MODELING TSUNAMI RUNUP DUE TO THE ANDAMAN TSUNAMI ALONG BEACHES IN MALAYSIA

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Abstract. *In an earlier paper, we modeled the propagation of waves across the deep oceans due to tsunami created from an earthquake that occurred around Banda Aceh, Indonesia on December 26, 2004 by means of an in-house tsunami propagation model TUNA-M2. Tsunami wave heights were simulated from the deep ocean up to offshore coasts at depths exceeding 50 m. In this paper, we develop an in-house model TUNA-RP, based upon the finite difference method, to guide the tsunami waves at these offshore locations onto the shallow coasts up to the beaches. The inputs for this model TUNA-RP are derived from the output of TUNA-M2 simulations, accounting for combinations of various source generation conditions reported. The synoptically simulated runup heights along certain beaches in Malaysia will be compared to the runup heights surveyed after the tsunami occurrence. Numerical stability and accuracy of the model will be briefly discussed in conjunction with other analytical and empirical formulations*

1 Introduction

An earthquake on the Richter scale of 9.3 occurred at 00:58:50 (UTC) on December 26 2004 off the west coast of northern Sumatra near the Province of Aceh, Indonesia. This earthquake created a large tsunami that inflicted tremendous damage to properties and the loss of around 300, 000 lives along the affected coastal regions. Tsunami is basically a wave with a median period and a long wavelength created by a large-scale abrupt vertical displacement of the seabed. Traveling with high speeds exceeding 100 m/s in the ocean, tsunami propagation is so fast as to render little time for the affected coastal areas to take protective measures. This paper first presents the results of simulation of the December 26 tsunami by means of the tsunami propagation simulation model TUNA-M2. Then the tsunami wave arriving at depth of 50 m or more will be guided by runup model TUNA-RP to the shore and beaches, the results of which will be synoptically compared with post tsunami survey of affected beaches in Penang, Langkawi and other beaches in Malaysia.

2 Shallow Water Equations

Under certain assumptions typically applicable to tsunami *propagation* in the ocean, the hydrodynamic equations for tsunami propagation can be depth averaged to give rise to the following three equations [12, 14, 15].

$$\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{1}$$

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x}\left(\frac{M^2}{D}\right) + \frac{\partial}{\partial y}\left(\frac{MN}{D}\right) + gD\frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M\sqrt{M^2+N^2} = 0 \tag{2}$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}\left(\frac{MN}{D}\right) + \frac{\partial}{\partial y}\left(\frac{N^2}{D}\right) + gD\frac{\partial \eta}{\partial y} + \frac{gn^2}{D^{7/3}} N\sqrt{M^2+N^2} = 0 \tag{3}$$

Discharge fluxes (M, N) in the x- and y- directions are related to velocities u and v by the expressions $M=u(h+\eta)=uD$, $N=v(h+\eta)=vD$, where h is the mean sea depth, η is the water elevation above mean sea level and D the total depth. The evolution of earthquake-generated tsunami waves has three distinct stages: generation [20], propagation and runup. There are several numerical models to simulate tsunami propagation, for example, the model TUNAMI-N2, developed by Imamura of Tohoku University [13] and the Boussinesq approximation [29]. The shallow water equation can also be solved by the finite element method [16]. A theoretical analysis of tsunami propagation relevant to this paper is available in Haugen et al. [11], Watts [26] and Teh et al. [28]. These analytical models are useful for understanding the basic features and characteristics of tsunami propagation in deep oceans. In this paper, a numerical simulation model for tsunami propagation TUNA-M2 is developed to simulate the December 26 tsunami on a meso scale of 1000 km by 1000 km, and a grid size of 1 km, and depths exceeding 50 m, with particular reference to the assessment of impact to coastal regions in Malaysia and Thailand (Figure 1). There are several scenarios of source dimensions as given in Table 1. Table 2 gives the maximum wave heights at five locations offshore where the depth is about 50 m, while Figure 2 provides the wave heights at the five locations over a period of time.

Table 1. Source size of the 26 December 2004 tsunami

Comment	Source size			Reference
	Length (km)	Width (km)	Displacement (m)	
Assumed by S. Ward	900	100	15	[2, 4]
	800	85	11	[7, 10]
Used by Istituto Nazionale Geofisica e Vulcanologia	700	100	20	[2]
	1200	200	13	[21]
	1300	150	20	[3, 23]
Sumatra segment Nicobar segment Andaman segment	420 325 570	240 170 160	5-20 (7) (5) <2	[17]
Sumatra segment Nicobar segment Andaman segment	200 670 300	150 150 150	20 20 20	[2]
	446	170	13.7 (+ve) 8.6 (-ve)	[30]
	443	170	10.7 (+ve) 6.6 (-ve)	[31]
	1200-1300	~150	15 (peak)	[1]
	1300	150	20	[3]
	500	150	20	[3, 27]

Table 2: Simulated maximum elevations and arrival times, with 1.4 m/s initial velocity

Location	Source size: 170 km × 260 km	
	Actual Simulation	
	Elevation (m)	Arrival time (hr)
(A) Penang	0.7	2.90
(B) Kantang	5.3	2.40
(C) Phuket	5.5	1.85
(D) Tasai	5.0	2.00
(E) Bokpyin	3.7	2.55

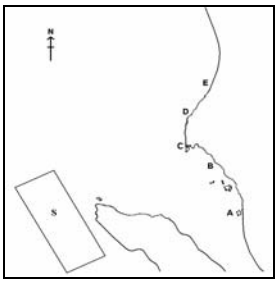

Figure 1: Study domain

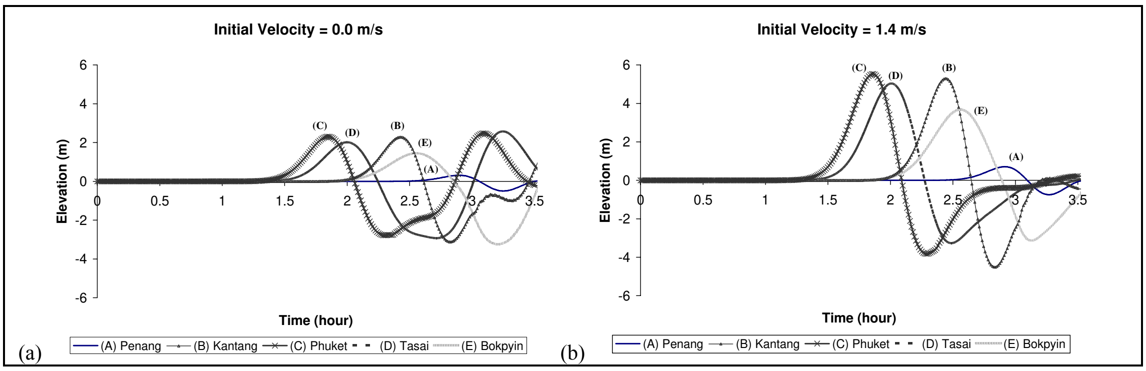

Figure 2. Simulated tsunami wave heights with initial velocity of 0.0 m/s (left) and 1.4 m/s (right) for actual simulations

3 Non-Linear Shallow Water Equations for Runup

The incoming tsunami waves at these depths of 50-100 m are then guided onto the beaches by means of the non-linear shallow water equations below [8, 18] to model wave *runup.* These equations are solved by the following finite difference approximation (6) to derive the runup model TUNA-RP.

$$\frac{\partial \eta}{\partial t}+\frac{\partial(u(\eta+h))}{\partial x}=0 \tag{4}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial \eta}{\partial x}=0 \tag{5}$$

$$\frac{\eta_i^j-\eta_i^{j-1}}{\Delta t}+\left(H_i+\eta_i^{j-1}\right)\frac{u_{i+\frac{1}{2}}^{j-\frac{1}{2}}-u_{i-\frac{1}{2}}^{j-\frac{1}{2}}}{\Delta x}+u_{i+\frac{1}{2}}^{j-\frac{1}{2}}\frac{H_{i+1}+\eta_{i+1}^{j-1}-H_i-\eta_i^{j-1}}{\Delta x}=0$$

$$\frac{u_{i+\frac{1}{2}}^{j+\frac{1}{2}}-u_{i+\frac{1}{2}}^{j-\frac{1}{2}}}{\Delta t}+u_{i+\frac{1}{2}}^{j-\frac{1}{2}}\frac{\left(u_{i+\frac{1}{2}}^{j-\frac{1}{2}}-u_{i-\frac{1}{2}}^{j-\frac{1}{2}}\right)}{\Delta x}+g\frac{\left(\eta_{i+1}^{j}-\eta_i^{j}\right)}{\Delta x}=0 \tag{6}$$

The incoming tsunami wave at the deeper part of the ocean is allowed to enter the near coastal region as a boundary condition in the form of a positive half sine curve with amplitude of 1.0 m (solitary wave). Various conceptual configurations of the bottom bathymetry are simulated. Figure 3 shows the progression of this solitary wave as it passes through a region with either a constant depth or a variable depth from offshore to the inner shore region. The solitary wave undergoes amplification as it passes from the deeper region to the shallower region. The amplification factors vary significantly depending on the bathymetry and wavelength, producing a maximum amplification of about 2.5. The range of amplifications appears to be below various amplification factors reported in the literature, which are based upon empirical formulae.

In Equation (7), the amplification factor (R/H, where H is the offshore wave heights and R is the runup heights) hovers around the value of about 3.1 [9], for a slope of 45°. On the other hand, the amplification factor (R/H) implied in Equation (8) varies according to the slope β and water depth d [24, 25].

$$\frac{R}{d}=3.1\left(\frac{H}{d}\right)^{1.15} \tag{7}$$

$$\frac{R}{d}=2.831\sqrt{\cot\beta}\left(\frac{H}{d}\right)^{5/4} \tag{8}$$

For a solitary wave that enters the shallow region at the location X_1 when t=0, the surface profile is defined as (9) below.

$$\eta(x,0)=\frac{H}{d}\operatorname{sech}^2\left(\sqrt{\frac{3}{4}\frac{H}{d}}\left(x-X_1\right)\right) \tag{9}$$

As the wave propagates into the region, the solutions considered from first-order of H/d are given below [32]. As the wave continues into shallower region, the values of depth d and celerity c decrease, while the wave height η and water velocity u increase.

$$\eta=H\operatorname{sech}^2\sqrt{\frac{3}{4}\frac{H}{d^3}}\left(x-ct\right) \tag{10}$$

$$c=\sqrt{gd}\left(1+0.5\left(H/d\right)\right) \tag{11}$$

$$u = \frac{\eta}{d}\sqrt{gd} \qquad (12)$$

Choi et al. [5] presented prognostic characteristics of tsunamis in the East Japan Sea based upon numerical simulation by means of linear long wave theory. Due to the lack of observed data, the concept of the synthetic catalogue is applied to generate possible tsunami scenarios. The use of synthetic approach to combine historical data with numerically generated hypothetical events has become popular in recent years [6, 19, 22] to evaluate the tsunami risk for various coastal seas. Based upon extensive analysis of data, Choi et al. [5] propose the following equation

$$H_{prog} = q\, H_{mean} \qquad (13)$$

with the coefficient q = 6 to provide an *upper bound* of tsunami wave height at each coastal location for tsunami risk assessment. However, the Equation (13) with the universal coefficent q = 2.5 may be used to predict maximum wave runup heights despite the large variations of the wave amplitudes, suggesting that the accuracy of such prediction is not expected to be too high. Based upon the finite difference approximation (6) of the non-linear shallow water equations (4) and (5), we obtained a series of runup amplifications that vary significantly over variations in bathymetry and wavelength. Figure 3 shows a few scenarios of this runup, indicating a maximum amplification factor of about 2.5. The tsunami wave height at deep-water depth of around 100 m simulated by TUNA-M2 for the 26 December 2004 tsunami is in the range of 0.7 to 1.0 m in coastal seas off Penang. Considering a maximum potential runup factor of about 2.5, this implies a runup heights of about 2.5 m along the beaches of Penang and northern Malaysia, which are lower than some of the observed runup heights between 3 to 4 m (Table 3). This lower value of simulated runup heights could be due to a combination of two factors. First, the runup factor simulated by TUNA-RP may be lower than the actual amplification factor or the propagation wave height simulated by TUNA-M2 at around 50 – 100 m depth may be lower due to the large grid size of 1000 m. Nevertheless, the simulated runup height is still within a factor of 2.0 of the observed runup heights.

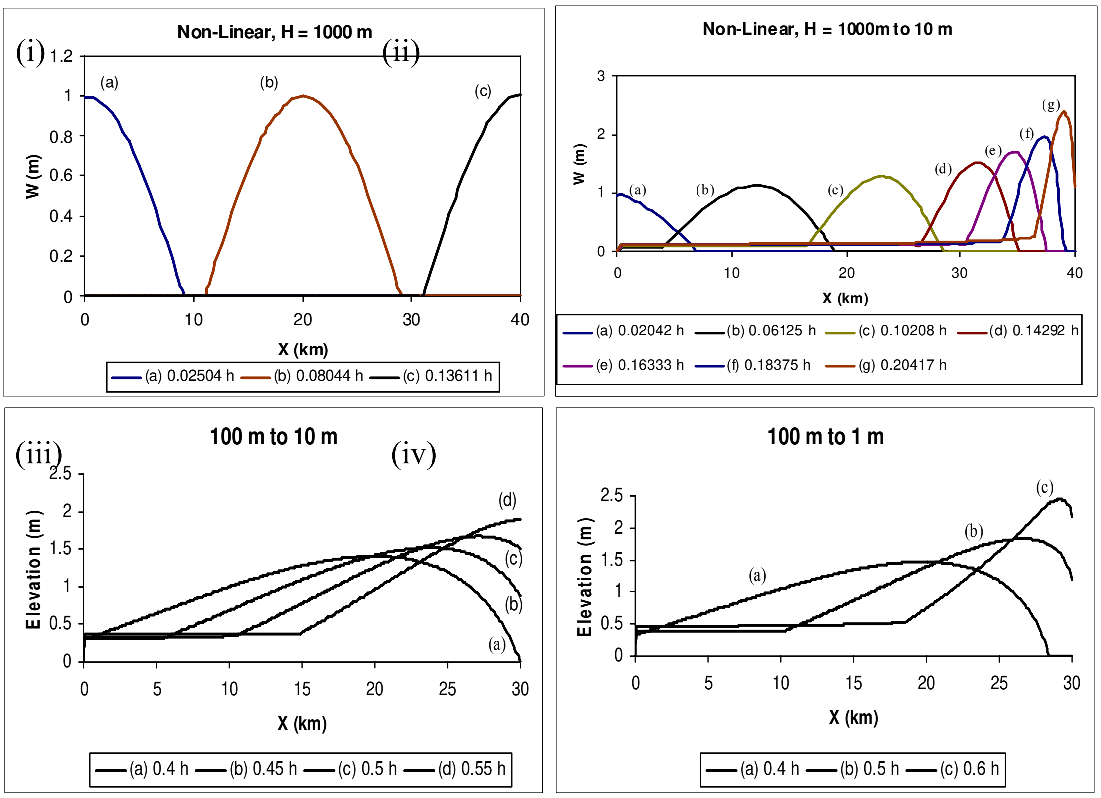

Figure 3. 1D runup (i) constant depth (upper left), (ii) to (iv) variable depth (upper right and bottom)

4 Conclusion

An in-house model TUNA-RP has been developed to simulate the runup of tsunami wave height along the beaches of Penang and north Malaysia. A maximum amplification factor of 2.5 is observed in the simulation with a maximum propagation height of around 1.0 m at off the coast of Penang with the depth of around 50 to 100 m. This amplification factor implies a maximum runup wave heights of 2.5 m, which is lower than wave heights observed after the tsunami occurrence in some places. This could be due to two factors, the first being the amplification factor of 2.5 is lower than the actual amplification factor. Further, the simulated propagation wave height at offshore location of 1.0 m may be lower than what actually happened. The relatively large grid size of 1000 m used in the meso scale model of 1000 km by 1000 km used in TUNA-M2 may have induced relatively large dispersion, hence reducing the amplitude of the wave height. Nevertheless, the simulated runup heights are still within a factor of 2.0 compared to runup height observed along the beaches. As regard the numerical stability of TUNA-RP, it is observed that the numerical scheme begins to show instability and eventually break-up when the runup wave height has approach an amplification factor of 2.5. Whether or not this is a numerical artifacts remain as an area of interest.

Table 3. Survey runup heights for the December 26 2004 tsunami

Location Name	Date (2005)	Latitude (N)		Longitude (E)		Runup Height (m)	Distance to Shore (m)
		Deg.	Min.	Deg.	Min.		
B. Ferringhi (Teluk Bayu)	20 Apr.	5	28.26	100	14.63	3.460	19.200
B. Ferringhi (Miami Beach)	20 Apr.	5	28.67	100	16.07	4.000	25.600
Tanjung Tokong	20 Apr.	5	27.62	100	18.48	3.650	35.800
Tanjung Tokong	20 Apr.	5	27.57	100	18.41	N/A	190.000
Tanjung Tokong	20 Apr.	5	27.70	100	18.50	2.610	18.300
Tanjung Bungah	21 Apr.	5	28.21	100	16.66	2.310	18.380
Tanjung Bungah	21 Apr.	5	28.20	100	16.65	2.940	36.200
Kuala Kedah	22 Aug	6	6.00	100	26.00	0.900	N/A
Yan (Kg. K.S. Limau)	22 Aug	5	53.00	100	21.00	1.227	12.900
Sg udang	22 Aug	5	48.00	100	22.00	1.500	N/A
Tanjung Dawai	22 Aug.	5	40.00	100	21.00	0.385	75.319
Kota K. Muda	22 Aug.	5	34.00	100	20.00	3.800	100.524
Kuala Kurau	23 Aug	5	0.00	100	25.00	1.930	N/A
Pantai Acheh	23 Aug.	5	24.00	100	11.00	2.505	13.400
Pantai Tengah (Lanai Hotel)	24 Aug	6	15.00	99	43.00	3.660	44.500
Pantai Chenang (Pelangi Hotel)	24 Aug.	6	17.00	99	43.00	3.749	54.720
Kuala Teriang	24 Aug	6	21.00	99	42.00	3.091	27.038
Pantai Kok (Mutiara Beach Resort)	24 Aug	6	21.00	99	40.00	2.246	50.840
Pantai Kok (Berjaya Hotel)	24 Aug	6	21.00	99	40.00	2.983	34.879

5 Acknowledgement

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