1. INTRODUCTION

Maintaining government spending on education has been a major goal in Indonesia, where the government has been struggling since 2003 to keep the allocation for education at or above 20% of the national and regional budget, based on Law no. 20 of 2003. Indonesia has this budget requirement for education because the government believes that it will enhance the country’s economic growth. A large body of research supports the belief that government spending on education influences economic growth significantly (Baldacci et al., 2008; Barro & Sala-I-Martin, 1995; Dauda, 2010; Gupta & Belassi, 2004; and Prasetya & Pangestuty, 2012).

While most researchers agree there is a relationship between the two, some do not consider that government spending must first enhance human capital, to improve education, as measured by enrollment rate (Baldacci et al., 2008; Gupta, Verhoeven, & Tiongson, 2002; and Rajkumar & Swaroop, 2007). Increased enrollment rates are likely to expand economic growth (Jalil & Idrees, 2013; McMahon, 1998; and Wolff, 2000).

Although some studies look at the effect of human capital on growth and use developing countries’ data, they cover many nations or long time span. No study has focused on one developing country’s case, especially Indonesia’s. As education spending will have different effect in different circumstances, it is important to study country specific cases. Our paper will attempt to fill this gap by focusing on Indonesia alone.

The paper will analyze the effect of government education spending on growth as an indirect process through enrollment rate. Thus, it aims to find the effect of that spending on enrollment rate and economic growth, and the direct effect of school enrollment rate on growth, specifically in Indonesia, using a panel data set of 26 provinces from 2000 to 2010.

This paper is organized as follows: section 2 provides the literature review, section 3 discusses the model and the dataset, section 4 presents the results, and section 5 offers the policy implication.

2. LITERATURE REVIEW

The relationship between government education spending and growth has been widely discussed among scholars with various results. Some researchers assert it has a positive and significant relationship, such as Barro & Sala-I-Martin (1995), Musila & Belassi (2004),
Baldacci et al., (2008), Dauda (2010), and Prasetya & Pangestuty (2012). On the other hand, Lotto (2011) found that, in Nigeria, the relationship is negative or not significant in the short run while possibly positive in the long run. Additionally, Blankenau and Simpson (2004) concluded that the relationship between education spending and economic growth exists because government provides financial support for formal schooling in many countries; however, there is no clear empirical validation of this relationship because education spending may crowd out other growth factors such as tax structure and government size.

Observing a strong relationship between economic growth and public spending on education, many scholars treat the latter as a direct factor of growth, such as Barro (1990), Blankenau and Simpson (2004), Musila and Belassi (2004), Pradhan (2009), Lotto (2011), and Prasetya and Pangestuty (2012). However, studies belonging to the Solow growth model formulated the problem differently. In that model’s original form, capital investment and population growth are the determinants of steady state level of per capita income. Mankiw, Romer, and Weil (1992) augment the Solow model by including accumulation of human capital as well as physical capital investment. Moreover, McMahon (1998) and Baldacci et al. (2008) believe education is a key component of human capital as the determinant of economic growth. From this point of view, government education spending should be treated as an indirect factor for economic growth through enrollment (Baldacci et al., 2008). Thus, we must first examine the process from government education spending to enrollment rate, and then from enrollment rate to economic growth.

The first process is the effect of government education spending on enrollment rate. Researches claim it is a positive and significant one (Baldacci et al, 2008, and Gupta, Verhoeven, and Tiongson, 2002); especially, if supported by good governance (Rajkumar and Swaroop, 2007). Using cross-section data, Rajkumar and Swaroop (2007) define educational attainment as the proportion of school-aged children who finish secondary school. However, Gupta, Verhoeven, and Tiongson (2002), define it as gross enrollment ratio in primary and secondary education; similarly, Mankiw, Romer and Weil (1992) use enrollment rate as a proxy of human capital accumulation.

In Baldacci et al. (2008), education is measured by the summation of primary and secondary enrollment rate and given a separate model from the general growth model. Baldacci et al. (2008) utilize panel data from 118 developing countries from 1971 to 2000 and apply one period of lag to education spending. They conclude education spending has positive, significant impact on human capital; and indirectly impacts on economic growth.

The second process is the relationship between enrollment rate and growth. Most researchers agree that enrollment rate enhances economic growth by enhancing human capital (Barro and Sala-I-Martin, 1995; Mankiw, Romer, and Weil, 1992; and Wolff, 2000). All of them use international data; for instance, Mankiw, Romer, and Weil (1992) use 98 countries, Barro and Sala-I-Martin (1995) use 97, and Wolff (2000) use 24 members of the OECD. A time series approach also proves that different levels of education positively and significantly affect to economic growth, for example Pakistan from 1960 to 2000 (Jalil & Idrees, 2013).

Using East Asian countries’ data from 1965 to 1990 and applying the ordinary least square with correction for heteroscedasticity and autocorrelation analysis for the data, McMahon (1998) finds that primary education spending from the government is very important in the early period. However, this spending matters less after primary education has generally been

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**Figure 1.** Process from education spending to economic growth
attained, at which point more should be spent on secondary and higher education. Similarly, Agiomirgianakis, Asteriou, and Monastiriotis (2002), employ a dynamic panel data approach to 93 countries from 1960 to 1987, and find the effect of education on growth increases linearly with the level of education.

This study puts together existing literature by relating government education spending, enrollment rate, and economic growth in two processes. First, we use the education equation to show the effect of government education spending on enrolment rate. Second, we use the Solow growth model with enrollment rate as a measurement of human capital. Finally, using both models and applying the chain rule, we obtain the effect of education spending on growth.

To be consistent with previous research related to growth, we employ a panel data approach in both growth and education equations to discern not only the time effect, but also the individual effect (Islam, 1995). Using panel data, we will see the individual effect in each province in Indonesia will in this study. Ours is a regional, not national, analysis.

3. MODEL AND DATA

3.1. Model

To obtain the impact of government spending on education on growth, this paper uses general specifications for real per capita income growth and enrollment rate as used in the existing literature. Baldacci et al. (2008) build a separate model for the education equation, using enrollment rate, as a determinant of economic growth. Therefore, we employ two models, a growth model and an education model. In both, we include lagged values of government spending and education variables.

Baltagi (2001) argues that panel approach is the most applicable method for growth model, since it does not require technology to be the same across individuals and provides a better dynamic effect for each. Islam (1995) adds that it lets us separate the effect of “capital deepening” and technological and institutional differences. Hence, we employ a panel approach for both models. Specification for each equation is shown below.

3.1.1. Education equation

The education equation follows the existing literature and makes available the dynamic effects of lagged spending. It is based on Baldacci et al. (2008) and Gupta, Verhoeven, and Tiongson (2002). They mainly use gross enrollment ratio (GER) as the education variable. They also use many variables to explain GER, such as initial income, education spending, child mortality, urbanization, share of female students, and repetition rate. However, due to limited time and data availability, this paper only used initial income as a control variable. Therefore, the general equation is as follows:

\[
GER_{it} = \gamma_{20} + \gamma_{21} \ln y_{it} + \gamma_{22} EduSpending_{it} + \eta_{2t} + \mu_{2i} + \varepsilon_{2it}
\]

This equation observes the impact of education spending on GER in each level of schooling where it is a proxy of human capital in the growth equation. The use of gross enrollment rate as a proxy is consistent with much of the literature examining the role of human capital on growth (Baldacci et al., 2008; Mankiw, Romer, & Weil, 1992).

Indonesia has four levels of education (see Appendix 1): pre-school, basic, secondary and higher education. In this paper, we used basic and secondary education without accounting for non-formal education. Basic education is primary school, while secondary education is consisted of lower and upper secondary school. GERs are taken for each level of schooling. Therefore, we had three education equations representing three levels of schooling. We used the GER of each schooling level as the education measurement. Some variables that will influence GER, based on the equation above are:

- Income level ($y$). An increase in per capita income tends to increase demand for education, making a positive coefficient. This variable is the logarithm of real per capita gross regional domestic product
(GRDP).

- Education Spending ($EduSpending$). This variable expresses education spending of provincial and municipal governments as a percent of GRDP. A one period lag account for the attainment time of the spending’s impact.
- $\eta_t$ and $\mu_i$ indicate the time-specific effect and province-specific effect, respectively.
- $\epsilon_{it}$ is error term.

### 3.1.2. Growth equation

Drawing upon Mankiw, Romer and Weil (1992), the growth equation is based on Solow’s neoclassical growth framework. It adds human capital accumulation to the Solow growth model. The growth per capita output equation is below (see Appendix 2 for the derivation):

$$g_{lt} = y_{10} + y_{11} \ln s_{kit} + y_{12} \ln s_{hit}$$

$$+ y_{13} \ln(n + g + \delta)_{it}$$

$$+ y_{14} \ln y_{i(t-1)} + \eta_{lt} + \mu_{lt}$$

$$+ \epsilon_{1lt}$$

where
- “ln” means “natural logarithm”.
- $g_{lt}$ is real GRDP growth.
- $S_{kit}$ denotes the investment ratio, in terms of gross fixed capital formation (GFCF) per GRDP. A higher investment ratio raises the stock of physical capital.
- $S_{hit}$ refers to the stock of human capital, represented by GER. The GER of each school's level will be estimated separately to better understand the progress in developing human capital for each province at different stages.
- $(n + g + \delta)$ is summation of population growth, technological and depreciation rate. Following Mankiw, Romer, and Weil (1992), we assume that $g + \delta$ is 0.05.
- $y_{i(t-1)}$ defines the lagged logarithm of per capita income. The negative direction of growth as initial per capita income increases shows that there is a conditional convergence to the steady state.
- $\eta_t$ and $\mu_i$ indicate the time-specific effect and province-specific effect, respectively.
- $\epsilon_{it}$ is error term.

**Table 1.** Group of provinces

<table>
<thead>
<tr>
<th>No</th>
<th>Name – Old Province</th>
<th>Name – New Province</th>
<th>Established</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Riau</td>
<td>a. Riau</td>
<td>a. 1958</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Riau Island</td>
<td>b. 2002</td>
</tr>
<tr>
<td>2</td>
<td>South Sumatera</td>
<td>a. South Sumatera</td>
<td>a. 1959</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Bangka Belitung</td>
<td>b. 2000</td>
</tr>
<tr>
<td>3</td>
<td>West Java</td>
<td>a. West Java</td>
<td>a. 1950</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Banten</td>
<td>b. 2000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. Gorontalo</td>
<td>b. 2000</td>
</tr>
<tr>
<td>5</td>
<td>South Sulawesi</td>
<td>a. South Sulawesi</td>
<td>a. 1960</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. West Sulawesi</td>
<td>b. 2004</td>
</tr>
<tr>
<td>6</td>
<td>Maluku</td>
<td>a. Maluku</td>
<td>a. 1958</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. North Maluku</td>
<td>b. 1999</td>
</tr>
<tr>
<td>7</td>
<td>Papua</td>
<td>a. Papua</td>
<td>a. 1969</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b. West Papua</td>
<td>b. 2001</td>
</tr>
</tbody>
</table>

*Note.* Statistic Indonesia
3.2. Data

We gathered a panel dataset for 26 provinces in Indonesia from 2000 to 2010, out of 34 provinces that exist in Indonesia today. Some new provinces are regrouping back into old provinces (before expansion) and therefore cannot provide comparable and consistent data (see Table 1 for group of provinces, and Table 2 for more description of the data).

Data for GRDP, GFCF, and GER are from Statistic Indonesia, while data for spending is from Ministry of Finance. At start, we had 286 data points, but only 260 were effective after regression due to limited availability of data (e.g. GER of Aceh in 2000 and 2001) and generating new variables that take into account year before 2000 (e.g. growth).

4. EMPIRICAL RESULT AND DISCUSSION

The aim of this study is to investigate the effect of education spending at a provincial level on economic growth indirectly through enrollment. Tables 2 and 3 show results for growth and education equations. Per previous discussions, we used the panel approach to estimate the education equation and growth model. The panel method has two kinds of effect, fixed and random. We employed the Hausman test before running the regression in order to

Table 2. Summary descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observation</th>
<th>Mean</th>
<th>St deviation</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real per capita GRDP growth</td>
<td>260</td>
<td>0.03</td>
<td>0.05</td>
<td>-0.32</td>
<td>0.47</td>
</tr>
<tr>
<td>Income level (log of real per capita GRDP)</td>
<td>286</td>
<td>1.19</td>
<td>0.64</td>
<td>0.72</td>
<td>3.79</td>
</tr>
<tr>
<td>Investment ratio (log of GFCF per capita GRDP)</td>
<td>286</td>
<td>-1.73</td>
<td>0.47</td>
<td>-3.56</td>
<td>-0.93</td>
</tr>
<tr>
<td>Primary gross enrollment ratio</td>
<td>284</td>
<td>1.08</td>
<td>0.04</td>
<td>0.91</td>
<td>1.18</td>
</tr>
<tr>
<td>Lower secondary gross enrollment ratio</td>
<td>284</td>
<td>0.81</td>
<td>0.09</td>
<td>0.51</td>
<td>1.01</td>
</tr>
<tr>
<td>Upper secondary gross enrollment ratio</td>
<td>284</td>
<td>0.57</td>
<td>0.13</td>
<td>0.29</td>
<td>0.95</td>
</tr>
<tr>
<td>Education spending (per GRDP)</td>
<td>286</td>
<td>0.04</td>
<td>0.04</td>
<td>0.00</td>
<td>0.37</td>
</tr>
<tr>
<td>Total Spending (in logarithm form)</td>
<td>286</td>
<td>14.86</td>
<td>0.86</td>
<td>11.93</td>
<td>16.81</td>
</tr>
<tr>
<td>Population growth</td>
<td>260</td>
<td>0.02</td>
<td>0.01</td>
<td>0.003</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: Ministry of Finance, Statistic Indonesia

Table 3. Education equation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Elementary GER</th>
<th>Lower secondary GER</th>
<th>Upper secondary GER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income level (t-1)</td>
<td>0.0878*** (0.0190)</td>
<td>0.0575*** (0.0381)</td>
<td>0.559*** (0.0716)</td>
</tr>
<tr>
<td>Education spending (t-1)</td>
<td>0.129** (0.0602)</td>
<td>0.116 (0.120)</td>
<td>1.115*** (0.226)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0877** (0.0341)</td>
<td>-0.318*** (0.0682)</td>
<td>-1.639*** (0.128)</td>
</tr>
<tr>
<td>Observation</td>
<td>259</td>
<td>259</td>
<td>229</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.025</td>
<td>0.025</td>
<td>0.407</td>
</tr>
<tr>
<td>Number of province</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
</tbody>
</table>

Standards errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
obtain the best panel approach. The results of the Hausman test (see Appendix 3), for both models, showed that applying fixed effect is more appropriate. In most cases, the coefficients are statistically significant and the directions areas expected (see Appendix 4 and 5 for more detail results).

**4.1. Education Equation**

Government spending on education positively affects GER after a one-year lag. Baldacci, et al (2008) found the similar result. The coefficient implies that an increase in education spending of 1% of GRDP would increase enrollment rates by a factor of 0.13 for primary, 0.12 for lower secondary, and 1.12 for upper secondary in the following year (see Table 3).

The income level also matters significantly to GER, especially last year income. The coefficient of income level means that an additional 1% in income will increase GER by 0.09, 0.06, and 0.56 point in elementary, lower secondary, and upper secondary school, respectively.

**4.2. Growth Equation**

The impact of different levels of schooling on economic growth may vary between nations (Jalil & Idrees, 2013); also, within regions of a nation. In the human capital augmented production function (growth equation), we substituted the human capital variable with GER of each level of schooling to measure the effect from each of them on growth. We separated primary, lower secondary, and upper secondary school into three equations to avoid the risk of multicollinearity, which would lead to incorrect inferences.

**Table 4. Growth equation**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Growth using</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2SLS</td>
</tr>
<tr>
<td>GRDP per capita (t-1, log)</td>
<td>-0.348***</td>
</tr>
<tr>
<td></td>
<td>(0.0933)</td>
</tr>
<tr>
<td>Investment ratio (t-1)</td>
<td>0.0645**</td>
</tr>
<tr>
<td></td>
<td>(0.0253)</td>
</tr>
<tr>
<td>Population growth (n+0.05)</td>
<td>-0.156</td>
</tr>
<tr>
<td></td>
<td>(0.237)</td>
</tr>
<tr>
<td>Primary GER (log)</td>
<td>2.498***</td>
</tr>
<tr>
<td></td>
<td>(0.857)</td>
</tr>
<tr>
<td>Lower secondary GER (log)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Upper secondary GER (log)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>(0.650)</td>
</tr>
<tr>
<td>Observation</td>
<td>260</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.123</td>
</tr>
<tr>
<td>Number of province</td>
<td>26</td>
</tr>
</tbody>
</table>

a Regression using primary gross enrollment ratio
b Regression using lower secondary gross enrollment ratio
c Regression using upper secondary gross enrollment ratio
Standards errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
All levels of GER were found to positively and significantly contribute to economic growth at a 1% statistical significance level, whether using 2SLS or IV regression, except for an insignificant coefficient of primary GER using IV regression. In the 2SLS regression, a 1% increase in the primary, lower secondary and upper secondary GER should raise the growth rate by 0.025, 0.028, and 0.003 points, respectively. In the IV regression, the instrument variables are education spending ratio in terms of GRDP, total value of education spending, initial income, and log of population growth. The coefficient of GER means that a 1% increase in the lower and upper secondary GER should increase the growth rate by 0.056 and 0.019 percentage points, respectively. In the IV regression, the multiplication of these effects will be the effect of education spending on growth.

4.3. The effect of education spending on growth through enrollment rate

We calculated the effect of government education spending on growth through enrollment rate based on the marginal effect of education spending on GER and the marginal effect of GER on growth. Based on the chain rule, the multiplication of these effects will be the effect of education spending on growth.

The table 5 shows that increasing 0.01 percent of education spending in terms of GRDP will increase growth by 0.003 point using 2SLS regression; while using IV regression, 0.01 percent increase of education spending will increase growth by around 0.002 point. Therefore, government spending has a positive impact on economic growth.

5. Conclusions and policy implication

This study attempted to examine the effect of government education spending on growth through school enrollment in Indonesia during the period 2000 to 2010. It has investigated using

### Table 5. Marginal Effect

<table>
<thead>
<tr>
<th>Marginal Effect</th>
<th>Elementary GER</th>
<th>Lower secondary GER</th>
<th>Upper secondary GER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education spending on GER</td>
<td>0.129</td>
<td>0.116</td>
<td>1.115</td>
</tr>
<tr>
<td>GER on growth (2SLS)</td>
<td>2.498</td>
<td>2.773</td>
<td>0.289</td>
</tr>
<tr>
<td>GER on growth (IV)</td>
<td>1.514</td>
<td>0.562</td>
<td>0.185</td>
</tr>
<tr>
<td>Education spending on growth (2SLS)</td>
<td>0.322</td>
<td>0.322</td>
<td>0.322</td>
</tr>
<tr>
<td>Education spending on growth (IV)</td>
<td>0.195</td>
<td>0.065</td>
<td>0.206</td>
</tr>
</tbody>
</table>

* Taken from Table 3

* Taken from Table 4

* Calculation based on chain rule \( \frac{dy}{dx} = \frac{dy}{dy} \cdot \frac{dy}{dx} \) where \( \frac{dy}{dy} \) is education spending on growth, \( \frac{dy}{dx} \) is education spending on GER, and \( \frac{dy}{dx} \) is GER on growth.

See Wooldridge (2002) chapter 2, p.45 on the summary of functional forms involving logarithm. For a level-log model, the interpretation of \( \beta \) is \( \frac{dy}{dx} = (\beta_1/100) \cdot (\Delta x) \). So, a one percent increase in \( x \) will raise \( y \) by \( (\beta_1/100) \).

Loc. cit.
fixed effect of panel data regression with 2SLS and instrument variable. The main findings are as follows:

1. For the education equation, standard panel data regression by using fixed effect approach clarifies that education spending has a positive relationship with enrollment ratio in all levels of education with 1% significant level in upper secondary school, 5% significant in primary, and not significant in lower secondary. The latter effect is because fewer labor forces are taken from lower secondary school.

2. For the growth equation, panel data regression using either 2SLS or the instrument variable shows that enrollment rate has a positive relationship with economic growth at a 1% statistical significant level, except for the instrument variable, and elementary school, where the relationship is insignificant. This is probably due to an already high elementary enrollment rate.

3. The chain rule confirms a positive effect of education spending on economic growth through school enrollment. Increasing that spending will increase that growth.

4. Indonesian provinces tend to show a conditional convergence to the steady state, including negative direction for the initial income level. Thus, all regions will have similar growth rate, even though the speed of convergence of each province may differ.

The policy implications of this study are as follows:

1. Having a wide variation of provinces in Indonesia and a large education budget, government should create an education system fostering collaboration between central government, local governments, schools, associations, and communities to support the implementation of policy.

2. We can learn from developed countries. The Japanese educational system, for instance, is built on four levels: central, prefectural, municipal, and scholastic. Those levels allow it to be relatively efficient and effective.

3. Education is considered a channel of technological transfer and a way to boost economic growth. To accelerate regional growth, especially in relatively diverse regions, government can provide talented citizens from relatively backward regions with scholarships, encouraging their education in more developed regions or even abroad. As a condition, citizens would have to return home and develop their region, becoming change agents toward a better condition.

4. West Nusa Tenggara was the province with the lowest economic growth surveyed. We suggest its local government allocate more of its budget to the education sector or other productive sectors such as health and infrastructure. this province's economic condition will therefore improve.

5. There is a one-year lag between increase in education spending and effect on school enrollment rate and growth. Government should allocate more resources to school infrastructure, school needs, or facilities such as classrooms, libraries, and books. The impact of government spending on education will probably increase since facilities of schools are well developed and maintained.

6. Provinces with low economic growth should increase the quality of not only formal schooling but also informal schooling. Better schools will push pupils to improve their knowledge, and skill. Later on, citizens of each province will be a good labor force or, even better, create jobs themselves. Therefore, every region can enhance their economic growth by enriching their society.

6. REFERENCES


Appendix 2. Derivation of growth equation

Following Mankiw, Romer, and Weil (1992) model, the equation started with Solow growth model by adding human capital accumulation and featuring Cobb-Douglas production function is:

\[ Y(t) = K(t)^{\alpha}H(t)^{\beta}[A(t)L(t)]^{1-\alpha-\beta} \quad 0 < \alpha + \beta < 1 \]

Where \( Y \) is output, \( H \) is human capital, \( K \) is physical capital, \( L \) is labor, and \( A \) is the level of technology. \( L \) and \( A \) are assumed to grow exogenously at rates \( n \) and \( g \) so that:

\[ (2) \quad L(t) = L(0)e^{nt} \]

\[ (3) \quad A(t) = A(0)e^{gt} \]

The intensive form of the Solow production function by defining all variables per effective labor is:

\[ (8) \quad \dot{Y}(t) = \dot{K}(t)^{\alpha}\dot{H}(t)^{\beta} \]

\[ \text{where} \quad \dot{Y}(t) = \frac{Y(t)}{A(t)L(t)}, \quad \dot{K}(t) = \frac{K(t)}{A(t)L(t)} , \quad \text{and} \quad \dot{H}(t) = \frac{H(t)}{A(t)L(t)} \]

Assuming both physical and human capital are accumulating factors, their equations of motion for are:

Physical capital:

\[ \dot{K}(t) = s_k Y(t) - \delta K(t) \]

\[ \dot{\dot{K}}(t) = \frac{\partial}{\partial t} \left( \frac{\partial K(t)}{A(t)L(t)} \right) \]

\[ \dot{L}(t) = \frac{K(t)A(t)L(t) - K(t)[A(t)L(t) + A(t)L(t)]}{[A(t)L(t)]^2} \]

\[ \dot{\dot{L}}(t) = \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)} \left[ \frac{A(t)L(t) + A(t)L(t)}{A(t)L(t)} \right] \]

\[ \dot{\dot{L}}(t) = \frac{s_k Y(t) - \delta K(t)}{A(t)L(t)} - (n + g)\dot{K}(t) \]

\[ \dot{\dot{K}}(t) = s_k \dot{Y}(t) - (n + g + \delta)\dot{K}(t) \]

Thus, the motion for physical capital is

\[ (9) \quad \dot{\dot{K}}(t) = s_k \dot{Y}(t) - (n + g + \delta)\dot{K}(t) \]
Human capital:

\[ H(t) = s_k Y(t) - \delta H(t) \]
\[ \dot{h}(t) = \frac{\partial}{\partial t} \left( \frac{H(t)}{A(t)L(t)} \right) \]
\[ \dot{h}(t) = \frac{H(t)A(t)L(t) - H(t)[A(t)L(t) + A(t)L(t)]}{[A(t)L(t)]^2} \]
\[ \dot{h}(t) = \frac{A(t)L(t)}{A(t)L(t)} \cdot \frac{H(t) - H(t)}{H(t)} - \frac{s_k Y(t) - \delta H(t)}{A(t)L(t)} - (n + g)\dot{h}(t) \]
\[ \dot{h}(t) = s_k \dot{y}(t) - \delta \dot{h}(t) - (n + g)\dot{h}(t) \]

Thus, the motion for human capital is

\[ \dot{h}(t) = s_h \dot{y}(t) - (n + g + \delta)\dot{h}(t) \]

Where \( s_k \) is saving rate of physical capital, \( s_h \) is saving rate of human capital, and \( \delta \) is depreciation rate that assumed to be the same for both physical and human capital.

In the steady state, physical and human capital per effective worker must be constant. This implies that solving for the steady-state can be done by finding the values for \( k \) and \( h \) which set the above equations of motion to zero. The steady-state conditions are then:

(11) \( s_k \dot{k}(t) = (n + g + \delta)\dot{k}(t) \)

(12) \( s_h \dot{y}(t) = (n + g + \delta)\dot{h}(t) \)

\[ s_h \dot{k}(t)^\alpha \dot{h}(t)^\beta = (n + g + \delta)\dot{h}(t) \]
\[ \dot{h}(t)^{\beta-1} = \left[ \frac{(n + g + \delta)}{s_h} \right] \dot{k}(t)^{-\alpha} \]

(13) \( \dot{h}(t) = \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{1}{1-\beta}} \dot{k}(t)^{\frac{\alpha}{1-\beta}} \)

Substituting equation (8) and (13) to equation (11), then:

\[ s_k \dot{k}(t) \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{1}{1-\beta}} \dot{k}(t)^{\frac{\alpha}{1-\beta}} = (n + g + \delta)\dot{k}(t) \]
\[ k(t)^{\alpha-1} \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{\beta}{1-\beta}} \dot{k}(t)^{\frac{\alpha}{1-\beta}} = \left[ \frac{(n + g + \delta)}{s_k} \right] \]
\[ \dot{k}(t)^{\alpha-1+\frac{\alpha}{1-\beta}} = \left[ \frac{s_k}{(n + g + \delta)} \right]^{\frac{1}{1-\beta}} \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{\beta}{1-\beta}} \]
\[ \dot{k}(t)^{\frac{\alpha}{1-\beta}} = \left[ \frac{s_k}{(n + g + \delta)} \right]^{\frac{1}{1-\beta}} \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{\beta}{1-\beta}} \]
\[ \dot{k}(t) = \left[ \frac{s_k}{(n + g + \delta)} \right]^{\frac{1-\beta}{1-\alpha-\beta}} \left[ \frac{s_h}{(n + g + \delta)} \right]^{\frac{\beta}{1-\alpha-\beta}} \]

(14) \( \dot{k}(t)^* = \left[ \frac{s_k^{1-\beta} s_h^\beta}{(n + g + \delta)^{1-\alpha-\beta}} \right]^{\frac{1}{1-\alpha-\beta}} \)
Substituting equation (14) and (15) to (8), we got:

\[ \hat{h}(t) = \left[ \frac{s_h}{(n + g + \delta)} \right]^{1/\beta} \left[ \frac{s_k^{1-\beta} s_h^{\beta}}{(n + g + \delta)^{1-\beta}} \right]^{\alpha/\beta} \]

Taking natural logarithm for equation (16) gives the steady state for output per effective labor is

Following Mankiw et al. (1992) and Islam (1995) by considering that the equation (17) explains the steady state behavior; \( y^* \) be the steady state level per effective worker; and let \( y(t) \) be its actual value at any time \( t \), the speed of convergence is given by:

The model suggests a natural regression to study the rate of convergence \( \hat{\epsilon} \). The equation (18) implies:

Where \( y(t) \) is income per effective labor at some initial time.

Substituting equation (14) to equation (13), then:

\[ \hat{h}(t)^* = \left[ \frac{s_h}{(n + g + \delta)} \right]^{1/(1-\alpha-\beta)} \]

Substituting equation (14) and (15) to (8), we got:

\[ \hat{y}(t) = \left[ \frac{s_k^{1-\beta} s_h^{\beta}}{(n + g + \delta)^{1-\beta}} \right]^{\alpha/\beta} \left[ \frac{s_h^{1-\alpha} s_h^{\alpha}}{(n + g + \delta)^{1-\alpha}} \right]^\beta \]

\[ \hat{y}(t) = \frac{s_k^{1-\beta} s_h^{\beta}}{(n + g + \delta)^{1-\beta}} \frac{s_h^{1-\alpha} s_h^{\alpha}}{(n + g + \delta)^{1-\alpha}} \]

\[ \hat{y}(t) = s_k^{1-\beta} s_h^{\beta} \frac{s_h^{1-\alpha} s_h^{\alpha}}{(n + g + \delta)^{(1-\alpha-\beta)}} \]

\[ \hat{y}(t)^* = s_k^{1-\beta} s_h^{\beta/\alpha (1-\beta)} \frac{\alpha}{1-\alpha-\beta} \ln s_h - \frac{\beta}{1-\alpha-\beta} \ln (n + g + \delta) \]

Taking natural logarithm for equation (16) gives the steady state for output per effective labor is

\[ \ln \hat{y}(t)^* = \frac{\alpha}{1-\alpha-\beta} \ln s_h + \frac{\beta}{1-\alpha-\beta} \ln s_h - \frac{\alpha + \beta}{1-\alpha-\beta} \ln (n + g + \delta) \]

Following Mankiw et al. (1992) and Islam (1995) by considering that the equation (17) explains the steady state behavior; \( \hat{y}^* \) be the steady state level per effective worker; and let \( \hat{y}(t) \) be its actual value at any time \( t \), the speed of convergence is given by:

\[ \frac{d \ln \hat{y}(t)}{dt} = \lambda [\ln(\hat{y}^*) - \ln \hat{y}(t)] \]

where \( \lambda = (n + g + \delta)(1 - \alpha - \beta) \)

The model suggests a natural regression to study the rate of convergence \( \lambda \). The equation (18) implies that:

\[ \ln \hat{y}(t_t) = (1 - e^{-\lambda t}) \ln \hat{y}^* + e^{-\lambda t} \ln \hat{y}(t_1) \]

\[ \tau = t_2 - t_1 \]

Where \( \hat{y}(t_1) \) is income per effective labor at some initial time.

Subtracting both side with \( \ln \hat{y}(t_1) \) yields

\[ \ln \hat{y}(t_2) - \ln \hat{y}(t_1) = (1 - e^{-\lambda t}) \ln \hat{y}^* + e^{-\lambda t} \ln \hat{y}(t_1) - \ln \hat{y}(t_1) \]

\[ \ln \hat{y}(t_2) - \ln \hat{y}(t_1) = (1 - e^{-\lambda t}) \ln \hat{y}^* - (1 - e^{-\lambda t}) \ln \hat{y}(t_1) \]
Substituting \( \dot{y}(t)^* \) into the equation (20):

\[
\begin{align*}
\ln \dot{y}(t_2) - \ln \dot{y}(t_1) &= (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln s_k + (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln s_h \\
&\quad - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln (n + g + \delta) - (1 - e^{-\lambda t}) \ln \dot{y}(t_1)
\end{align*}
\]

Reformulate equation (21) in terms of income per capita. Note that income per capita per effective labor is:

\[
\dot{y}(t) = \frac{Y(t)}{A(t)L(t)} = \frac{Y(t)}{A(0)e^{\sigma L(t)}}
\]

Taking the logarithm of the equation above, thus,

\[
(22) \quad \ln \dot{y}(t) = \ln y(t) - \ln A(0) - gt
\]

Replace all \( \dot{y}(t) \) in the equation (21) using the equation (22) gives

\[
\begin{align*}
\ln y(t_2) - \ln A(0) - gt_2 - (\ln y(t_1) - \ln A(0) - gt_1) &= (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln s_k + (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln s_h \\
&\quad - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln (n + g + \delta) - (1 - e^{-\lambda t})(\ln y(t_0) - \ln A(0) - gt_1)
\end{align*}
\]

Equation (23) is the steady state level of growth in terms of output per capita. Thus, in the Solow model the growth income is a function of saving rate of physical capital (s_k), saving rate of human capital (s_h), population growth, technological and depreciation rate (n + g + \delta), and initial level of income \( \dot{y}(t_1) \).

Following the conventional notation of panel data that applied by Islam (1995), the equation is:

\[
(24) \quad g_{it} = \gamma_0 + \gamma_1 \ln s_{kt} + \gamma_2 \ln s_{ht} + \gamma_3 \ln (n + g + h)_{it} + \gamma_4 \ln y(t_1)_{i(0)} + \eta_t + \mu_i + \epsilon_{it}
\]

where:

\[
\begin{align*}
g_{it} &= \ln y(t_2) - \ln y(t_1) \\
\mu_i &= (1 - e^{-\lambda t}) \ln A(0) \\
\eta_t &= (t_2 - e^{-\lambda t} t_1) g \\
\gamma_1 &= (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \\
\gamma_2 &= (1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \\
\gamma_3 &= -(1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \\
\gamma_4 &= -(1 - e^{-\lambda t})
\end{align*}
\]

and \( \epsilon_{it} \) is the transitory error term that varies across countries and time periods and has mean equal to zero.