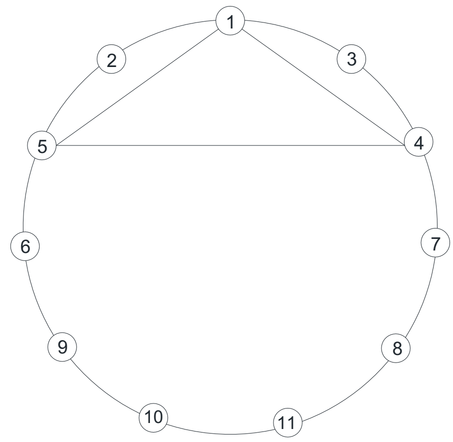


Fig. 3

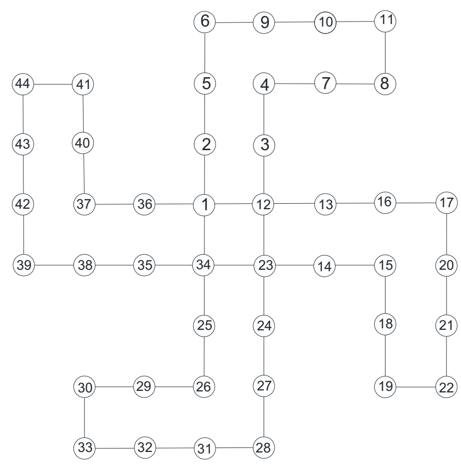


Fig. 4

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## IV. CONCLUSION

Difference cordial labeling for different cycle graphs have been discussed. As all cycle graphs are difference cordial by adding one or more chords in cycle $C_n$. We proved that the resultant graphs are also difference cordial.

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