DESIGN OF REDUCED-ORDER μ-CONTROLLER FOR FLEXIBLE STRUCTURES

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Abstract

In this paper, μ-controller based on a structured singular value is used to reduce the vibration of the flexible structures having the transverse and torsion vibrations. Meanwhile, in designing control system using μ-controller yields a controller with the order at least equal to the plant order. Such controller often can not be used in practical application, thus the order of controller is reduced by using the weighting balanced realization.

Keywords: μ-controller; flexible structures, weighting balanced realization.

1. Introduction

In the vibration control of a flexible structure, it is difficult to identify its modal shape and physical parameters. Moreover, the instability of the closed-loop system due to perturbations of the higher-order modes, so called spillover, often arises. For this reason, demands for the robust control which guarantees stability against perturbations such as modeling errors, parameters variation are very strong. Meanwhile, in regard to the flexible structures having multi directional degrees of freedom, there are many occasions, where the control of transverse and torsion vibrations is required simultaneously. In such a case , the securing of robustness is even more difficult because of perturbations due to the transverse-torsion coupling. Nevertheless, there are cases in which the achievement of robust stability is not enough. To be more specific, a demand for keeping the control performance at a target level even under the influence of a perturbation, we call it as robust performance has been becoming stronger with the recent progress in studies on robust control.

In this paper, μ-controller based on a structured singular values is used to reduce the vibration of the flexible structure having the transverse and torsion vibrations. The structure has four-stories and is tower-like in shape. Each story is modeled such that it has a single-degree-of freedom in the transverse direction and one more degree-of-freedom in the angle of torsion around the center of the story, which yields the whole structure, has 8 degree-of-freedom.

Design of control system using the μ-controller, in the first iteration we obtain the 30-order controller. For the better results, more iteration is needed and it is caused to increase the order of controller. Since the high-order controller is impractical in the real implementation, then the order of controller can be reduced up to the 8^{th}-order using the weighted balanced realization. In this case, the robust stability and robust performance can be maintained by the reduced-order controller.

2. Model of Structure

The structure has four-stories and is tower-like in shape. To simplify the modeling processes, some assumptions are made. Each story is modeled such that it has a single-degree-of-freedom in transverse direction (the same direction as the excitation) and one more degree-of-freedom in the angle of torsion around the centroid of the story, which yields that the whole structures has 8 degrees-of-freedom. This structure has long and short spans symmetric with respect to the central axis, but has a deviation on the right and the long side on the third story due to an auxiliary mass, which thereby creates a coupling between the transverse and torsion vibration. The mass distribution of each story is homogeneous and the stiffness of four columns are supposed to be the same in the direction of the excitation at all stories. On this condition, the distance from the centroid to the spring of the right side of i th-story and the distance from the centroid to the spring of the left side of the i th-story are equal and all the cross terms have no value.

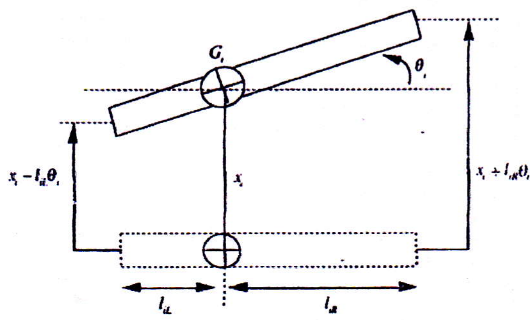

Fig 1. Transverse and torsion displacement of the i th-story of structure

Consider the transverse and torsion vibrations of the i th-stories as shown in Figure 1. x_i and θ_i are the transverse and torsion displacements of the i th-stories. l_{iL} and l_{iR} are the distances from the centroid to the spring of the left and right sides of the i th-stories. m_i and I_i are the mass and moment of inertia. k_{iL} and k_{iR} are the spring constants of the left and the right sides of the i th-stories.

The total kinetic energy of the structure can be written as follows

$$T(t)=\sum_{i=1}^{4}\tfrac{1}{2}\left\{m_i\left(\dot{x}_i(t)+\dot{z}(t)\right)^2+I_i\dot{\theta}_i^2(t)\right\} \tag{1}$$

and the total potential enegy is

$$V(t)=\sum_{i=1}^{4}\tfrac{1}{2}\begin{bmatrix} k_{iR}\left\{\left(x_i(t)+l_{iR}\theta_i(t)\right)-\left(x_{i-1}(t)+l_{i-1R}\theta_{i-1}(t)\right)\right\}^2+ \\ k_{iL}\left\{\left(x_i(t)+l_{iL}\theta_i(t)\right)-\left(x_{i-1}(t)+l_{i-1L}\theta_{i-1}(t)\right)\right\}^2 \end{bmatrix} \tag{2}$$

where $x_0=l_0=\theta_0=0$.

By using the Langrange's equations

$$\frac{d}{dt}\left(\frac{\partial L(t)}{\partial \dot{q}_i(t)}\right)-\frac{\partial L(t)}{\partial q_i(t)}=f(t),\ i=1,2,3,4 \tag{3}$$

where $q_i(t)=\left[x_i(t)\quad \theta_i(t)\right]^T$ and $L=T-V$, the dynamics of the structure can be written in the second-order matrix differential equations as follows

$$M_p\ddot{x}_0(t)+C_p\dot{x}_0(t)+K_px_0(t)+d_p\ddot{z}(t)+b_pf(t)=0 \tag{4}$$

where $x_0(t)=\left[x_1(t)\quad x_2(t)\quad x_3(t)\quad x_4(t)\quad \theta_1(t)\quad \theta_2(t)\quad \theta_3(t)\quad \theta_4(t)\right]^T$, M_p, C_p, and K_p are the inertia, the damping and the stiffness matrices of the structure respectively. d_p is the disturbance vector for the excitation acceleration $\ddot{z}$, and b_p is the input matrix for the control force.

In the structure, two active dynamic absorbers are mounted at the top of the structure on both edges in parallel with the vibration control devices. Each absorber is composed of an auxiliary mass and a moving coil to drive the mass. The equation of the circuit is described as

$$\begin{aligned} &L_a\dot{i}(t)+R_ai(t)+K_e\dot{x}_a(t)=e(t) \\ &f(t)=K_fi(t) \end{aligned} \tag{5}$$

where x_a is the strokes of the actuator, e is the control voltage, f is the control force, L_a is the inductance, R_a is the resistance, K_e is the induced voltage, K_f is the thrust constant and i is the current. Meanwhile, the equations of motion of the absorbers are written by

$$\begin{aligned} &m_{aR}\ddot{y}_{aR}(t)=-f_R(t)-K_{aR}x_{aR}(t) \\ &m_{aL}\ddot{y}_{aL}(t)=-f_L(t)-K_{aL}x_{aL}(t) \end{aligned} \tag{6}$$

where m_a is the auxiliary mass, K_a is constant and

$$\begin{aligned} &y_{aR}(t)=x_{aR}(t)+x_4(t)+l_{fR}\theta_4(t)+z(t) \\ &y_{aL}(t)=x_{aL}(t)+x_4(t)+l_{fL}\theta_4(t)+z(t) \end{aligned} \tag{7}$$

Subscripts R and L denote the right and the left sides.

For control analysis and design purposes, the model of structure and absorbers are transformed into state space form. By the combination of the state variables of structure, absorbers, strokes, and defining the state vector x as

$$x=[x_{aR} \quad x_{aL} \quad \dot{x}_{aR} \quad \dot{x}_{aL} \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad \theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad \dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4 \quad \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \quad \dot{\theta}_4 \quad i_R \quad i_L]^T$$

the state equation of the control object can be written by

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Be(t)+D\ddot{z}(t) \\ y(t) &= Cx(t) \end{aligned} \tag{8}$$

3. Design of Controller

Structured singular value μ has been newly introduced in order to cover the defective points of the conventional design of H-infinity control. What is meant by a structured singular value, in contrast to the conventional singular value σ which is defined for a closed loop system, is a value that is defined by the following equation, relative to a system M obtained by the loop-closing connection of a controller K and a block structure Δ:

$$\mu(M)=\frac{1}{\min\{\overline{\sigma}(\Delta) \mid I-M\Delta=0\}} \tag{9}$$

It has been guaranteed theoretically that in this case, robust performance is realized within an error built into a generalized plant, as long as the value of μ is less than one. Then the object of the μ design is to obtain an internally stable controller. However, there are no known methods at present to obtain such a controller analytically, and it is, moreover, difficult to uniquely determine the value of μ itself. For this reason, the method used at present in μ-synthesis involves the iteration of the disturbance strength scaling, the design of the H-infinity controller and the calculation of the μ values. This computation method is called D-K iteration. Although the convergence of the D-K iteration is not guaranteed, this is the only method of solution at the present time. The procedure of the D-K iteration is as follows:

(i) Design an H-infinity controller for the plant of the equation (8).

(ii) Using the controller obtained by the step (i) and a block structure Δ describing the connection of a closed loop transform, the generalized plant as a closed loop system, and calculate μ for this system M.

(iii) Compute the scaling matrix D which minimizes μ (in general, D is a function of frequency), approximate it by using rational functions and multiply D to the generalized plant.

(iv) Design H-infinity controller again for the generalized plant which has been scaled (this is called a μ-controller in here).

(v) Repeat steps (ii) through (iv) until the condition μ is satisfied.

4. Reduced-order Controller

In this section we introduce the procedure to reduce the order of controller[3] obtained by μ-synthesis. The state space equation of the control system obtained in Section 3 can be written in the form

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}y(t) \\ u(t) &= \hat{C}x(t) \end{aligned} \tag{10}$$

Suppose the control system with the n th-order, minimal, asymptotically stable system to be balanced with respect to the asymptotically stable input and output weightings $W_i(s)$ and $W_o(s)$, respectively. The transfer function of the weightings input and output is as follows:

$$\begin{aligned} W_i(s) &= H_i(sI - F_i)^{-1}G_i + D_i \\ W_o(s) &= H_o(sI - F_o)^{-1}G_o + D_o \end{aligned} \tag{11}$$

The frequency weighted balanced realization algorithm as follows

(1) Given

a. $\hat{A}, \hat{B}, \hat{C}$ with $\mathrm{Re}[\lambda(\hat{A})] < 0$

b. F_i, G_i, H_i, D_i with $\mathrm{Re}[\lambda(F_i)] < 0$

c. F_o, G_o, H_o, D_o with $\mathrm{Re}[\lambda(F_o)] < 0$

(2) Solve for U and Y from

$$\begin{bmatrix} \hat{A} & \hat{B}H_i \\ 0 & F_i \end{bmatrix}\begin{bmatrix} U & U_{21}^T \\ U_{21} & U_{22} \end{bmatrix} + \begin{bmatrix} U & U_{21}^T \\ U_{21} & U_{22} \end{bmatrix}\begin{bmatrix} \hat{A}^T & 0 \\ H_i^T\hat{B}^T & F_i^T \end{bmatrix} + \begin{bmatrix} \hat{B}D_i \\ G_i \end{bmatrix}\begin{bmatrix} D_i^T\hat{B}^T & G_i^T \end{bmatrix} = 0$$

$$\begin{bmatrix} \hat{A}^T & \hat{C}^T G_o^T \\ 0 & F_o^T \end{bmatrix}\begin{bmatrix} Y & Y_{12} \\ Y_{13}^T & Y_{22} \end{bmatrix} + \begin{bmatrix} Y & Y_{12} \\ Y_{13}^T & Y_{22} \end{bmatrix}\begin{bmatrix} \hat{A} & 0 \\ G_o\hat{C} & F_o \end{bmatrix} + \begin{bmatrix} \hat{C}^T D_o^T \\ H_o^T \end{bmatrix}\begin{bmatrix} D_o\hat{C} & H_o \end{bmatrix} = 0$$

(3) Solve for eigenvalues and eigenvectors of UY

$UY = T\Lambda T^{-1}$, $\Lambda = diag\{\lambda_i\}, \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$.

$\Sigma = diag\{\sigma_i\}, \sigma_i = \sqrt{\lambda_i}, i = 1,2\cdots n$.

$F = T^{-1}\hat{A}T, G = T^{-1}\hat{B}, H = \hat{C}T$.

(4) Partitioning of the frequency weighted balanced realization

$$F = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix}, G = \begin{bmatrix} \hat{B}_1 \\ \hat{B}_2 \end{bmatrix}, H = \begin{bmatrix} \hat{C}_1 & \hat{C}_2 \end{bmatrix}$$

(5) The r th-order reduced order controller is then given by $K_r(s) = \hat{C}_1(sI - \hat{A}_{11})\hat{B}_1$, where the portioning was done such that $\hat{A}_{11}$ is $r \times r$ and $\sigma_{r+1} < \sigma_r$.

5. Simulation Results

In the simulation, μ -synthesis was carried out by the using of the computer aided control system design tool Matlab [1]. In this case, the following 3-block structure was used as the block of structure Δ in consideration of three loops, (1) sensitivity of strokes to observation disturbances, (2) sensitivity of the transverse displacement and torsion angle to shaking accelerations, and (3) robust performance with respect to reduced-order errors. Moreover, it is conjectured that the order of approximation of the D matrix is raised, the robust performance is improved, instead, the order of the μ -controller ends up increasing as well. Coupling relations of perturbations can be described, so that it is possible to describe more accurately.

As shown in Fig.1 and Fig. 2 , from the impulse response of the transverse and torsion displacements, the performance of the 8th-order controller are effective to reduce the vibration of structure.

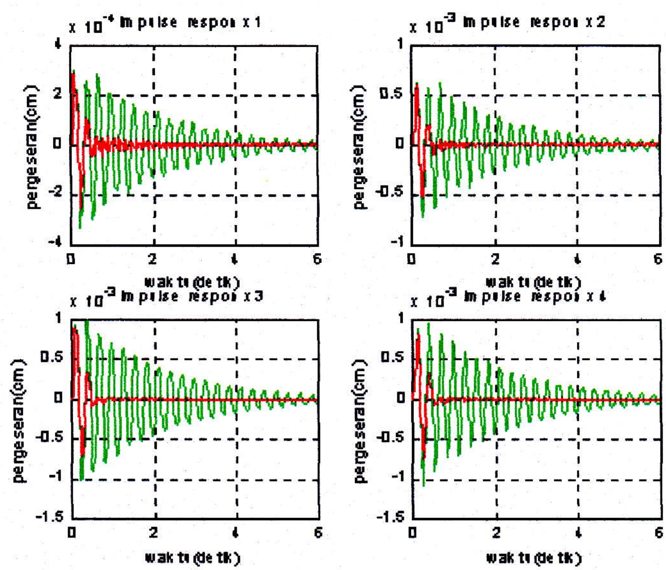

Fig. 2 Time response of the transverse displacement controlled(bold), no controller(dash)

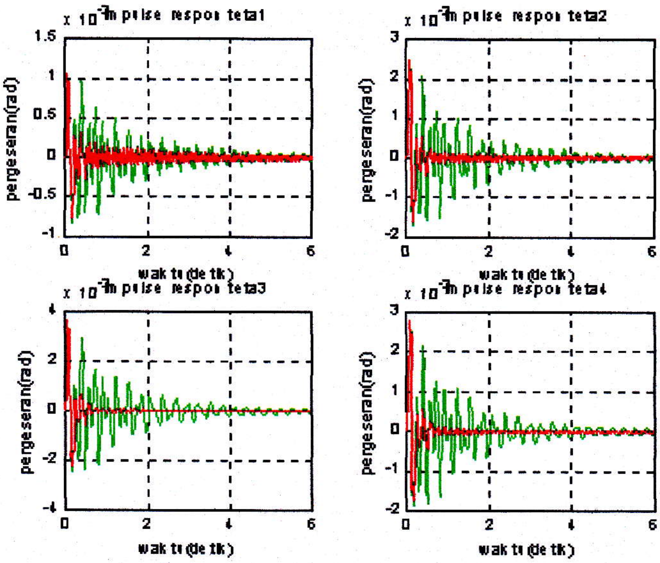

Fig.3 Time response of the torsion displacement
Controlled(bold), no controller(dash)

6. Conclusion

This paper provided modeling and designing of controller for flexible structure having the transverse-torsion coupled vibration modes. In designing of controller, the μ -controller is utilized and the robust performance can be improved. The order of controller can be reduced up to 8^{th} order by using the weighting balanced realization.

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