



*The Development of Yield Management Model to Optimize The Selling Income on Multi-Leg Flight by Considering Distance and Operational Cost*

**Pengembangan Model Manajemen Untuk Optimasi Pendapatan Penjualan pada Penerbangan Multi-Leg dengan Mempertimbangkan Jarak Tempuh dan Biaya Operasional**

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**ABSTRACT / ABSTRAK**

*This study is aimed at applying yield management to optimize the airlines selling income by determining or allocating the number of passengers in multi-leg flight in which each leg has two fares classes. Basically, yield management is one of allocating and controlling passenger's seats methods to optimize the airlines income. Normally, the existing yield management model will only consider the fare, demands and capacity variables. Other variables such as operational and distance will not be covered. When those variables are covered, they are considered as constant variables that will not affect the income optimization. The development of yield management model in this study is applying the linear program model approach, in order to obtain an optimum solution in the multi-leg flight. The approach will produce a decision variable; i.e. the number of passengers that will be allocated in each leg for business and economy fares; and objective function; i.e. maximum income and multi-leg flight route being served.*

*The result of the study is a smaller solution, both for the decision variable and objective function. However, the solution is more valid because it considers more aspects, which are ideally more suitable to the actual condition of the flight operational.*

Penelitian ini bertujuan mengaplikasikan model *yield management* untuk mengoptimasikan pendapatan penjualan perusahaan penerbangan dengan cara pengalokasian jumlah penumpang pada penerbangan multileg dimana masing-masing leg mempunyai dua jenis kelas tarif. Pada dasarnya, *yield management* adalah metode pengalokasian dan pengendalian kursi penumpang untuk mengoptimasikan pendapatan perusahaan penerbangan.. Pengembangan model *yield management* dalam studi ini menggunakan pendekatan model program linier, untuk mendapatkan solusi optimum dalam penerbangan multileg. Pendekatan ini akan menghasilkan suatu variabel keputusan yaitu jumlah penumpang yang akan dialokasikan pada setiap leg untuk harga tiket bisnis dan ekonomi; dan fungsi tujuannya adalah memaksimalkan pendapatan dan rute penerbangan multileg yang dilayani. Hasil penelitian menunjukkan bahwa solusi yang lebih kecil, baik untuk variabel keputusan maupun untuk fungsi tujuan. Meskipun demikian, solusi ini lebih valid karena mempertimbangkan banyak aspek, dimana idealnya lebih sesuai dengan kondisi aktual dari operasi penerbangan.

## Introduction

One of the passengers' seats controlling and allocating strategies that can be used to maximize income is *yield management* methodology, which also known as *revenue management*. It is a system, technique or method designed to maximize selling of the passengers' seats on an optimum price and airlines incomes [Chen, et al., 1991].

The *yield management* model is first introduced by Littlewood in 1972 [Chen, et al., 1999], to allocate the capacity of a *single leg* flight with *two fare classes* analysis model, i.e. *discounted fare* and *full fare*, by assuming that the discounted fare is booked first. Littlewood found a technique to set *booking limit*, the number of discounted fare tickets being sold. Littlewood shows that there is a trade off between (a) income and discounted fare sales toward (b) decrease income expectation and full fare sales, thus, the seats will be sold under the discounted fare [McGill & Ryzin, 1999]. The result of the study is named Littlewood regulations, i.e. consumers with discounted fare will be accepted when the income from the discounted fare passengers is more than the marginal income expectation from the full fare sales which is formulated as follows:

$$R_{discount} > R_{full} \times Prob(Full\_fare\_demand > Remaining\_seat\_number)$$

In 1988, Belobaba applied a probability decision mode on yield management, to solve the multiple-fare class single leg problem. Belobaba [1988] used a heuristic procedure to

manage the booking limits for various types of lowest fare class. The procedure is a development of the Littlewood method which is known as EMSR (the expected marginal seat revenue).

The yield management technique is continuously developed. In 1993 Brummele & McGill developed a passengers' seats allocation on single leg flight by attracting the consumers in sequence with low and high fares and making an optimum condition set. Brummele & McGill [1993] showed that the fixed sales limit policy which maximizing the income expectation can be classified by applying a condition set on sub-differential and income expectation function. The condition set is related to the demand probability distribution on varied fare class. Using the EMSR method will reveal the calculation of optimum protection for full fare class.

Talluri & Ryzin [1988] suggested a method by inserting a probability demand into the DMP (deterministic mathematical programming) model on multi-leg cases and called it RLP (randomized linier programming). Tallun & Ryzin showed that the bid prices which are based on the DMP model will be best applied when the seats capacity in each leg and flight network is almost unlimited. They proved that when leg capacity and sales volume are extremely high, the bid price control will be optimum. Tallun & Ryzin simulated a sequence of demand realization for each itinerary and fare class in the flight network.

Chen, Gunther, and Johnson [1999] developed a new approach on yield management issue by using a linear program to determine the function value of and the Markov decision issue. Netessine & Shumsky [2000] conducted a study on passengers' seats allocation by considering competition factors. Bitran & Caldentey [2002] formulated a yield management problem with stochastic model, whereas Cooper & Gupta [2003] considered the Markov decision process model on yield management issue.

Thus, out of the conducted studies, especially McGill & Ryzin [1999] who conducted an observation on 190 study results on yield management issue, it can be concluded that the yield management is used to maximize income by considering variables including ticket demands, tickets fares/prices, leg, capacity, and airlines competition factors, whereas the distance and operational cost variables are rarely discussed. When they are discussed, the result is an assumption that the distance and operational cost

will not affect the income optimization with yield management model

Since the objective of airlines operation is to get a maximum income, the study is aimed at the development of yield management model by inserting distance and operational cost variables to optimize the airlines income during passengers seats allocation.

## Literatur Review

### Yield Management Model

The term yield management or revenue management has been widely used in service industries to describe limited resources allocation, such as passengers seats in airlines industry, rooms in hotel industry, cars in car rental agencies, etc.. The purpose of yield management [McGill & Ryzin, 1999] is to maximize profit by selling passengers seats on an optimum price/fare. By applying yield management each flight is expected to be full and each passenger pays with a maximum price on each affordable class.

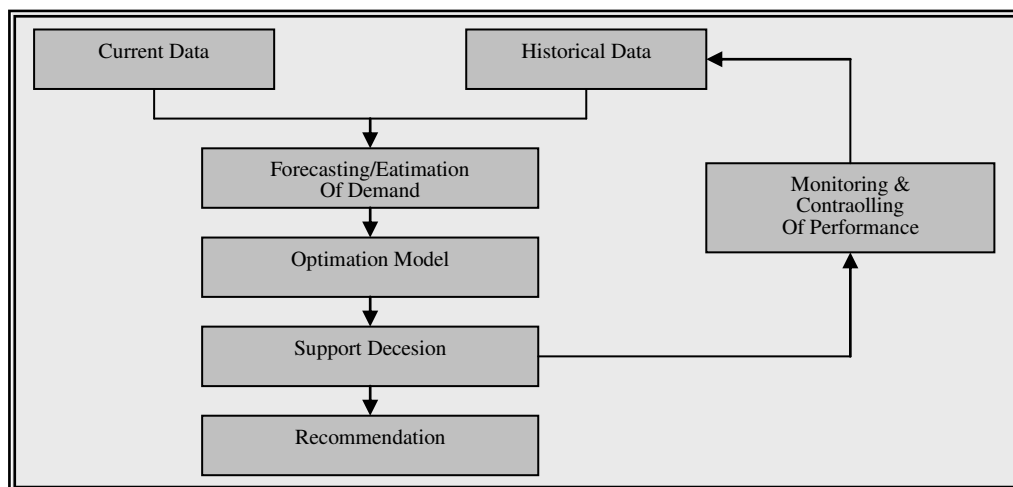


Figure 1. The Standard Yield Management Model [Daudel, 1994]

Applying the yield management model will optimize income by considering the fare and demand variables on each leg as well as the plane capacity that is used to serve certain route/leg obtained.

This method can be conducted by using the Daudel systematic phases model [1994] (Figure 1). It aims at meeting the number of passengers (demands) in each type of fare class in each leg that will provide a maximum income to the airlines company.

When an airplane flies to transport goods from one place to another, it Inserting the variables to the yield management model will provide the income value. Thus, the variables will be converted into unit cost per passenger. In the study, the modified yield management model can be conducted by inserting the distance and operational cost variables as in Figure 2.

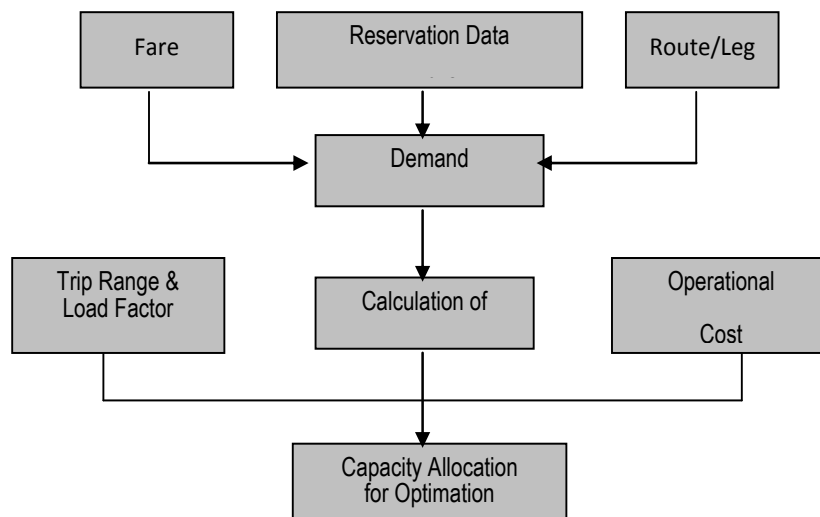


Figure 2. Modified Yield Management Model

When an airplane flies to transport goods from one place to another, it needs fuel. The number of used fuel is inversely proportioned to the distance, whereas the number of transported cargo is equivalent to the distance [Nugraha, 2003]. The fuel and travel time variables will affect the number of required cost. In general, the cost is called airplane total operational cost which consists of two cost categories, i.e. direct and indirect operational cost.

In order to optimize income by applying modified yield management model, the steps taken are almost the same as the general yield management model. However, it differs in its passengers' seats allocation in each leg for each offered fare class. The total optimum income value will be obtained by considering the distance and operational cost which is converted in costs, i.e. unit cost per passenger.

Based on the conceptual model as well as the problem solving model diagram, the objective function of the developed model is to maximize the airlines profit or income of the airlines companies, which serve the route which consists of several legs (multi-leg).

The income optimizing phases of the yield management model that will be conducted in this study will be categorized into two, i.e. the standard yield management model (YMStd) and the modified yield management (YMod). The different between both procedures that the considered additional variables. The conducted optimizing procedure is started by making preliminary demand model for each leg and fare class is based on the historical reservation data to determine the fare of each flight leg, the model of unit cost per passenger, make passengers' seats allocation model, income model, and income/ profit optimization model.

### (1) Booking Estimation Model

The aim of the formulation of booking estimation model is to find out the number of demands in each fare class in each flight leg in the future. For this research, the mathematical model of booking estimation will be conducted by considering the demand characteristics which are probable and determine. The probability demand is used during the determination of the protection level on the fill fare, whereas the determination and fixed demand will be used in allocating the passengers' seats allocation when determining the booking limit in the discount fare.

In the probability demand, if there is a random  $X$  variable in a certain leg which is continuously observed, the first probability distribution calculation is its relative function (probability mass function) which is a fraction between the number of demands on the  $t$  observation period. Then, its probability distribution can be calculated/stipulated with a cumulative distribution function which is an accumulation of relative frequency up to the  $t$  period.

If demands is  $X = (X_1, X_2, \dots, X_n)$  and its relative frequency is

$$P(X) = \sum_{X_i \leq X} P(X_i), X \geq 0. \quad [1]$$

$P(X)$  is the mass probability function on the observation period. Thus, the  $X$  probability distribution is the cumulative distribution function, i.e.  $P(X) = \sum_{X_i \leq X} P(X_i)$ ,  $X \geq 0$ .

### (2) Booking Limits Model

Fill and discounted fare demands on between legs are free. Thus, sales limitation is applied to both fares to determine the booking limit. The limitation for fill fare can be done by calculating the protection level, whereas for discounted fare can be done by calculating the booking limit.

To determine the number of seats allocated for the discounted fare and protected seats for full fare, the used formula will be different for the single leg as well as the multi leg. The formula for the single leg flight

can also be used for the multi leg in addition to the between leg allocation formula.

On the multi leg flight, each demand for each ODF should be immediately be decided whether to accept or refuse the tickets sales demands. If  $\phi_i$  is the  $D_i$  cummulatibe distribution function and  $X_i$  is the number of seats with a fare class of  $i = 1$  or  $2$ , in which  $X_2 = C - X_1$  and  $F_F > F_D$ , then the protection level of the I class should meet:  $\frac{F_D}{F_F} = P(D_i > X_i^{opt})$  [2]

### 3) Capacity Allocation Model

The determination of the seats allocation in this research is done with the no control model. In the no control model, there should not be any passengers' seats allocation on each fare class. In addition, the consumer/demands will be accepterd based on the First Come First Served principle so that the whole plane capacity is filled. However, on the case study that will be conducted for the PT. Garuda Indonesia with its Boeing737-400, the passengers seats allocation is done based on the cabin distribution.

Assuming that the passengers' seats sales are determined based on the two types of fare classes or com-partment configuration, i.e. C compartment with FF fare and Y compartment with FD fare, then the logarithm for the compartment allocation with the no control model which is based on the First Come First Served principle is described in Figure 3.

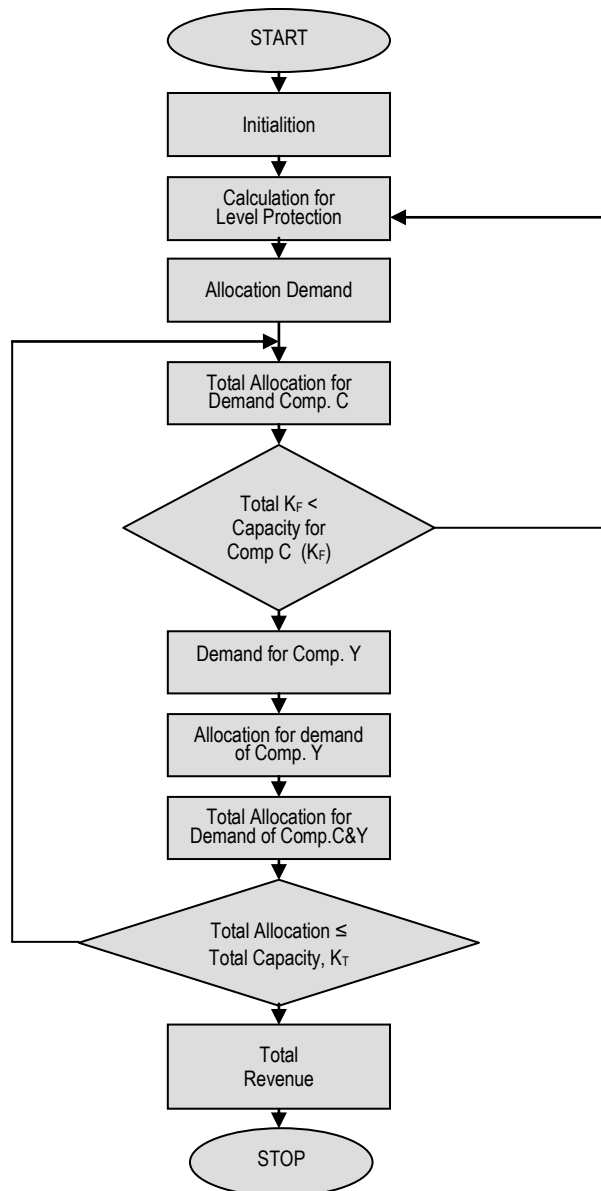
The algorithm in Figure 3, will be used for the standard yield manage-ment model, whereas the modified yield management model will be add with the distance and other operational cost

variables which are converted in the unit cost per passenger.

The algorithm steps that will be conducted covers two phases which consists of protection level calculation (simulation with Pascal programming language is applied for this study), whereas the second phase i.e. the compartment allocation for the economy fare, mathematical programming, i.e. linear program is applied.

The conducted algorithm steps are as follow:

1. Input parameter initialization for the whole served leg and route; including: capacity of plane (K), compartment business class ( $K_F$ ), fares (economy and fill fares), number of legs, and demands pattern.
2. Randomly generating demands for the fill fare for the whole leg.
3. Whole legs protection level calculation with  $K_F$  compart-ment capacity limit.
4. Fare class allocation by considering the shadow price of each leg; in which when the shadow price  $>$  the fare for the alternative leg, then the booking demand will be accepted, and when the shadow price  $<$  the fare, then the booking demands will be declined.
5. Checking the total capacity on the allocation result; if the total capacity has not temporarily full and the demands are still coming, then return to step 4; if the total of the demands allocation (the fill + economy fare demands allocation)



> the plane capacity, then the allocation will seized.

- The total income calculation, i.e. the total income of the fill + economy fares. The fill fare income = the number of demands of the fill fare times the fill fare value.

By using the linear program for the YM standard model, the steps of the capacity allocation data processing are conducted on the algorithm 4, after the sales

protection level is obtained when logarithm 3 is applied. After the sales protection level is obtained, the mathematical model is made as an input for the linear program method, i.e. determining the decision variables, value and objective function variable, and constraint, both the functional and the non-negative constraints.

**a. Identification of Booking Protection level**

Based on the said algorithm, the allocation is conducted based on the compartment, and then the booking limit will be first calculated for C compartment class, i.e. by calculating the protection level. The protection level will be calculated using equation (1) and equation (2), which the protection level and the historical data of demand will be obtained from fare ratio.

From the equation (1), i.e.  $P(X) = \sum_{X_i \leq X} p(X_i)$ , with  $X > 0$  and equation (2), i.e.

$$\frac{F_D}{F_F} = P(D1 > X_i^{opt}), \text{ the protection level}$$

for C compartment, i.e.  $X^*$  can be obtained by using the following equation:

$$P(X) \leq \frac{F_D}{F_F}.$$

**b. Determining the decision variable**

After the protection level of booking has been decided, the calculation is continued by allocating the passenger seat capacity by using linear programming. In this case, the capacity allocation is the selection of decision variable values in order to obtain the optimum yield.

The decision variable in this research is the number of booking in each leg for every fare class (X), so that the number

of decision variable is the combination of a set of the point of arrival-departure (number of legs) and multiplied by the number of fare classes. If the number of nodes is n of its pair, the number of legs/ segments is  $C_r^n$ . Because there is f types of fare classes, the number of decision variable is the number of the combination of  $C_r^n$ , leg multiplied by f fare classes. In the case study of this research the flight routes comprise four cities/nodes ( $n = 4$ ), the pairs of cities/nodes are two ( $r = 2$ ), the fare class are two classes ( $f = 2$ ), and then the numbers of decision variable are 12. In other words, the decision variable is X which  $i = 1, 2, 3, \dots, 12$ .

**c. Determining the objective function**

The objective function is a comprehensive measurement of performance. This optimized objective function is the multiplication of the optimum number of the decision variable and the increase value and every decision variable.

In this research, the objective function is the maximized revenue, which is the multiplication of decision variable (the number of bookings) and the decision variable value (the cost of the fare charged for each class in each leg, i.e. the discount fare and the fill fare). If R is the revenue,  $F_f$  is the cost of the fare, and  $X_i$  is the decision variable, the objective function that needs to be obtained in this research is:

$$\text{Maximize } R = \sum_{i=1}^i \sum_{f=1}^f F_f \cdot X_i$$



#### d. Determining the constraints

In this research, the constraints comprise the constraint of plane capacity, constraint of C compartment capacity, limitation of booking toward the plane capacity when the plane is flying over a particular leg, and the constraint of load factor/trip range. The main constraint in this research is the constraint of the plane capacity. If  $K$  is the plane capacity, then the booking limit is  $< K -$  the protection level; or otherwise, the protection level is  $< K -$  the booking limit.

#### Case Study

The case study that will be conducted for the PT. Garuda Indonesia with its Boeing737-400, for multi leg route, i.e. CKG-DPS-TIM-DJJ (Cengkareng-Denpasar-Timika-Djayapura) and CKG-UPG-BIK-DJJ (Cengkareng-UjungPandang-Biak-Djayapura).

#### 1. The Booking Estimation

To establish the optimum allocation of seat number for each fare class in each flight leg, the booking estimation of the same flight based on the fare classes is required. The booking estimation is conducted using the estimation method based on the historical data of booking and the running request of booking, or based on the estimation of the load factor.

The future booking estimation of each flight will be set approximately the average load factor of the passenger for each fare class, toward the capacity of each compartment. The plane capacity is 124 seats, divided according to compartment, i.e. 22 seats for C

compartment (fill fare or  $F_F$ ) and 102 seats for Y compartment (discount fare or  $F_D$ ).

In order to obtain the average value of booking request distribution (the number of passenger) in each leg/segment the data in Picture 4 above and the multiplying factors (% SLF) are used. Thus, the booking average of each leg in both flight routes for the fill fare class (C compartment) and the discount fare class (Y compartment) can be set. The request (load factor) distributions as well as the capacity data are available for each fare class, so that there are 100%, 75%, 50%, and 25% booking estimation from each compartment capacity.

#### 2. The Fare Calculation toward the Trip Range

To obtain a fare cost that can be changed based on the calculation estimation that will be performed, a fare calculation model using the trip range method based on the relation between the trip range and fare will be made. By using a regression model, the  $F_F$  (full fare) and  $J$  (trip range) formulation model is as follows:

$$F_F = 6207,989853 \times J^{0,801275}$$

Based on the above formulation model of the fare toward the trip range, the fare cost of each leg based on the trip range for the CKG-DPS-TIM-DJJ flight route and CKG-UPG-BIK-DJJ flight route with the discount fare for as many as 75%, 50%, and 25% of the full fare are as in Table 2 and Table 2.

### 3. The calculation of the booking protection level

The capacity allocation to accept/refuse the discount fare booking is performed by using the calculation of the booking protection level. The calculation of the booking protection level, to allocate the capacity of the fill fare class, is performed by accumulating the values of the arrival probability, and then the result of the accumulation is compared to the ratio of the discount fare and the fill fare. If the value of the accumulation is bigger (>) than the ratio of the fare, the allocation of the fill fare is provided as many as the number of the booking arrival accumulation.

For the calculation of the booking protection level, the data of the discount fare, the fill fare, and the load factor distribution are required. Then the calculation of the fare and the load factor data, the data that will be used as the input in the calculation of the protection level, will be set in each leg, in each route.

#### The capacity allocation by using the standard YM model

The capacity allocation is divided into two, i.e. the capacity allocation of C compartment seat capacity (the fill fare class) and the capacity allocation of the discount fare class booking (the Y compartment). In this research, the

Table 1. The fare of each leg in CKG-DPS-TIM-DJJ flight

RUTE : CGK-DPS-TIM-DJJ					
Leg	Trip Range (NM)	Fill Fare (Rp)	Skenario : Discount Fare (Rp)		
			75%	50%	25%
CGK-DPS	613	1,062,853	797,140	531,427	265,713
CGK-TIM	2,020	2,763,417	2,072,563	1,381,709	690,854
CGK-DJJ	2,127	2,880,102	2,160,077	1,440,051	720,026
DPS-TIM	202	436,688	327,516	218,344	109,172
DPS-DJJ	1,921	2,654,357	1,990,768	1,327,179	663,589
TIM-DJJ	287	578,615	433,961	289,308	144,654

Source : PT. Garuda Indonesia & Analysis Result

Table 2. The fare of each leg in CKG-UPG-BIK-DJJ flight route.

RUTE : CGK-UPG-BIK-DJJ					
Leg	Trip Range (NM)	Fill Fare (Rp)	Skenario : Discount Fare (Rp)		
			75%	50%	25%
CGK-UPG	828	1,352,371	1,014,278	676,186	338,093
CGK-BIK	2,058	2,804,994	2,103,746	1,402,497	701,249
CGKDJJ	2,127	2,880,102	2,160,077	1,440,051	720,026
UPG-BIK	1,021	1,599,588	1,199,691	799,794	399,897
UPG-DJJ	1,329	1,975,848	1,481,886	987,924	493,962
BIK-DJJ	298	596,391	447,293	298,196	149,098

Source : PT. Garuda Indonesia & Analysis Result

capacity allocation to accept/refuse the booking of the fill fare class (the C compartment) is performed by using the calculation of the booking protection level, whereas for the discount fare class (the Y compartment) it is performed using the no control model with the linier programming.

In calculating the booking protection level, the capacity allocation of the fill fare class is performed by accumulating the values of the arrival probability (the booking of the passenger seats), and then the result of the accumulation is compared to the ratio of the discount fare and the fill fare. For the allocation of the passenger seats of the discount fare class, there is no exclusion (no control) or using the 'first come first serve' (FCFS) principle.

The capacity allocation using the no control model is performed to determine the number of request that will be accepted or refused and the request for each discount fare class, in each flight leg. The determination of allocation is conducted based on the load factor pattern of each fare class in each flight leg, by considering the protection level of the seat booking of the fill fare class that has been previously allocated, in order to maximize the revenue.

The capacity allocation using the no control model is performed to determine the number of request that will be accepted or refused and the request for each discount fare class, in each flight leg. The determination of allocation is conducted based on the load factor pattern of each fare class in each flight leg, by considering the

protection level of the seat booking of the fill fare class that has been previously allocated, in order to maximize the revenue

To calculate the capacity allocation comprehensively, either for the fill fare class or the discount fare class, the data that have been set in the calculation of the booking protection level, added by the data of the calculation result of the obtained booking protection level are required. The setting of the data that will be completed comprises:

- a. The setting of the discounted fare comprises the discounted fares as many as 75% of the full fare, 50% of the full fare, and 25 % of the full fare.
- b. The setting of the load factor (the request) for the discounted fare comprises 80% of the load factor, 70% of the load factor, and 60% of the load factor.
- c. The setting of the load factor (the request) for the fill fare comprises 100% of the capacity.
- d. 75% of the capacity, 50% of the capacity, and 25% of the capacity.

The allocation of the passenger seats in order to obtain the optimal solution will use a method of the linier mathematical programming model. To solve the problem of the linier programming, the problem must be formulated in the form of equation, comprising the decision variable that is the problem whose values must be determined, perfor-mance rating must be stated in the mathematical function known as the objective function, as well as the restrictions known as the constraints.

As an illustration to solve the problem of the capacity allocation using the linier programming for both routes, i.e. CKG-DPS-TIM-DJJ flight route and CKG-UPG-BIK-DJJ flight route, the calculation sequence/formulation for both routes will be identical (since the calculation uses the same scenario data), the difference occurs as the objective function formulation based on the fare is different. The sequence/formulation of the calculation of the capacity allocation for the scenario 1 is as follows:

**Decision variable**

Decision variable that will be decided is the number of booking for each leg in each fare class. In the case study of this research the flight route comprises four cities/nodes, so that there are six legs/segments in that flight route (Figure 4), whereas there are two fare classes for each leg, so that there are 12 decision variables (Figure 4), i.e.:  $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}$ .

Maksimasi

$$R = F_{F_{1-2}} X_1 + F_{D_{1-2}} X_2 + F_{F_{1-3}} X_3 + F_{D_{1-3}} X_4 + F_{F_{1-4}} X_5 + F_{D_{1-4}} X_6 + F_{F_{2-3}} X_7 + F_{D_{2-3}} X_8 + F_{F_{2-4}} X_9 + F_{D_{2-4}} X_{10} + F_{F_{3-4}} X_{11} + F_{D_{3-4}} X_{12} \quad (6)$$

**Determining the objective function**

The objective function R that will be optimized is to maximize the revenue for flight that serves a route comprising four cities, which the amount of fare of one leg, from one point of departure to one point of arrival, is varied, based on some matters especially the trip range of the plane.

The formulation of the objective function to solve the problems of a.

From the fare calculation, the formulation of the objective function for both routes that has scenario data 1, i.e. 75% discount fare, 80% load factor of discount fare, and 100% load factor of fill fare, is as follows:

Route : CKG-DPS-TIM-DJJ

Maximize

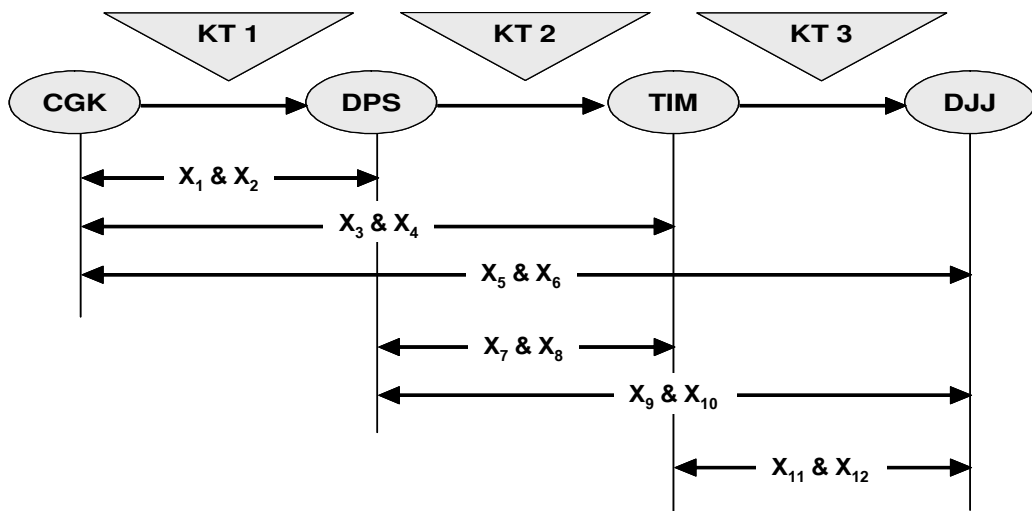


Figure 4 : Decision Variable in CKG-UPG-BIK-DJJ flight route.

$$R = 1062853X_1 + 791140X_2 + 2763413X_3 + 2072563X_4 + 2880102X_5 + 2160077X_6 + 436688X_7 + 327516X_8 + 2654357X_9 + 1990768X_{10} + 578615X_{11} + 433961X_{12} \quad (7)$$

Route : CKG-UPG-B1K-DJJ

Maximize :

$$R = 1352371X_1 + 1014278X_2 + 2804994X_3 + 2103346X_4 + 2880102X_5 + 2160077X_6 + 1599588X_7 + 2199691X_8 + 1975848X_9 + 1481886X_{10} + 596391X_{11} + 447293X_{12} \quad (8)$$

### Determining the constraints

Based on the route in which there is a probability that the plane will load or unload passengers, the constraints that should be estimated in this research comprise the constraint of plane capacity, constraint of C compartment capacity, constraint of load factor of fill fare, and constraint of load factor of discount fare.

Constraints of *seat factor* :

1.  $X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \leq 124$
2.  $X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} - X_1 - X_2 \leq 124$
3.  $X_5 + X_6 + X_9 + X_{10} + X_{11} + X_{12} - X_1 - X_2 - X_3 - X_4 - X_7 - X_8 \leq 124$

Constraints of C Compartement :

1.  $X_1 + X_3 + X_5 \leq 22$
2.  $X_3 + X_5 + X_7 + X_9 - X_1 \leq 22$
3.  $X_5 + X_9 + X_{11} - X_1 - X_3 - X_7 \leq 22$

Constraints of demands for fare class (level protection) :

$$X_1 \leq 13; X_3 \leq 14; X_5 \leq 14; X_7 \leq 12; X_9 \leq 15; X_{11} \leq 14$$

Constraint of demands for discount fare (based on *load factor*):

1.  $X_i \leq 82$ , untuk *load factor* = 80%, dimana  $i = 2, 4, 6, 8, 10$ , dan 12
2.  $X_i \leq 72$ , untuk *load factor* = 70%, dimana  $i = 2, 4, 6, 8, 10$ , dan 12
3.  $X_i \leq 62$ , untuk *load factor* = 60%, dimana  $i = 2, 4, 6, 8, 10$ , dan 12

Constraints for non negative :

$$X_i \geq 0, \text{ dimana } i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \text{ dan } 12$$

### The capacity allocation using the modified YM model

In the capacity allocation of the modified YM model, the algorithm of the capacity allocation is similarly conducted as the calculation of the capacity allocation of the standard YM model.

The capacity allocation of the modified YM model will calculate the variables of trip range and operational cost represented to the unit cost per passenger/flight. The difference between these two models is on the value of the objective function that not only calculates the fares, but also calculates the operational costs per unit of passenger for each leg and considers the constraints comprising the additional capacity as well as fuel.

In order to obtain the unit cost per passenger required in the capacity allocation of the modified YM model, the calculation and the formation of the model comprising the model of passenger fare regarding the trip range,

the model of load factor regarding the trip range, the model of trip time and trip fuel regarding the trip range, as well as the estimation of plane operational cost based on the components of variable cost and fixed cost are conducted.

In transporting the load from one point of departure to one point of arrival, the plane needs trip fuel during the flight period. In other words, the trip fuel affects the cost spent to transport that load.

If the amount of the load is smaller than the maximum capacity of the plane, the amount of trip fuel needed by the plane to cover the same trip range will be smaller than when the plane is fully loaded. By assuming that the weight of a passenger and his/her baggage is 90 kg, and by the fact that the maximum payload  $W_{p0} = 15.772$  kg,

carried by the plane can be calculated by using the equation, so that the correction factor of the trip fuel can be stated as:

$$W_{p0} = K \cdot W_{pb} + W_{pF}, \text{ so}$$

$$W_{pF} = W_{p0} - K \cdot W_{pb}$$

$$W_{pF} = 15.772 - 124 \cdot 90 = 4.612 \text{ kg}$$

and for the model formation and selection, the equation of the relationship between the load and the trip range is:

$$k_f = 9,24009 \cdot 10^{-9} \times (35,488 + 11,160LF_p + 4612 \cdot LF_b)^{1,70941}$$

$$W_p = 22.346,33634 - 5,33965 \cdot J_{Max}$$

Since  $W_{p0} = 15.772$  kg, the relation-ship between the load and the maximum trip range and vice versa is as follows (12) and (13).

$$J_{Max} = \begin{cases} 2.586,539 - 0,0422 \cdot W_p & \text{if } W_p < 11.017,759 \text{ kg} \\ 4.184,981 - 0,1873 \cdot W_p & \text{if } (11.017,759 < W_p < 15.772) \text{ kg} \\ 1.231,230 & \text{if } W_p = 15.772 \text{ kg} \end{cases} \quad (12)$$

$$W_{Max} = \begin{cases} 15.772 \text{ kg} & \text{if } J_{Max} < 1.231,230 \text{ NM} \\ 22.346 - 5.340 \cdot J_{Max} & \text{if } (1.231,230 < J_{Max} < 2.121,596) \text{ NM} \\ 61.293,145 - 23,697 \cdot J_{Max} & \text{if } (2,121,596 < J_{Max} < 2.586,539) \text{ NM} \end{cases} \quad (13)$$

Tabel 3. Variable Cost and Multiplying Load Factor

Component of Variable Cost	Total (Rp/trip)	Multiplying Load Factor
Pax Comisión	29.518	LF x K
Catering	10.509	LF x K
On Borrada Service	17.299	LF x K
Reservation	12.769	LF x K
Fuel	3.166	Muatan (kg)

Source : PT. Garuda Indonesia & Analysis Result

and the seat capacity = 124 seats, the maximum weight of cargo that can be

In order to be able to calculate the plane operational cost, the operational cost component based on variable cost

and fixed cost should be recognized. Since the cost component will be calculated mainly on the variable cost in the further calculation, the variable costs either affected by the carried load (the number of passengers), or variable cost and trip time estimation are as follow:

By using the above calculation model, the extents that will be used as the new constraints or objective function values are obtained for CKG-DPS-TIM-DJJ route and CKG-UPG-BIK-DJJ route. After putting the parameter of both routes into the above calculation model, the algorithm calculation of the capacity allocation is done by the calculation sequence/formulation as follows:

### Decision variable

Decision variable that will be decided is the number of booking for each leg in each fare class. The number of decision variables for both YM model is 12 for each model, i.e.:  $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11},$  and  $X_{12}$ .

### Determining the objective function

The objective function R that will be optimized is to maximize the revenue. The objective function R that will be optimized is to maximize the revenue for flight that serves a route comprising four cities, which the amount of fare of one leg, from one point of departure to one point of arrival, is varied, based on some matters especially the trip range of the plane.

Tabel 4. Fare and Operational Cost Component

Component	L E G						
	1 - 2	1 - 3	1 - 4	2 - 3	2 - 4	3 - 4	
<b>CGK-DPS-TIM-DJJ Route</b>							
Business Fare	1,062,853	2,763,417	2,880,102	436,688	2,654,357	578,615	
	0.75	797,140	2,072,563	2,160,077	327,516	1,990,768	433,961
Economic Fare	0.50	531,427	1,381,709	1,440,051	218,344	1,327,179	289,308
	0.25	265,713	690,854	720,026	109,172	663,589	144,654
<b>CGK-UPG-BIK-DJJ Route</b>							
Business Fare	1,352,371	2,804,994	2,880,102	1,599,588	1,975,848	596,391	
	0.75	1,014,278	2,103,746	2,160,077	1,199,691	1,481,886	447,293
Economic Fare	0.5	676,186	1,402,497	1,440,051	799,794	987,924	298,196
	0.25	338,093	701,249	720,026	399,897	493,962	149,098
<b>Operational Cost Component</b>							
Catering Cost	10,509	10,509	21,018	31,527	10,509	21,018	10,509
Other Variabel Cost	59,586	59,586	59,586	59,586	59,586	59,586	59,586
Fuel Cost	3,166	284,940	284,940	289,940	284,940	289,940	284,940

Source : PT. Garuda Indonesia & Analysis Result

$$\text{Maximize } R = \sum (\text{Fare} - \text{Operational\_Cost}) \cdot X_i \quad (14)$$

or Maximize

$$\begin{aligned} R = & (F_{F_{1-2}} - B_{Op1})X_1 + (F_{D_{1-2}} - B_{Op2})X_2 + (F_{F_{1-3}} - B_{Op3})X_3 \\ & + (F_{D_{1-3}} - B_{Op4})X_4 + (F_{F_{1-4}} - B_{Op5})X_5 + (F_{D_{1-4}} - B_{Op6})X_6 \\ & + (F_{F_{2-3}} - B_{Op7})X_7 + (F_{D_{2-3}} - B_{Op8})X_8 + F_{F_{2-3}} X_7 \\ & + F_{D_{2-3}} X_8 + F_{F_{2-4}} X_9 + F_{D_{2-4}} X_{10} + F_{F_{3-4}} X_{11} + F_{D_{3-4}} X_{12} \quad (15) \end{aligned}$$

When the trip range variable, especially the operational cost variable, increases, the objective function formulation will also change. A change from only considering the fare factor for each leg in each class to considering the operational cost per passenger unit, comprising the cost of pax commission, catering, reservation, on board services, and fuel trip usage, is occurred.

Those costs will be grouped based on the frequency of those costs charged to each passenger, i.e. the costs charged only on the ticket purchasing such as the costs of pax commission, reservation, on board services and the costs charged for several times according to the number of legs passed through by the passenger to get to his/her destination such as the catering cost. The cost for trip fuel will depend on the number of payload (the passenger and his/her baggage). From the above formula the objective function value can be formulated, by decreasing every variable cost on the applied fare, as follows.

From the calculation result using the data of fare and operational cost for each leg, the objective function value for CKG-DPS-TIM-DJJ route or the

objective function value for CKG-UPG-BIK-DJJ route can be obtained. By putting that objective function value into the formula, the objective function formulation for both routes by using the data scenario can result in 75% discounted fare.

#### Determining the Constraints

The constraints will change with the increase in the trip range and payload variables, so that the constraints for the modified YM model having a data scenario in which there is 75% discounted fare, will be the same as standard YM model comprising, the constraints of the plane capacity (seat factor), the constraints of the C compartment, the obstacles/limitations of booking for fill fare class (protection level), and the obstacles/limitations of booking for discount fare class (based on load factor). The difference between these constraint is only on the constraint of the payload toward the trip range that is calculated, by assuming that the capacity of cargo is 20% and the payload weight of the passenger and his/her baggage is 90 kg, the payload per passenger = 108 kg so that the constraint of the payload comprises:



- (i)  $108X1 + 108X2 + 108X3 + 108X4 + 108X5 + 108X6 \leq 15572$
  - (ii)  $108X3 + 108X4 + 108X5 + 108X6 + 108X7 + 108X8 \leq 15572$
  - (iii)  $108X5 + 108X6 + 108X9 + 108X10 + 108X11 + 108X12 \leq 12807^*$
- Note: \*) is a requirement (15)

**The Optimization of Problem Solution**

The solution optimization is conducted in a linier program model by using a computer program, i.e. QS (Quantitative System) Version 3.0. The result (solution) obtained for the standard YM method and the modified YM method for CGK-DPS-TIM-DJJ route and CGK-UPG-BIK-DJJ route is presented in the following table. In the solution table, the fare scenario column contains the discounted fares being set, i.e. 75%, 50%, and 25% of the normal fare. The  $LF_{TEkom}$  column designed in 80%, 70%, and 60% is the scenario of the load factor for the passenger of the discount fare class. The  $LF_{TBsn}$  column designed in 100%, 75%, 50%, and 25% is the scenario of the load factor for the passenger of the fill fare class.

**Analysis and Discussion**

**a. The result and the ratio of the optimized solution**

The results of the optimized solution whether using the standard YM model (YMStandard) or the modified YM model (YMModified), which are presented in the tables below, are the solution ratios obtained by using the standard YM model and the modified YM model, for each route by using 36 scenario data.

The results of the above optimized solution comprise:

- a) The number of all passengers carried in one flight,
- b) The number of the passengers in each segment comprising three segments (determined based on the obstacle of plane and compartment capacity),
- c) The revenue obtained and the allocation result in one flight, and
- d) The average of the revenue per passenger (yield).

The ratio of the result of the optimized solution using Standard YM and Modified YM according to CGK-DPS-TIM-DJJ Scenario. The ratio of the result of the optimized solution using Standard YM and Modified YM according to CGK-UPG-BIK-DJJ Scenario.

**b. The analysis and discussion of the influence of additional variable**

In the formulation of the objective function, there are differences in the value of the objective function in which the modified YM method has calculated the component variable of the operational charged to each passenger (such as the costs of catering, pax commission, on board services, and reservation) as well as the operational cost of the trip fuel charged on each kg of the payload

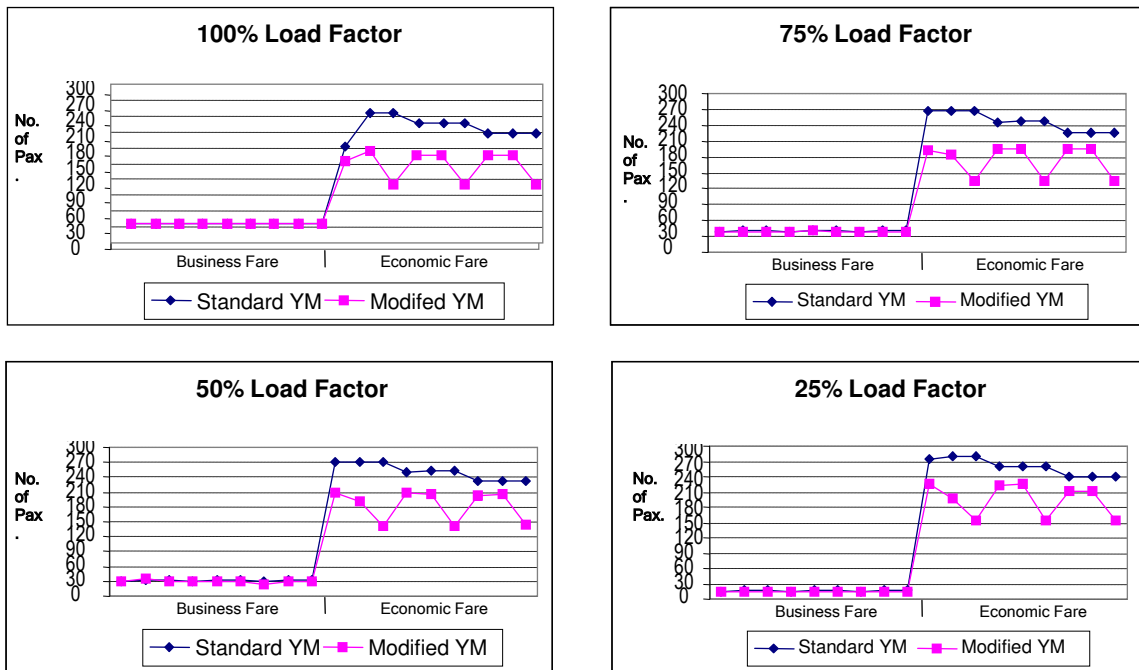


Figure 5. Comparison of optimized solution using Yield Management (Standard vs Modified Model) according to CGK-DPS-TIM-DJJ scenario.

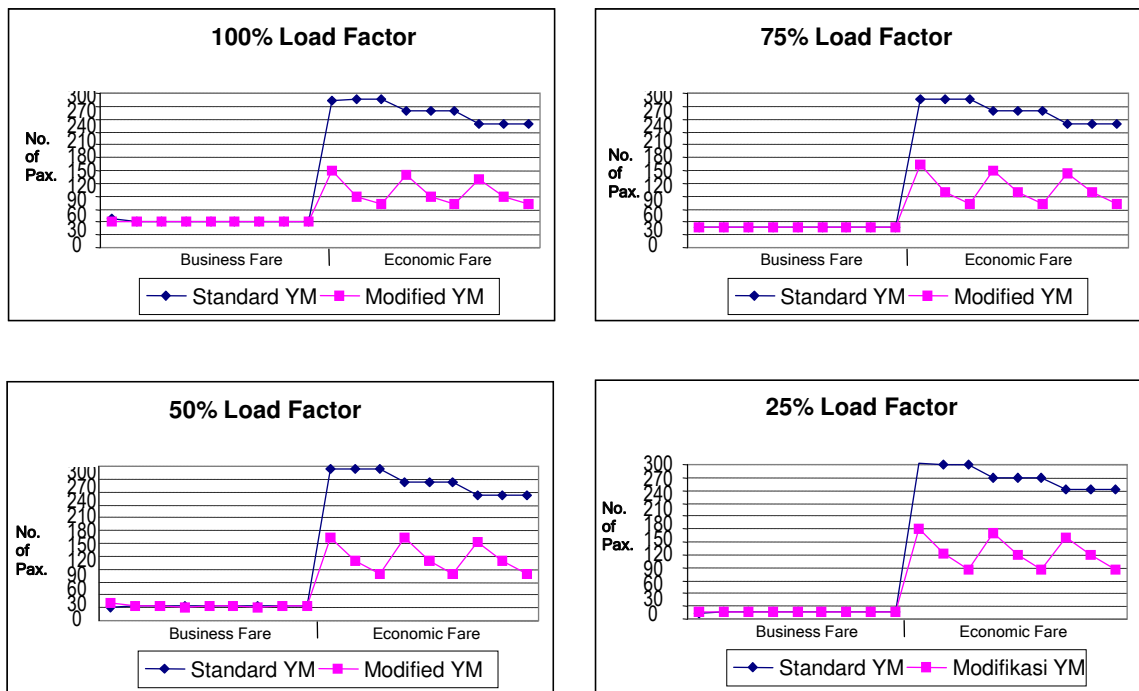


Figure 6 : Comparison of optimized solution using YM Standar Model Vs YM Modifikation according to CGK-UPG-BIK-DJJ scenario.

The influence of these operational costs to the objective function will result in the reduction of the objective function value of the modified YM method, so that the alternative of the decision of the passenger selection based on fare classes that will be served in each leg will change.

In the formulation of the constraints, there is a difference between the standard YM method and the modified YM method due to the increase of the constraint of the payload toward the trip range. The constraint of the payload toward the trip range is determined based on the actual trip range of each leg (point of departure). To analyze the impact of the new additional variable in the standard YM method, the analysis and discussion are conducted by testing the optimized result of the standard YM method toward the limitation of modified YM method.

#### **c. The analysis of the solution result testing**

The solution testing of the standard YM method toward the constraint of the modified YM is performed by

putting the value of the optimal decision variable into each payload limitation, toward the trip range grouped in three segments. The optimized solution of the decision variable either for the standard YM method or the modified YM method is multiplied by the payload weight of each passenger in accordance with the segment when the plane flies.

From the result of this testing, it is shown that the constraint of the payload, either for the CKG-DPS-TIM-DJJ route or CKG-UPG-BIK-DJJ route, in the segment 1, segment 2, or segment 3, this technical requirement can still be met or, in other words, it does not significantly affect the revenue optimization (there are 36 types of data scenario).

#### **d. Validating the developed method**

One way to have model validity is by testing the data consistency. This data consistency testing is done after completing the rechecking on the problem formulation and there is no failure found in it.

The validating/testing of the data consistency-based model is done by performing the revenue calculation manually (conventionally), using the optimized solution (decision variable) that has been obtained using linier program. Manually, the decision variables, which are the optimal solution and linier program-ming method, are multiplied by each fare in the same scenario to obtain the sum of the revenue.

The validation testing using this way is conducted to observe the data consistency being used. If there is no difference in the revenue calculation result between calculation and linier programming or manual calculation, the data being used in the modeling process are valid (consistent), otherwise if there is the difference, the data or the model is invalid (inconsistent). If the data/model is invalid, it is necessary to

look for the wrong data sources and types being used.

And from this data testing result to examine the model validity, a significant difference on the sum of the revenue done manually or by linier programming is not found. The existing difference is not too significant because it is too small, caused by the rounding off of the decision variable.

## **Conclusion and Suggestion**

### **Conclusion**

From the result of the development of the yield management model for multi leg flight by adding the variables of trip range and operational cost, it can be concluded that:

In the case of the flight of the single leg route, the yield management that only considers the variables of booking, fare, and plane capacity will not create any problem if it is applied; whereas in the case of the flight of the multi leg route, it will create a problem if the number of booking in each leg is even or bigger than the plane capacity.

The settlement formulation of the optimized solution of the yield management using linier programming method after the adding of the variables of trip range and operational cost will experience changes in the objective function and limitation, in this matter the limitation relates to the payload.

The result of the optimized solution is the decision variable, i.e. the number of passenger of a certain fare class obtained using the standard yield management model (without the addition/ consideration of the trip range and operational cost variables),

which if it is put into the formulation of the modified yield management model, it will contravene several obstacles/limitations so that the result of the optimized solution is invalid.

The objective function, i.e. the sum of the revenue obtained from one flight to serve a certain route (the accumulation and revenue of each leg) using the standard yield management model is bigger than the total revenue obtained by using the modified yield management model, yet the yield (the average revenue from each passenger) is bigger if using the modified yield management model.

### **Sugestion**

The development of the yield management model in this research still has many shortcomings to be improved. Therefore, the research can be developed further to:

1. In this research the variables added to the yield management model are the variables of trip range (related to payload and trip fuel) as well as operational cost, especially catering cost, so that there is a possibility to add other variables.
2. The improvement of the booking estimation for both fill fare class and discount fare class or for the problems with more than two fare classes (in this research, it is assumed that there are only two fare classes which it is in accordance with the types of the compartment of Boeing 737-400 operated by PT. Garuda Indonesia to serve the route studied as the case in this research).

3. is completed by using a computer program which does not separate In the problem solution procedure, the calculation/processing is separately completed. It will be better if the calculation/processing the prediction of the booking estimation, the calculation of the protection level, and the allocation of the passenger seat entirely. The most probable and important development that will give many benefits to the airlines is to prepare a yield management system (information) that integrates all aspects of marketing, ticket booking service, and fare strategy planning/determination.

4. In the matter of model testing, this research uses a testing process based on the optimized solution ratio (the revenue) between the standard YM model and the modified YM model, as well as performs the testing to the data consistency. It will be better if the actual (historical) data, for example, and the data of several flights are used. However, it is not completed in this research since there is no actual data; so that the scenario data suitable based on the load factor data are used.

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