# THE QUADRATIC ASSIGNMENT PROBLEM : SOME NEW RESULT AND GENERALIZATION

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## Abstract

The Quadratic assignment problem is a combinatorial problem of deciding the placement of facilities in specified locations in such a way as to minimize an function expressed in terms of distances between locations and flows between facilities. The approach presented in this paper is to treat is as a large quadratic programming problem with integer restrictions on the variables. The integer requirements are initially relaxed, and the nearest feasible solutions in then sought. Schemes for accomplishing this are described, and their applicability to more general problems is discussed.

## 1. Introduction

It is well know the quadratic assignment problem (QAP) can be used for tackling problems which frequently occur in facilities location, plant layout and backboard wiring. However, the QAP has proved to be much more difficult to solve than the linier case. This difficulty has made it an area of investigation by numerous researchers such as Steinberg (1961), Nugent et.al(1968), Gaschutz and Ahrens (1968), Elshafei (1977), Burkard and Stratman (1978) and Murtagh et. al. (1982). Most of these researchers treated the QAP as combinational problem. State of the art on QAP can be found in Burkard et. al(1998) and Commander (2003).

The significant success of the large-scale optimization software, MINOS developed by Murtagh and Saudferss (1977,1978,1980) suggests that the QAP should be treated as large quadratic program rather than use a combinational method. This paper describes research result in approaching the solution of the QAP using the MINOS nonlinear programming system. The approach adopted is to relax the integer requirements initially and once a non-integer optimal solution is found, the nearest integer feasible solution is then sought. Strategies for finding the integer feasible solution are based on computational experience on a variety solution dominated by the constraints.

To obtain a good feasible starting point a heuristic procedure proposed by Murtagh et.al (1982) could be used. Nevertheless, other results obtained from the literature could be very fruitful in providing a good feasible starting point.

The following gives a brief description of MINOS non-linier optimization system. The quadratic assignment problem and computational experience with it are presented in section 3 and section4, respectively. Generalization to other problems are discussed is section 5.

## 2. The MINOS Nonlinear Optimization System

MINOS is a in-core, fortran-based optimization system for the solution of large-scale linier and nonlinear programming problems involving sparse linier or nonlinear constraints.
In many real-life problems it turns out most of the variables are linier and only a small percentage of the variables are involved non linearly in the objective function and/or the constraints. Therefore, the standard problem to be solved by MINOS is expressed in the form

Minimize $\quad f^0(\underline{x}) + c^T\underline{x} + \underline{d}^T\underline{y}$ (2.1a)

Subject to $\quad f(\underline{x}) + A_1\underline{y} \quad (m_1 \text{ rows})$ (2.1b)

$A_2\underline{x} + A_3\underline{y} \quad (m_2 \text{ rows})$ (2.1c)

$$\underline{\ell} \leq \begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix} \leq \underline{u} \quad (m = m_1 + m_2) \tag{2.1d}$$

were $f(\underline{x}) = [f^1(\underline{x}),....f^{m_1}(\underline{x})]^T$. Then $n_1$ "nonlinear" variables are designed by $\underline{x}$ and occur nonlinearly in either the objective function $f^0(\underline{x})$ or the first $m_1$constraints. The $n_2$" linier" variables $\underline{y}$ will generally include Ś full set of m slack variables, so that the equality and inequality constraints can be accommodated in ( 2.1b,c )by appropriate bounds in (2.1d). Despite the problem being expressed in that manner, the MINOS code is still very effective for large problems which are entirely nonlinear. The solution process consists of sequence of "major iterations " At the start of each major iteration, the nonlinear constraints are linearized at some base point $\underline{x}_k$ and nonlinearities are adjoined to the objective function with lagrange multipliers.

Define $\quad f(\underline{x}, \underline{x}_k) = f(\underline{x}_k) + J(\underline{x}_k)(\underline{x} - \underline{x}_k)$ (2.2)

Where $J(\underline{x})$ is the $m_1$ x $n_1$ Jacobian matrix whose i j th element is

$$\partial f^i / \partial x_j.$$

Where kth major iteration of the process, the following linearly constrained subproblem is formed:

$$\text{Minimise } L(\underline{x}, \underline{y}, \underline{x}_k \underline{\lambda}_k, \rho) = \begin{array}{c} f^0(\underline{x}) + \underline{c}^T + \underline{d}^T\underline{y} - \underline{\lambda}_k^T(f - \tilde{f}) \\ +^{1/2}\rho(f - \tilde{f})^T(f - \tilde{f}) \end{array} \tag{2.3a}$$

$$\text{Subject to } \quad \underline{\tilde{f}} + A_1 y = \underline{b}_1 \tag{2.3b}$$

$$A_2\underline{x} + A_3 y = \underline{b}_1 \tag{2.3c}$$

$$\underline{\ell} \leq \frac{\underline{x}}{\underline{y}} \leq \underline{u} \tag{2.3d}$$

The objective function L is a Modified augmented lagrangian. The vector $\underline{\lambda}_k$ is an estimate of the Lagrange multiplier for the nonlinear constraints and modified quadratic penalty function. The latter parameter enhances the convergence properties if $\underline{x}_k$ is far remove from the optimal, and the $\underline{\lambda}_k$ are taken as the optimal values at the solution of previous subproblem. As the sequence of major iterations approaches the optimal as measured by the relative change is successive estimates of $\underline{\lambda}_k$ and the degree to which the nonlinear constrains are satisfied at $\underline{x}_k$) the penalty parameter $\rho$ is reduced to zero and quadratic rate of convergence of the subproblem is achieved.

The linearly constrained subproblem constrain matrix equations is the from of $A\underline{x} = \underline{b}$, in which we may partition the variables by introducing the notion of superbasic as follows:

$$A\underline{x}\,[\,B \;\; S \;\; N\,] \begin{bmatrix} \underline{x}_B \\ \underline{x}_S \\ \underline{x}_N \end{bmatrix} = \underline{b} \qquad (2.4)$$

The nonbasic variables $\underline{x}_N$, are at one other of their bounds and stay there for the next step $\Delta\underline{x}$. The superbasic variables $\underline{x}_S$ are free to move in any direction and provide the driving force to minimize the function, while the basic variables $\underline{x}_B$ must follow to satisfy the equation

$$\Delta\underline{x} = \begin{bmatrix} -B^{-1}s \\ I \\ O \end{bmatrix} \Delta\underline{x}_S \qquad [2.6]$$

the matrix on the right-hand side of(2,6) serves as a "reduction" matrix and premultiples the gradient vector, and also pre-and post-multipliers the hessian matrix to yield a Newton step over the subspace of superbasic variables.

For optimization of the reduced function a factorization $R^TR$ (R upper triangular) of a quasi-Newton approximation to the reduced hessian is used. Stable numerical methods based on orthogonal transformations are used, and sparsity is the constraints is maintained by storing and updating an LU factorization of B.

Apart from the usual revision and restart options, MINOS also allows the user to specify starting point.

## 3. The Quadratic Assignment Problem

this is a combinatorial problem of deciding the placement of facilities in specified locations in such a way to minimize a quadratic objective function. Consider the problem of locating n facilities in n given locations. If the flow $f_{ik}$ between each pair of facility I and facility k and the unit transportation cost (or distance) $d_{j\ell}$ between locations j and $\ell$ are known, then the problem is defined to be.

$$\text{Minimize} \quad \Phi = {}^{1/2} \sum_{i=1}^{n} \sum_{k=1}^{n} \sum_{\ell=1}^{n} f_{ik}\, d_j\, x_{ij}\, x_{k\ell} \qquad (3.1)$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{ij} = 1 \quad I=1,\ldots\ldots.n \qquad (3.2)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j=1,\ldots\ldots n \tag{3.3}$$

$$0 \leq x_{ij} \leq 1 \tag{3.4}$$

$$x_{ij} \text{ integer}$$

Matrices $[x_{ik}]$ and $[d_{j\ell}]$ are assumed to be symmetric. The assignment variable $x_{ij}$ has a value 1 facility $i$ is at location $j$, and is zero otherwise. The constraints reflect the fact that each location can be assigned to only one facility, and each facility can be assigned to only one location.

In order to solve the problem using MINOS effectively we ignore the integrality requirement and then examine any variables which take non-integer values at the solution. This means we treat the problem (3.1)-(3.4) above as a large quadratic programming problem and the code exhibits a superlinear rate of convergence. Obviously, there are $n^2$ nonlinear variables in the problem but the constraints are particularly sparse, therefore the MINOS can tackle the problem easily.

Generally the QAP is a non-convex problem so any solution obtained will necessarily be a local optimum and non a global optimum.

A simple heuristic (Murtagh et. al, 1982) ranks the facilities in decreasing order of frequency of use and locations in increasing order of distance and makes an initial assignment by pairing them literature (Gaschutz and Ahrens, 1968 ; Elshafei, 1977) and these should approach using MINOS.

## 4. Computational Experience

We have investigated several large QAP problems and the results are significantly successful. The size of the problems range from the 19x19 hospital layout problem cited by Elshafei(1977), the 20x20 facility location problem of Nugent(1968) to the 36x36 backboard wiring problem cited by Steinberg (1961). Despite the fact that the problems are large, the number of superbasic variables present at the optimum solution are small. For example, the Elshafei problem had the potential for 323 superbasics, but only 6 superbasics were present at the optimum. The Nugent problem had 8 superbasics at the optimum, whereas there should potentially be 360. Similarly, the Steinberg problem had the potential for 1224 superbasics but only had 3 present at the optimum.

Firstly, by using MINOS we had the optimum " Continuous" solution as shown in Table 1 the 20x20 Nugent's problem and in the Tables 2 and 3 for the 36x36 Steinberg's problem with different measures Of distance. The starting point for the search process was the solution obtained by Nugent et. Al (1968) for nugent's problem and the solution obtained by Gaschutz and Ahrens (1968) for Steinberg's problem.

The original intention was to adopt a branch-and-bound approach for obtaining an optimal integer-feasible solution. Using this approach a sequence of problems are solved in which a selected variable that was not integer-valued at the solution would have integer bounds placed either side of its value, giving rise to two further problems with one of these new bounds in each. Clearly good heuristics for variable selection and deciding which problem to solve and which to place in a "master list" of unsolved problems are necessary in such an approach in order to reduce what could become a massive computing load. Also, with zero –one variables as present in this particular problem imposing such bounds on a specific variable at a definite value and optimization takes place with the others.

Computational experience on the three large problems obvioted the need for such an approach however. A glance at the continuous solutions obtained by MINOS for the Steinberg problem in Table 2 and 3 suggests that the integer-feasible solution is "obvious"in that variables ($x_{ij}$) that are not integer-valued occur generally in pairs for any facility (i), with such values that it is clear which of the pair should be rounded to unity and which to zero. The continuous solution obtained for the nugent problem show in table 1 also exhibits this behaviour, except that four facilities (i=10,12,15 and 20) have more than two non-zero associated variables. A heuristic approach used to resolve this difficulty was to rank the variables in increasing order of "integer infeasibility" (i.e. their proximity to one or zero) and make the assignment in this order.

The results obtained using this approach are shown in tables 4, 5 and 6. it can be seen that the results compare well with the best published value; improving on them in one instance and being slightly worse in another. It should be noted that the computing times involved were approximately one tenth that of the combinatorial approaches in the previously published results.

## 5. Generalization to other Non-linear Integer Problems

The danger in using heuristics in seeking an optimum is that any solution obtained is "sub-optimal" and there is no easy way of measuring how much one is sub-optimal. It can be argued that they branch-and-bound approach used in linier programming is rigorous, in that eventually the optimal integer-feasible solution is found. However in practice the branching process is usually terminated before the exact optimum is reached, but at least it is possible to determine a bound on how far the current best solution is from the optimum (Murtagh, 1981).

In the case of nonlinear programming the problem is generally non-convex. Heuristics the are used in the branch-and-bound approach in selecting the variable on which to perform the branching operation by imposing upper and lower bounds, and also in selecting the problem to be solved. The non-convex nature of the problem means that the heuristics used in the branch-and-bound approach have no guarantee of eventually providing a global integer-feasible solution, so there is an argument for using simpler and more direct heuristics in the nonlinear case.

It cannot be expected that the nearest integer-feasible solution can be so easily found in the general case as it is for the QAP problems examined in this paper. The difficulty of solving a single nonlinear program (as compared to a linear program) suggests that implementation of a branch and bound approach would consume an excessive amount of computing resources, quite apart from there being no assurance that the global integer optimum would be found. We are currently investigating the following approach which we see as a compromise between the two extremes of simple heuristic rounding and branch and bound.

1. Obtain the "continuous" optimum using MINOS.
2. With the Lagrange multipliers of the problem held fixed at their values at the solution of step 1., minimize the Lagrangian function using discrete steps on the variables required to be integer. (This is an unconstrained problem.)
3. perform a local ( constrained) search on the integer variable which have a high reduced cost.

The last two of the above steps are expressed in quite general terms, and the specifics of how each is to be accomplished is the object of our research. Preliminary results on separable nonlinear functions have been encouraging, and we intend to pursue the use of structure further.

## 6. Conclusions

Computational experience with the QAP problems described in this paper indicates that efficient solution- s of large problems can be obtained by combining large-scale nonlinear programming capability with appropriate heuristics. Although the use of heuristics gives no assurance of yielding the optimal integer-feasible solution, it is possible to measure the deterioration from the optimum continuous solution. The results obtained for this class of problem show near-optimal solutions were obtained in minimal computing time.

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Tabel Results for 20x 20 QAP (NUGENT et. al)

| VARIABLE | ACTIVITY | VARIABLE | ACTIVITY |
|---|---|---|---|
| $x_{1,12}$ | 0.75195 | $x_{11,18}$ | 0.51949 |
| $x_{1,17}$ | 0.24805 | $x_{12,12}$ | 0.06818 |
| $x_{2,8}$ | 1.0 | $x_{12,13}$ | 0.51949 |
| $x_{3,11}$ | 0.62792 | $x_{12,18}$ | 0.41233 |
| $x_{3,17}$ | 0.37208 | $x_{13,1}$ | 0.67861 |
| $x_{4,3}$ | 0.48051 | $x_{13,6}$ | 0.32139 |
| $x_{4,4}$ | 0.54949 | $x_{14,19}$ | 1.0 |
| $x_{5,2}$ | 1.0 | $x_{15,9}$ | 0.48596 |
| $x_{6,1}$ | 0.32139 | $x_{15,10}$ | 0.12930 |
| $x_{6,6}$ | 0.67861 | $x_{15,14}$ | 0.38774 |
| $x_{7,7}$ | 1.0 | $x_{16,20}$ | 1.0 |
| $x_{8,3}$ | 0.48051 | $x_{17,5}$ | 1.0 |
| $x_{8,13}$ | 0.48051 | $x_{18,15}$ | 1.0 |
| $x_{9,16}$ | 1.0 | $x_{19,4}$ | 0.42774 |
| $x_{10,11}$ | 0.37208 | $x_{19,10}$ | 0.57226 |
| $x_{10,12}$ | 0.17987 | $x_{20,4}$ | 0.05277 |
| $x_{10,17}$ | 0.37987 | $x_{20,9}$ | 0.51404 |
| $x_{10,18}$ | 0.06818 | $x_{20,10}$ | 0.29845 |
| $x_{11,14}$ | 0.48051 | $x_{20,14}$ | 0.13474 |
| OBJECTIVE VALUE | 796.79 | | |

Tabel 2 Results for 36x36 QAP
Case A: $d_{ij}=(x_1\text{-}x_j)^2 + (y_i\text{-}y_j)^2$

| VARIABLE | ACTIVITY |
|---|---|
| $x_{1,24}$ | 0.035568 |
| $x_{1,33}$ | 0.96432 |
| $x_{2,18}$ | 1.0 |
| $x_{3,8}$ | 1.0 |
| $x_{4,16}$ | 1.0 |
| $x_{5,7}$ | 1.0 |
| $x_{6,6}$ | 1.0 |
| $x_{7,24}$ | 0.962132 |
| $x_{7,25}$ | 0.03568 |
| $x_{8,17}$ | 1.0 |
| $x_{9,35}$ | 1.0 |
| $x_{10,25}$ | 0.96432 |
| $x_{10,34}$ | 0.03568 |
| $x_{11,5}$ | 0.51950 |
| $x_{11,14}$ | 0.48050 |
| $x_{12,5}$ | 0.48050 |
| $x_{12,14}$ | 0.51950 |
| $x_{13,15}$ | 1.0 |
| $x_{14,13}$ | 1.0 |
| $x_{15,33}$ | 0.03568 |
| $x_{15,34}$ | 0.96432 |
| $x_{16,36}$ | 1.0 |
| $x_{17,27}$ | 1.0 |
| $x_{18,26}$ | 1.0 |
| $x_{19,22}$ | 1.0 |
| $x_{21.3}$ | 0.67533 |
| $x_{21,11}$ | 0.32467 |
| $x_{22,2}$ | 0.67533 |
| $x_{22,3}$ | 0.32467 |
| $x_{123,12}$ | 1.0 |
| $x_{24,1}$ | 1.0 |
| $x_{25,2}$ | 0.32467 |
| $x_{25,11}$ | 0.67533 |
| $x_{26,10}$ | 1.0 |
| $x_{27,4}$ | 1.0 |
| $x_{28,32,}$ | 1.0 |
| $x_{29,31}$ | 1.0 |
| $x_{30,30}$ | 1.0 |
| $x_{31,29}$ | 1.0 |
| $x_{32,21}$ | 1.0 |
| $x_{33,19}$ | 1.0 |
| $x_{34,20}$ | 1.0 |
| $x_{35,28}$ | 1.0 |
| $x_{36,9}$ | 1.0 |

Tabel 3 Results for 36x36 QAP
Case : $d_{ij}=|x_1\text{-}x_j|^2 + |y_i\text{-}y_j|^2$

| VARIABLE | ACTIVITY |
|---|---|
| $x_{1,24}$ | 0.38889 |
| $x_{1,25}$ | 0.30111 |
| $x_{126}$ | 0.31000 |
| $x_{2,26}$ | 1.0 |
| $x_{2,18}$ | 1.0 |
| $x_{3,8}$ | 1.0 |
| $x_{4,6}$ | 1.0 |
| $x_{5,7}$ | 1.0 |
| $x_{6,6}$ | 0.61111 |
| $x_{7,24}$ | 0.38889 |
| $x_{7,25}$ | 1.0 |
| $x_{8,17}$ | 1.0 |
| $x_{9,34}$ | 0.31000 |
| $x_{10,25}$ | 0.69000 |
| $x_{11,5}$ | 0.60005 |
| $x_{11,14}$ | 0.39995 |
| $x_{12,5}$ | 0.39995 |
| $x_{12,14}$ | 0.60005 |
| $x_{13,15}$ | 1.0 |
| $x_{14,32}$ | 1.0 |
| $x_{15,33}$ | 1.0 |
| $x_{16,35}$ | 0.869557 |
| $x_{16,36}$ | 0.13043 |
| $x_{17,27}$ | 0.12043 |
| $x_{17,36}$ | 0.86957 |
| $x_{18,27}$ | 0.86957 |
| $x_{18,35,}$ | 0.13043 |
| $x_{19,21}$ | 0.82271 |
| $x_{19,22}$ | 0.17729 |
| $x_{20,23}$ | 1.0 |
| $x_{21,3}$ | 0.25326 |
| $x_{21,12}$ | 0.74674 |
| $x_{21,3}$ | 0.74674 |
| $x_{21,12}$ | 0.25326 |
| $x_{22,13}$ | 1.0 |
| $x_{24,2}$ | 0.78315 |
| $x_{24,10}$ | 0.21685 |
| $x_{25,2}$ | 0.21685 |
| $x_{25,10}$ | 0.17212 |
| $x_{25,11}$ | 0.61103 |
| $x_{25,10}$ | 0.61103 |
| $x_{25,11}$ | 0.38897 |
| $x_{27,4}$ | 1.0 |
| $x_{28,21}$ | 0.17729 |

| VARIABLE | ACTIVITY | VARIABLE | ACTIVITY |
|---|---|---|---|
| | | $x_{28,22}$ | 0.82271 |
| | | $x_{29,30}$ | 0.35704 |
| | | $x_{29,31}$ | 0.64296 |
| | | $x_{30,30}$ | 0.64296 |
| | | $x_{30,31}$ | 0.35704 |
| | | $x_{31,28}$ | 0.73684 |
| | | $x_{31,29}$ | 0.26316 |
| | | $x_{32,10}$ | 1.0 |
| | | $x_{33,19}$ | 0.49211 |
| | | $x_{33,28}$ | 0.26316 |
| | | $x_{33,29}$ | 0.24474 |
| | | $x_{34,19}$ | 0.50789 |
| | | $x_{34,29}$ | 0.49211 |
| | | $x_{35,9}$ | 1.0 |
| | | $x_{36,1}$ | 1.0 |
| Optimal $\varphi$ | 7777.89 | Optimal $\varphi$ | 4627.00 |

Table 4 Test Example NUGENT

Dimension : 20
Best Published Objective value :1287 (1)
Present method objective value : 1361
Solution :

| Set $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Assigned $j$ | 1 | 2 | 8 | 11 | 4 | 2 | 6 | 7 | 3 | 16 | 17 | 18 | 13 | 1 | 19 | 14 | 20 | 5 | 15 | 10 | 9 |

Table 5

Dimension : 36 case A: $d_{ij} =(x_i-x_j)^2=(y_i-y_j)^2$
Best published objective value :7926 [1]
Present method objective value :7929
Solution :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 24 | 22 | 21 | 27 | 11 | 6 | 5 | 3 | - |
| 26 | 25 | 23 | 14 | 12 | 13 | 4 | 8 | 2 |
| 33 | 34 | 32 | 19 | 20 | 7 | 10 | 18 | 17 |
| - | 31 | 30 | 29 | 28 | 1 | 15 | 9 | 16 |

Table 6

Dimension : 36 Case B : $d_{ij} = | x_i - x_j |^{2} = | y_i - y_j |^{2}$
Best published objective value :4802 [1]
Present method objective value :4784
Solution :

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| - | 24 | 22 | 27 | 11 | 6 | 5 | 3 | - |
| 26 | 25 | 21 | 23 | 12 | 13 | 4 | 8 | 2 |
| 34 | 32 | 19 | 28 | 20 | 7 | 1 | 10 | 18 |
| 31 | 33 | 30 | 29 | 14 | 15 | 9 | 16 | 17 |