Study of State Estimation Using Weighted Least Squares Method

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Abstract—*Power state estimation constitutes the core of the on-line security analysis function. The challenge number one of a state estimator is to provide the optimal estimates of system state with minimum of measurement data. This paper describes weighted least squares state estimation method and investigates how the efficiency of WLS state estimation changes according to 4 parameters: number of measurements, measurement type, measurement weight and level of noise. Different simulation cases are tested on 3-bus system and IEEE 14-bus system. The results show that accurate estimates of system state can be obtained with minimum of measurement data on condition to choose a good combination of accurate measurements with a minimum of voltage measurements and power injection measurements and these data should be properly distributed throughout the system. For best results, the two factors (weight and noise) must be combined to obtain the best estimation. Indeed, the most accurate measurements (lower level of noise) should have greater weight compared to bad measurements (higher level of noise), specially voltage measurements due to their big impact.*

Keywords—Level of noise, Measurement type, Measurement weight, Number of measurements, Voltage measurement,weighted least squares state estimation method.

I. INTRODUCTION

Electric power system deals with the generation, transmission, and distribution of electric energy. The efficient and optimum economic operation and planning, along with security of electric power systems, have always occupied an important position in the power industry. In order to achieve these objectives, it is essential for power engineers to accurately monitor the power system operating states. An essential tool for monitoring the power system is state estimation. In energy control centers, power system state estimation is carried out in order to provide best estimates of what is happening in the system based on real-time measurement and a predetermined system model. It is required in the critical operational functions of a power grid such as real-time security monitoring, load forecasting, economic dispatch, and load frequency control.

Most of network applications use the real-time data provided by the state estimator. Therefore, an optimal performance of state estimation output is the ultimate concern for the system operator. This need is particularly more in focus today due to deregulated and congested systems and smart grid initiatives. The output of the state estimator nearly represents a true state of the system. However, discrepancies may occur due to incomplete measurements, meaning many variables are not measured or data is not available, inaccurate network parameters, and errors in measurements [1].

Most state estimation programs in practical use are formulated as overdetermined systems of non-linear equations and solved as weighted least-squares(WLS) problems [2].

This paper describes Weighted Least Squares method for state estimation of power system, investigates its characteristics and observes the effect of 4 parameters (Number of measurements, measurement type, measurement weights and level of noise) on the quality of state estimation. Both simple power system case (3 bus) and a larger power system IEEE 14 bus test cases are utilized.

II. WLS METHOD

The starting equation for the WLS state estimation algorithm is:

$$z = h(x) + e \quad (1)$$

where: z is the (mx1) measurement vector; x is an (nx1) state vector to be estimated; h is a vector of nonlinear functions that relate the states to the measurements; and e is an (mx1) measurement error vector. Clearly, m must be grater then n in order to have measured the n states and have additional information to provide redundancy, m>n.

The measurement errors ei are assumed to satisfy the following statistical properties:
First, the errors have zero mean: E(ei) = 0, i = 1, ..., m
Second, the errors are assumed to be independent, such that the covariance matrix is diagonal.

ISSN: 2349-6495(P) | 2456-1908(O)

Cov(e) = E (e, eT) = R = diag{σ12 , σ22 , ….,σm2} (2)

The solution to the state estimation problem can be formulated as a minimization of following objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}} = [z - h(x)]^T R^{-1} [z - h(x)] \quad (3)$$

To find the minimization of this objective function the derivative should be set to zero. The derivative of the objective function is denoted by g(x):

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \quad (4)$$

where: H(x)= $\frac{\partial h(x)}{\partial x}$ called the measurement Jacobian matrix. Ignoring the higher order terms of the Taylor series expansion of the derivative of the objective functions yields an iterative solution as shown below:

$$x^{k+1} = x^k + [G(x^k)]^{-1}[[H(x^k)]^T [R]^{-1}[z - h(x^k)]] \quad (5)$$

Where the gain matrix, G, is defined as:

$$G(x^k) = \frac{\partial g(x)}{\partial x} = H^T R^{-1} H \quad (6)$$

For the first iteration of the optimization the measurement function and measurement Jacobian should be evaluated at flat voltage profile, or flat start. A flat start refers to a state vector where all of the voltage magnitudes are 1.0 per unit and all of the voltage angles are 0 degrees. In conjunction with the measurements, the next iteration of the state vector can be calculated again and again until a desired tolerance is reached [3,4].

The flowchart [5] of WLS method is shown in figure 1:

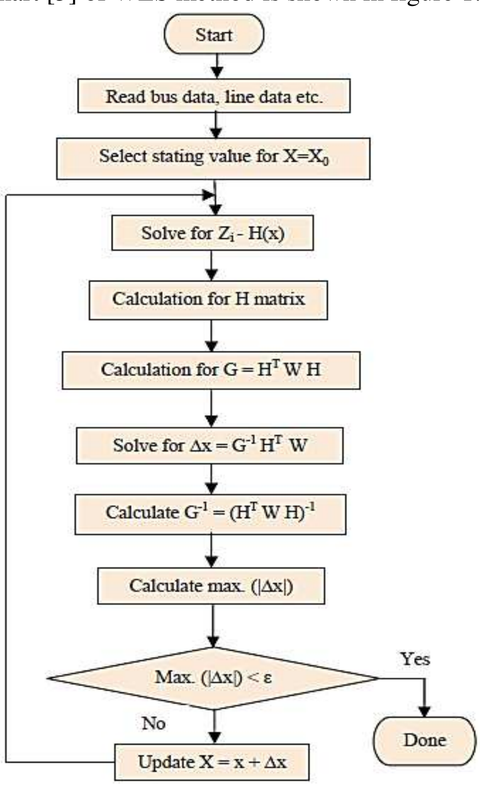

Fig. 1: The flow chart of WLS Method

III. SIMULATION RESULTS

This section presents a study of WLS state estimation characteristics through the observation of the effect of the 4 following parameters (Number of measurements, measurement type, measurement weight and level of noise) on the accuracy of state estimation. The simulations are tested on two systems: a simple 3-bus system and IEEE 14-bus system presented below:

- Case Study Utilizing a Three-Bus System

A simple case study of 3-bus system is shown in Figure2. Bus 1 is the reference bus, bus 2 is the load bus, and bus 3 is the generator bus. The network data are shown in the same figure.

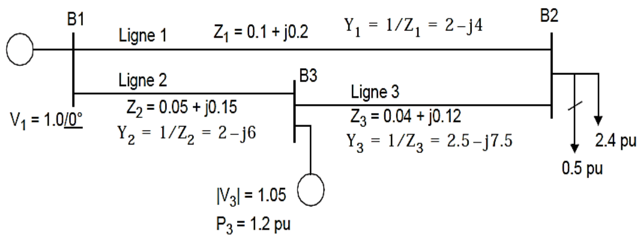

Fig.2: Case Study of 3-bus system

- Case Study Utilizing IEEE 14-bus system

The system is shown in figure 3. The network data files can be downloaded from Power Systems Test Case Archive [6].

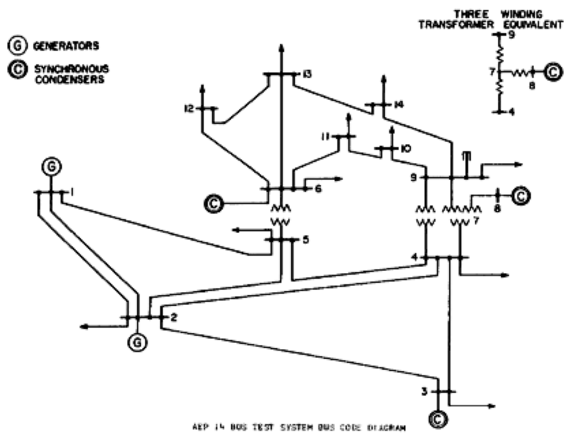

Fig. 3: IEEE 14 Bus Test case

For both test cases the measurement data are chosen from Newton Raphson load flow results [7,8] and consist of three kinds of measurements: voltage magnitudes, real and reactive power injections, real and reactive power flows. Weight of all measurements is assumed 1. The true values of voltage magnitude and angle are from Newton Raphson load flow results. To compare the state estimate accuracy of the following simulations, mean absolute percentage error (MAPE) is introduced as follows [9]:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \times 100\% \quad (34)$$

Where, At is the actual value and Ft is the calculated value. A smaller value of MAPE indicates a more accurate state estimation result.

3.1. Effect of number of measurements:

The robustness of state estimation can be guaranteed only if the number of the available measurements is high enough and properly distributed throughout the system. A measure of the number of measurements may be denoted by the redundancy factor η, which is defined as [9]:

$$\eta = \frac{Dimension of z}{Dimension of x} = \frac{m}{n} = \frac{m}{2N-1} \quad (35)$$

We will analyze the influence of the degree of redundancy through 5 different cases for both 3-bussystem and IEEE 14-bus system. The comparison is set according to the two state variables: voltage magnitude(MPAEV) and voltage angle(MAPEθ).

3.1.1 Simulation results for 3 bus system:

Table.1:Cases studies with different number of measurements

	m	η	V	Pinj	Qinj	Pflow	Qflow
Case1	21	4,2	3	3	3	6	6
Case2	15	3	3	0	0	6	6
Case3			0	3	3	6	3
Case4	5	1	1	0	0	2	2
Case5	<5	All possible combinations are tried					

The results are shown below in table2 and 3:

Table.2:WLS state estimation of voltage magnitude / different redundancy degree/3 bus system

Bus ID	True Value (P,U)	Estimated value of voltage magnitude by WLS (P.U) / 3 bus System				
		Case1	Case2	Case3	Case4	Case 5
1	1	1	1,000002	Gain matrix is Singular	1	Gain matrix is Singular
2	0,8898	0,889791	0,889795		0,889796	
3	1,05	1,05	1,050004		1,050006	
MAPEV (%)		0,0004	0,0004		0,0003	
Number of Iterations		5	5		5	

Table.3:WLS State estimation of voltage angle/ Different redundancy degree/3bus system

Bus ID	True Value (degree)	Estimated value of voltage angle by WLS (degree)/ 3 bus System				
		Case1	Case2	Case3	Case4	Case 5
1	0,0000	0,000000	0,000000	Gain matrix is Singular	0,000000	Gain matrix is Singular
2	-13,3116	-13,311451	-13,311332		-13,311422	
3	-4,2380	-4,237860	-4,237850		-4,237997	
MAPEθ (%)		0,0033	0,0035		0,0001	
Number of Iterations		5	5		5	

For the latest case (Case5), all possible combinations of measurements types were tried but all have not converged, this verify the observability condition (m>=n). It is a necessary but not sufficient condition. In fact, we have a counter-example: Case 3, even if the number of measurements (m=15) is greater than number of states (n=5), the algorithm has not converged. On the other hand, for case 2 with the same number of measurements the algorithm has converged, indicating that WLS state estimation is affected by the combination of measurements types chosen.

3.1.2. Simulation results for IEEE 14- bus system:

Table.4: Cases studies with different number of measurements

	m	η	V	Pinj	Qinj	Pflow	Qflow
Case1	120	4,4	14	14	14	38	40
Case2	95	3,5	1	8	8	38	40
Case3	41	1,5	1	8	8	12	12
Case4	27	1,0	1	6	6	7	7
Case5	<27	All possible combinations are tried					

The results are presented in figures 4 and 5:

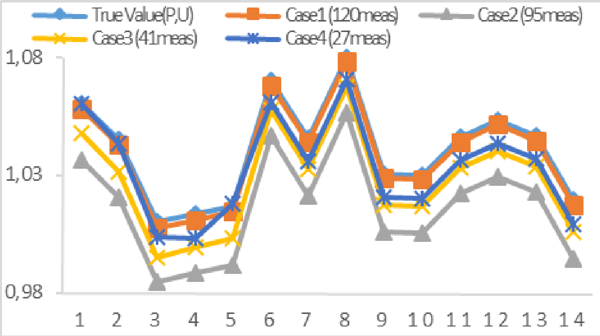

Fig.4: Comparison of WLS state estimation of voltage magnitude with different redundancy

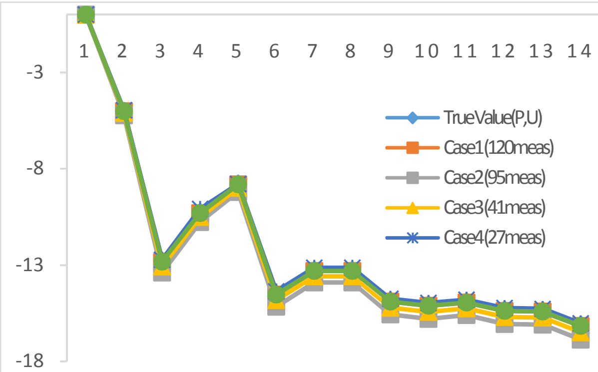

Fig.5: Comparison of WLS state estimation of voltage angle with different redundancy degree

The nearest case to the true value is case 1, that's obvious because we have a large number of measurements (m=120).

As seen, increased redundancy improves the accuracy of the estimation, but not uniformly. In fact, although case 4 has a lower redundancy degree a better solution is obtained compared with cases 2,3 which have a higher η.

We conclude, that the WLS state estimation is affected not only by the number of measurements but also by other correlated factors like measurement type, measurement location, measurement error….

In the following point, we will study the effect of measurement type.

3.2. Effect of measurement type:

The WLS algorithm is tested on 6 different cases of combinations of measurements types for both 3 bus system and IEEE 14 bus system.

3.2.1. Simulation results for 3 bus system:

Table.5:Cases studies with different combinations of measurements types

	m	η	V	Pinj	Qinj	Pflow	Qflow
Case1	21	4,2	3	3	3	6	6
Case2	18	3,6	0	3	3	6	6
Case3	15	3	3	0	0	6	6
Case4	12	2,4	0	0	0	6	6
Case5	9	1,8	3	3	3	0	0
Case6	6	1,2	0	3	3	0	0

The results are shown in tables 6 and 7 below:

Table.6: WLS State estimation of voltage magnitude /different combinations of measurements types/3bus system

Bus ID	True Value(P,U)	Case1	Case2	Case3	Case4	Case5	Case6
1	1	1	0,999993	1,000002	1,000004	1	0,99998
2	0,8898	0,889791	0,889783	0,889795	0,889798	0,889788	0,889766
3	1,05	1,05	1,049994	1,050004	1,050006	1,049998	1,049979
MAPEV (%)		0,0004	0,0011	0,0004	0,0004	0,0005	0,0026
Number of Iterations		5	6	5	6	5	6

Table.7: WLS State estimation of voltage angle /different combinations of measurements types/3bus system

Bus ID	True Value (degree)	Case1	Case2	Case3	Case4	Case5	Case6
1	0	0	0	0	0	0	0
2	-13,3116	-13,311451	-13,311649	-13,311332	-13,311276	-13,311519	-13,312072
3	-4,238	-4,23786	-4,237919	-4,23785	-4,237833	-4,237857	-4,238018
MAPEθ (%)		0,0033	0,0019	0,0035	0,0039	0,0034	0,0004
Number of Iterations		5	6	5	6	5	6

Tables 6 and 7 shows that for cases (2,4 and 6) where voltage measurements are missing, more number of iterations (6 iterations) is required and estimation accuracy is lower compared with the other cases. So, we deduce that the presence of voltage measurements isnecessary for an efficient execution of the program without problems.

3.2.2. Simulation results for IEEE 14- bus system:

Table.8: Cases studies with different combinations of measurements types

	m	η	V	Pinj	Qinj	Pflow	Qflow
Case1	120	4,4	14	14	14	38	40
Case2	106	3,9	0	14	14	38	40
Case3	92	3,4	14	0	0	38	40
Case4	78	2,9	0	0	0	38	40
Case5	42	1,6	14	14	14	0	0
Case6	28	1,0	0	14	14	0	0

The results are presented in figures 6 and 7.

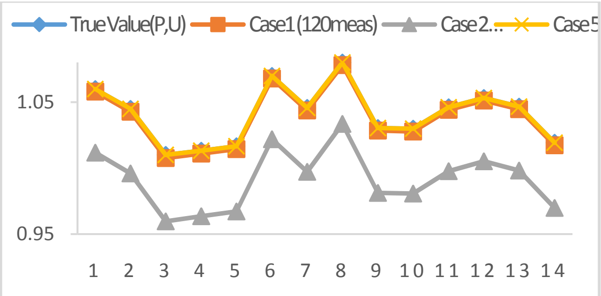

Fig 6: Comparison of WLS state estimation voltage angle according to different combinations of measurements types

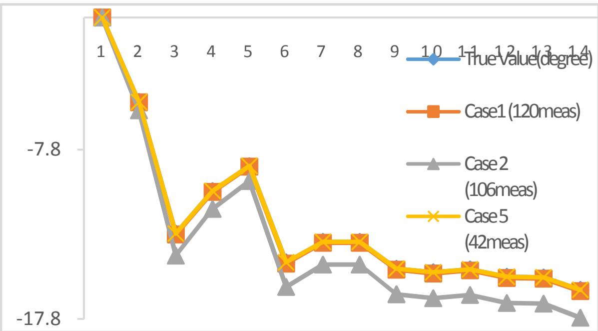

Fig 7: Comparison of WLS state estimation voltage angle according to different combinations of measurements types

For cases 3 and 4, Gain matrix is singular,so the program doesn't converge. In case 6, only power injection measurements are utilized, the program diverges: Number of iterations >1000 and MAPE> 100%. It means that the presence of power injection measurements without voltage measurements may lead to convergence problems.

A good accurate solution is obtained in case 5. Therefore, the combination of voltage measurements with power injection measurements, is better than the combination of voltage measurements with power flow measurements. As result, an optimal combination should necessary contains a minimum number of voltage and power injection measurements with some power flow measurements.

In practice we usually use a redundancy factor η >=1,5. So, to be closer to the reality, we will consider in the next studies, two models as follow:

- For 3 bus system: 7 measurements are taken throughout the network (1V, 2 Pinj, 2 Pflow, 2 Qflow).- For 14 bus system: 41 measurements are taken throughout the network (1V, 8Pinj, 8Qinj,12Pflow, 12 Qflow).

3.3. Effect of measurements weights:

As defined previously the measurement error covariance matrix R is a diagonal matrix of measurement variances constituted by weights. So, Wi(weight) =1/ $\sigma^2 i$, where $\sigma^2 i$ assumed error variance of measurement "i".

In this point, we will study the effect of measurements weights on the state estimation by WLS.

In the previous simulations, we supposed that all measurements had the same weight which was set to 1. Now, two simulations will be presented: one is setting the same weight for the different measurements, we only change his value. Another, different weights are tried according to the type of measurements.

3.3.1. Same weight for all measurements:

3 simulations are tried with different weights (σ=1, σ=0.1 and σ=0.001), for the two systems (3 bus and IEEE 14 bus).The 3 bus system results are shown in table 9 and table10.

Table.9:Estimated value of voltage magnitude with the same weight for all measurements

Bus ID	True Value(P,U)	σ=1	σ=0,1	σ=0,001
1	1	1,000001	1,000001	1,000001
2	0,8898	0,889797	0,889797	0,889797
3	1,05	1,050007	1,050007	1,050007
MAPEV (%)		0,0004	0,0004	0,0004
Iterations		5	5	5

Table.10: Estimated value of voltage angle with the same weight for all measurements

Bus ID	True Value (degree)	σ=1	σ=0,1	σ=0,001
1	0	0	0	1,000001
2	-13,3116	-13,31149	-13,31149	0,889797
3	-4,238	-4,237957	-4,237957	1,050007
MAPEθ (%)		0,001	0,001	0,0004
Iterations		5	5	5

According to those results, we deduce that Whatever the value of the weight as it is the same for all measurements, the result doesn't change. The same results are obtained for IEEE 14 bus system.

3.3.2. Different weights according to the type of measurement:

6 cases are tested for 3 bus and 14 bus systems; results are shown below.

3.3.2.1. Simulation results for 3 bus system

Table.11: Estimated value of voltage magnitude with different weights according to measurements type

Bus ID	True Value(P,U)	Measurements Variance [V, Power injection, Power flow]					
		[0.1, e-6, e-6]	[e-6, 0.1, e-6]	[e-6, e-6, 0.1]	[0.1, 0.1, e-6]	[0.1, e-6, 0.1]	[e-6,0.1,0.1]
1	1	1,000117	1	1	1,000004	1,000001	1
2	0,8898	0,889919	0,889796	0,889796	0,889801	0,889797	0,889796
3	1,05	1,050116	1,050006	1,050006	1,05001	1,050007	1,050006
MAPEV (%)		0,0121	0,0003	0,0004	0,0005	0,0004	0,0003
Iterations		6	5	5	5	5	5

Table.12:Estimated value of voltage angle with different weights according to measurements type

Bus ID	True Value (degree)	Measurements Variance [V, Power injection, Power flow]					
		[0.1, e-6, e-6]	[e-6, 0.1, e-6]	[e-6, e-6, 0.1]	[0.1, 0.1, e-6]	[0.1, e-6, 0.1]	[e-6,0.1,0.1]
1	0	0	0	0	0	0	0
2	-13,3116	-13,307962	-13,311422	-13,311513	-13,311301	-13,311492	-13,311507
3	-4,238	-4,237056	-4,237997	-4,237947	-4,237964	-4,237942	-4,237961
MAPEθ (%)		0,0223	0,0001	0,0013	0,0008	0,0014	0,0009
Iterations		6	5	5	5	5	5

In the first case, voltage magnitude measurement has the lowest weight compared with power injection and power flow measurements. As seen in the tables 11 and 12, this weight combination requires a higher number of iterations for convergence and yields the biggest deviation from the true value for both states: voltage magnitude and voltage angle. On the other hand, when voltage measurement has a greater weight, the results are better.

3.3.2.2. Simulation results for IEEE 14 bus system:

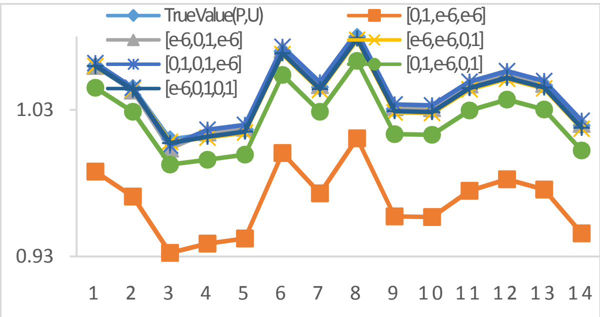

Fig.8: State estimation of voltage magnitude with different weights according to measurements type

In those figures 8 and 9, it is noticed that the simulations result of the first case are far from the true values (MAPEV=7% and MAPE θ= 16%). Also, the convergence of that case requires a higher number of iterations: 11 counter only 5 iterations for the other cases. This joins the results of 3 bus system simulations.

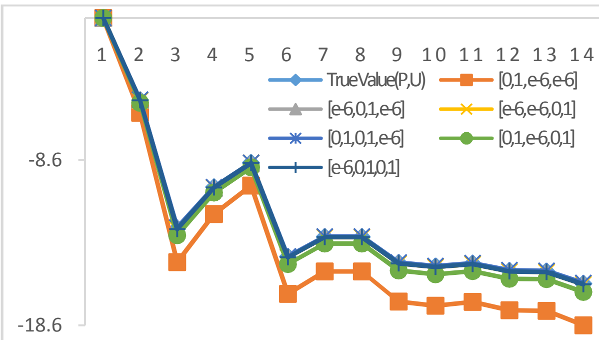

Fig.9: State estimation of voltage angle with different weights according to measurements type

We conclude that voltage magnitude error produce a large deviation on the state estimation. So, it would be interesting to choose voltage measurements with small errors and to minimize noise disturbance which could affect measurements' quality for this type.

3.4. Effect of level of noise:

In this point we will study the effect of noise on the accuracy of WLS state estimation. Two simulation cases are tested: one assuming the same level of noise for all measurements type and the other by changing the level of noise according to the type of measurement.

3.4.1. Same level of noise for all measurements:

3.4.1.1. Simulation results for 3 bus system:

Table.13: Estimated value of voltage magnitude with the same level of noise for all measurements

Bus ID	True Value (P,U)	Without noise	1% noise	3% noise	6% noise
1	1	1,000001	1,009977	1,029931	1,059867
2	0,8898	0,889797	0,899511	0,91896	0,948179
3	1,05	1,050007	1,059945	1,079826	1,109657
MAPEV(%)		0,0004	1,0121	3,037	6,0764
Iterations		5	5	5	5

Table.14: Estimated value of voltage angle with the samelevel of noise for all measurements

Bus ID	True Value (degree)	Without noise	1% noise	3% noise	6% noise
1	0	0	0	0	0
2	-13,3116	-13,311	-13,1620	-12,8726	-12,4613
3	-4,238	-4,2379	-4,1997	-4,1252	-4,0182
MAPEθ (%)		0,001	0,9036	2,6614	5,1857
Iterations		5	5	5	5

The accuracy of the state estimation changes proportionally to the level of noise applied to the measurements.

3.4.1.2. Simulation results for IEEE 14 bus system:

As seen in figures 10 and 11, the more the level of noise applied to measurements is important (6%), the more the deviation between estimation results and the true values is bigger (MAPE V=4%).

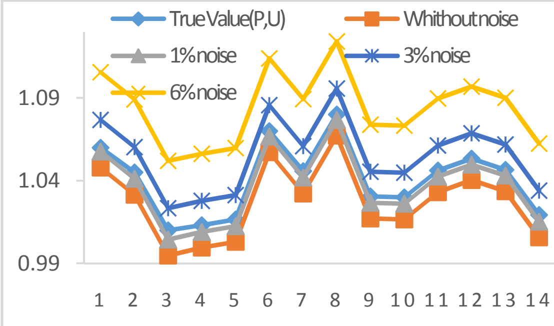

Fig.10: Estimated value of voltage magnitude with the same level of noise for all measurements

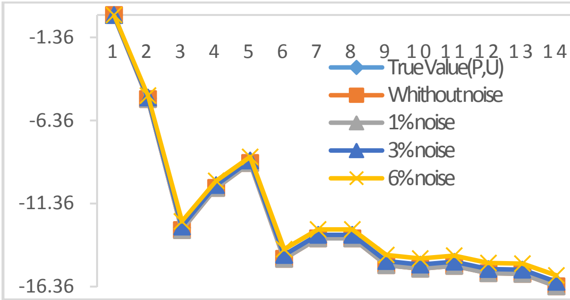

Fig.11: Estimated value of voltage angle with the same level of noise for all measurements

3.4.2. Different level of noise according to the measurements type:

6 different cases are studied, to assess the effect of noise applied to the various measurement types with different levels on the estimation quality.

3.4.2.1. Simulation results for 3 bus system:

Table.15: WLS state estimation of voltage magnitude with different level of noise for measurements according to their type

Bus ID	True Value (P,U)	%Noise [V, Power injections, Power flow]					
		[6%, 1%, 1%]	[1%, 6%, 1%]	[1%, 1%, 6%]	[6%, 6%, 1%]	[6%, 1%, 6%]	[1%,6%,6%]
1	1	1,059797	1,012466	1,007988	1,061857	1,058127	1,01011
2	0,8898	0,952286	0,900441	0,894964	0,952605	0,948169	0,895545
3	1,05	1,107075	1,062506	1,060556	1,109166	1,107887	1,062749
MAPEV (%)		6,146	1,2112	0,7948	6,2929	5,9619	0,957
Iterations		4	5	5	5	5	5

Table.16:WLS State estimation of voltage magnitude with different level of noise for measurements according to their type

Bus ID	True value (degree)	%Noise [V, Power injections, Power flow]]					
		[6%, 1%, 1%]	[1%, 6%, 1%]	[1%, 1%, 6%]	[6%, 6%, 1%]	[6%, 1%, 6%]	[1%,6%,6%]
1	0	0	0	0	0	0	0
2	-13,3116	-11,800027	-13,743553	-13,307602	-12,332684	-11,916925	-13,904794
3	-4,238	-3,843578	-4,24089	-4,344744	-3,88559	-3,974423	-4,388224
MAPEθ (%)		9,3068	0,0682	2,5187	8,3155	6,2194	3,5447
Iterations		4	5	5	5	5	5

As noticed in tables 12 and 13, for cases (1,4 and 5) voltage measurements are affected with high level of noise (6%) compared with other types, therefore the deviation from the true value is important for both states (voltage magnitude and voltage angle). On the other hand, for cases (2,3 and 6), where voltage measurements are the less affected by noise, the accuracy of estimation is better.

3.4.2.2. Simulation results for IEEE 14 bus system:

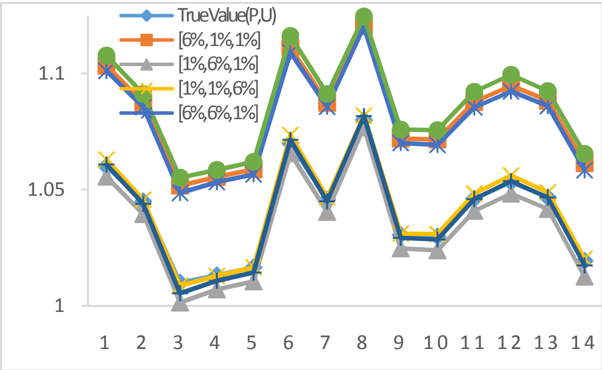

Fig 12: WLS state estimation of voltage magnitude with different level of noise according to measurements type

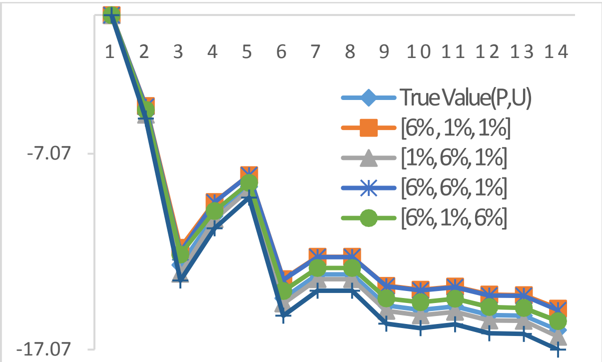

Fig 13: WLS state estimation of voltage angle with different level of noise according to measurements type

For voltage magnitude, the difference between cases is more pronounced than voltage angle's estimation. The results are similar to those obtained in the case of assigning different weights to measurements by type. In fact, when voltage measurement is affected by a higher level of noise 6% (Cases: 1,4 and 5), the error become more important for both states MAPEV and MAPEθ. On the other hand, with a lower noise 1%(cases 2,3 and 6) the error is small.

We conclude that the two factors (weight and noise) must be combined to obtain the best estimation. Indeed, the most accurate measurements (lower level of noise) should have greater weight compared to bad measurements (higher level of noise).

IV. CONCLUSION

This paper describes Weighted Least Squares state estimation method, investigates its characteristics and observes the effect of 4 parameters (Number of measurements, measurement type, measurement weight and level of noise) on the quality of state estimation.

The simulations show that increased redundancy improves the accuracy of the estimation, but the effect is not uniform. In fact, satisfying solution may be obtained without redundancy η=1, on the other hand the system may be unobservable even with high degree of redundancy which means that state estimation is affected by other correlated factors as measurement type, measurement location, measurement error…

The results show also the importance of voltage measurements compared with the other types: therefore, their presence is indispensable for an efficient execution of WLS state estimation program without problems and they should be accurate as possible because voltage measurement error produces a large deviation in final results.

The study of the effect of measurement weights and noise, depicts that those factors must be combined to obtain the best estimation. Indeed, the most accurate measurements (lower level of noise) should have greater weight compared to bad measurements (higher level of noise).

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